DESIGN OF EQUIRIPPLE FIR FILTER USING PARKS-MCCLELLAN REMEZ ALGORITHM

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Abstract - This paper design the Finite Impulse Response Digital Filters using Optimal Equiripple Method. It can be seen that the stopband attenuation is good and it has narrow transition width; these characteristics can be modified by increasing the filter order or by changing the filter specifications. Thus the resulting filter response have ripple of equal magnitude in both passband and stopband. Also, this method is very powerful, very flexible and offers lower filter order that meet the design specifications than the other methods. The Remez algorithm also called the Remez exchange algorithm or Park-McClellan procedure. The Remez algorithm in effect goes a step beyond the minimax approximation algorithm to give a slightly finer solution to an approximation problem. The advantage of this algorithm is to obtain minimum length and to reduce computational cost and reduce noise by increasing efficiency of the filter.

***Key Words*:** Remez, Park-Mc Clellan, Lowpass filter, High pass filter.

1. INTRODUCTION

In designing FIR filter, most important parts are approximation and realization. Transfer function can be calculated in four steps after taking specification in approximation stage as, usually in the frequency domain, desired or ideal response is chosen. Filter class is chosen which is allowed (e.g. the tap for a FIR filter). Approximation quality is chosen. Lastly, best algorithm is selected which is used to find the transfer function implementation of the above transfer function in the form of circuit (blocks) or program (coding) is done by selecting the structure of filter, this stage is called as realization. Filter structure selection is important part in implementation on FPGA because of area and speed. Hardware implementation part in pre modulation cannot afford more area because of less space in on flight [4]. There are three types of FIR filter design techniques,

In planning FIR filter, most significant parts are estimation and acknowledgment. Move capacity can be determined in four stages subsequent to taking detail in estimate stage as, as a rule in the frequency area, wanted or ideal reaction is picked. Filter class is picked which is permitted (for example the tap for a FIR filter). Estimation quality is picked. In conclusion, best calculation is chosen which is utilized to observe the exchange work Implementation of the above move work as circuit (squares) or program (coding) is finished by choosing the design of filter, this stage is called as acknowledgment. Filter structure choice is significant part in execution on FPGA in view of region and speed. Equipment execution part in pre adjustment can't manage the cost of more region as a result of less space in on flight [4]. There are three sorts of FIR filter plan strategies,

a) Windowing technique
b) Frequency sampling
c) Optimal design technique

We cannot achieve minimum order of filter with window design technique because it is a simple and convenient design technique for higher order filters. Rectangular, Blackman, Hamming, Hanning, Kaiser, Flat-top and Gaussian are some of the design techniques which are mostly used [5].

Frequency sampling design technique is the simplest and most direct technique if the desired frequency response is specified. In this technique desired frequency response can be obtain by sampling the frequency response which is provided by the previous method [4]. There are many optimal design techniques where we can specify pass and stop bands. Some of these techniques are equiripple and least square methods. Most important type of optimal design technique is Parks – McClellan algorithm [6]. In this paper this algorithm is still optimized such that pass band error is reduced.

We can't accomplish least request of filter with window plan method since it is a basic and advantageous plan strategy for higher request filters. Rectangular, Blackman, Hamming, Hanning, Kaiser, Flat-top and Gaussian are a portion of the plan strategies which are generally utilized [5].

Frequency testing plan procedure is the easiest and most direct method in the event that the ideal frequency reaction is indicated. In this procedure wanted frequency reaction can be acquire by inspecting the frequency reaction which is given by the past strategy [4]. There are numerous ideal plan methods where we can determine pass and stop groups. A portion of these procedures are equiripple and least square techniques. Most significant kind of ideal plan strategy is Parks -

McClellan calculation [6]. In this paper this calculation is as yet enhanced with the end goal that pass band blunder is diminished.

2. LITERATURE REVIEW

Generally, the design of FIR filters follows two main principles, i.e., the specification on the response error and the low implementation complexity. we describe the sparsity of the filter coefficients using the k-maximum function, we estimate the frequencies at which the magnitude of the response error is maximized when constructing linear problems in the proposed algorithm. To address the nonlinearity and non convexity of the resulted optimization problem, we transform it into a piecewise linear concave optimization (PLCO) problem [1].

A major drawback of FIR filter is the large number of arithmetic operations involved during the implementation which limits its speed and demands more power. This has motivated researchers to lean on the field of hardware efficient low-power filter design and accordingly this field has been enriched with number of valuable contributions from many scientists all over the world [2].

FIR filters are generally characterized by their impulse response coefficients indicating the multiplication constants with the input signals. These multipliers are power and area consuming devices and thus make the filter unbefitting in portable wireless devices like mobile phones, tablets, laptops etc. One of the most efficient ways to reduce the complexity of digital filter is to confine the tap coefficients to assume values in the form of sums of signed-powers-of-two (SPT)[3]and so on.

3. DIGITAL FILTER

Filters are generally utilized in signal processing and correspondence frameworks in applications, for example, filter equalization, noise suppression, radar, sound handling, video handling, biomedical signal handling, and investigation of monetary and monetary information. For instance in a radio receiver band-pass filters, or tuners, are utilized to extricate the transmissions from a radio filter. In a sound realistic adjuster the information signal is separated into various sub-band signals and the addition for each sub-band can be differed physically with a bunch of controls to change the apparent sound sensation. In a Dolby framework pre-sifting and post separating are utilized to limit the impact of commotion. In hi-fi sound a repaying filter might be remembered for the preamplifier to make up for the non-ideal frequency reaction qualities of the speakers. Filters are additionally used to make perceptual general media impacts for music, films and in broadcast studios. The essential elements of filters are one of the followings:

(a) To restrict a signal into a recommended frequency band as in low-pass, high-pass, and band-pass filters.

(b) To break down a signal into at least two sub-bands as in filter banks, realistic balancers, sub-band coders, frequency based multiplexers.

(c) To change the frequency range of a signal as in phone filter equalization and sound realistic balancers.

(d) To demonstrate the input and output relationship of a framework, for example, media transmission filters, human vocal plot, and music synthesizers.

4. EQUIRIPPLE FILTER

An equiripple filter is simply a filter with ripples of equal height. The magnitude response of actual digital filters may exhibit ripples. For example, the magnitude response of a finite impulse response low pass filter may have ripples close to its cutoff frequency, because the typical filter construction will use continuous functions (e.g., with the Fourier transform) to approximate a discontinuous ideal magnitude response. These ripples are the manifestation of the Gibbs phenomenon. The fact that equiripple filters have ripples of equal height should not mean much. It is more important that the design of equiripple filters is such that the height of these ripples can be controlled. This itself is not unique of equiripple filters.

5. REMEZ ALGORITHM

The best filters contain an equiripple characteristic in their frequency response magnitude and the elliptic filter (or Cauer filter) was optimal with regards to the Chebyshev approximation. When the digital filter revolution began in the 1960s, researchers used a bilinear transform to produce infinite impulse response (IIR) digital elliptic filters. They also recognized the potential for designing FIR filters to accomplish the same filtering task and soon the search was on for the optimal FIR filter using the Chebyshev approximation.

It was well known in both mathematics and engineering that the optimal response would exhibit an equiripple behavior and that the number of ripples could be counted using the Chebyshev approximation. Several attempts to produce a design program for the optimal Chebyshev FIR filter were undertaken in the period between 1962 and 1971.[1] Despite the numerous attempts, most did not succeed, usually due to problems in the algorithmic implementation or problem formulation. Otto Herrmann, for example, proposed a method for designing equiripple filters with restricted band edges.[1] This method obtained an equiripple frequency response with the maximum number of ripples by solving a set of nonlinear equations. Another method introduced at the time implemented an optimal Chebyshev approximation, but the algorithm was limited to the design of relatively low-order filters.

6. PROPOSED ALGORITHM

Parks-McClellan method which is also known as the equiripple, Optimal, or Minimax method with the Remez exchange algorithm is used to find an optimal equiripple set of coefficients to design an optimal linear phase filter. This is a standard method for the FIR filter design which minimizes the filter length for a particular set of design constraints. This method is used to design linear phase, symmetric or antisymmetric filters of any standard type. Better filters result from minimization of maximum error in both, the stopband and the passband of the filter which leads to equiripple filters. The proposed algorithm is used to design linear-phase FIR filters based on the Chebyshev (or minimax) error criterion. The minimization of the Chebyshev norm is useful because it permits the user to explicitly specify band-edges and relative error sizes in each band. We will see that linear-phase FIR filters that minimize Chebyshev error criterion can be found with the REMEZ algorithm or by linear programming techniques. Both these methods are iterative numerical algorithms and can be used for very general functions D(w) and W(w) (although many implementations work only for piece-wise linear functions). The REMEZ algorithm is not as general as the linear programming approach, but it is very robust, converges very rapidly to the optimal solution, and is widely used. Parks and McClellan proposed the use of the REMEZ algorithm for FIR filter design and made programs available. Note that the weighted error function is given by

$$E(\omega) = W(\omega)\,(A(\omega) - D(\omega)).$$

The amplitude response of a type I FIR filter is given by

$$A(\omega) = \sum_{n=0}^{M} a(n)\cos(n\omega).$$

To understand the Remez exchange algorithm, first note that can be written as

$$W(\omega_i)\,(A(\omega_i) - D(\omega_i)) = (-1)^i\,\delta$$

$$A(\omega_i) - D(\omega_i) = \frac{(-1)^i\,\delta}{W(\omega_i)}$$

or

$$\sum_{k=0}^{M} a(k)\cos(k\omega_i) - \frac{(-1)^i\delta}{W(\omega_i)} = D(\omega_i) \quad \text{for} \quad i = 1,\ldots,R.$$

If the set of extremal points in the alternation theorem were known in advance, then the solution could be found by solving the system of equations. In matrix form, these L+2L 2 simultaneous equations become

$$\begin{pmatrix} 1 & \cos(\omega_0) & \cos(2\omega_0) & \cdots & \cos(L\omega_0) & \frac{1}{W(\omega_0)} \\ 1 & \cos(\omega_1) & \cos(2\omega_1) & \cdots & \cos(L\omega_1) & \frac{-1}{W(\omega_1)} \\ \vdots & \vdots & \ddots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \ddots & \vdots \\ 1 & \cos(\omega_{L+1}) & \cos(2\omega_{L+1}) & \cdots & \cos(L\omega_{L+1}) & \frac{\pm(1)}{W(\omega_{L+1})} \end{pmatrix} \begin{pmatrix} h(L) \\ h(L-1) \\ \vdots \\ h(1) \\ h(0) \\ \delta \end{pmatrix} = \begin{pmatrix} A_d(\omega_0) \\ A_d(\omega_1) \\ \vdots \\ \vdots \\ \vdots \\ A_d(\omega_{L+1}) \end{pmatrix}$$

or

$$W\begin{pmatrix} h \\ \delta \end{pmatrix} = A_d$$

A more efficient variation of this method was developed by Parks and McClellan, and is based on the Remez exchange algorithm. To understand the REMEZ exchange algorithm, we first need to understand Lagrange interpoloation.

Now A(ω)is an LLth-order polynomial in x=cos(ω)x ω, so Lagrange interpolation can be used to exactly compute A(ω)from L+1L 1 samples of A(ωk), k=[0,1,2,...,L]k 0 1 2 ... L.

Thus, given a set of external frequencies and knowing δ, samples of the amplitude response A(ω)A ω can be computed directly from the

$$A(\omega_k) = \frac{(-1)^{k(1)}}{W(\omega_k)}\delta + A_d(\omega_k)$$

Note that Equation is a set of L+2simultaneous equations, which can be solved for δ to obtain

$$\delta = \frac{\sum_{k=0}^{L+1} \gamma_k A_d(\omega_k)}{\sum_{k=0}^{L+1} \frac{(-1)^{k(1)}\gamma_k}{W(\omega_k)}}$$

$$\gamma_k = \prod_{i=i\neq k,0}^{L+1} \frac{1}{\cos(\omega_k) - \cos(\omega_i)}$$

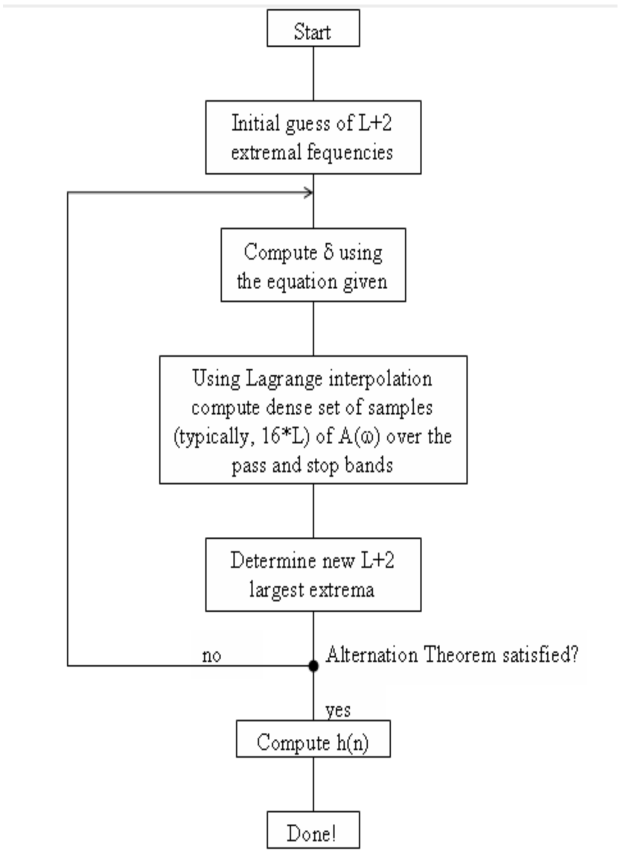

Figure 1: Flowchart of Proposed Algorithm

The algorithm proceeds by iteratively

1. The initialization step: The algorithm can be initialized by selecting any R frequency points between0and for which the weighting function W(w) is not zero. For example, one can choose the initial frequencies to be uniformly spaced over the region where W(w) > 0.

2. The interpolation step: The interpolation step requires solving the linear system. It can be treated as a general linear system, however, there do exist fast algorithms for solving the system 3. Updating the reference set: After the interpolation step is performed, the weighted error function is computed, and a new reference set w1, ...,wR is found such that the satisfy the following update criteria

The initial reference set can be taken to be R points uniformly spaced (excluding regions where W(w) is zero). Convergence is achieved when

$$\frac{||E_k(\omega)||_\infty - |\delta_k|}{||E_k(\omega)||_\infty} < \epsilon,$$

Estimation of the Filter Order: When designing a FIR digital filter, there is always the target in the minds of engineers that meets the design requirements. Therefore, the order of the FIR filter (number of coefficients) should be estimated from its specifications using suitable formula. Numerous authors have advanced methods for estimating the minimum value of the filter order N from FIR filter specifications: passband edge frequency, stopband edge frequency passband ripple, and stopband ripple.

$$N \cong \frac{-20\log\left(\sqrt{\delta_p\delta_s}\right) - 13}{14.6\,(f_s - f_p)/F_s}$$

FIR LPF can be designed by using the more precise method which is the optimal equiripple method, using the weight functions for low pass filter; the pass band and stop band ripple then:

$$A_p = 0.01\ dB \Rightarrow \delta_p = 5.8 * 10^{-4}$$

$$A_s = 40\ dB \Rightarrow \delta_s = 0.01$$

And:

$$N \cong \frac{-20\log\left(\sqrt{\delta_p\delta_s}\right) - 13}{14.6\,(f_s - f_p)/F_s} \approx 27$$

7. RESULT

From the above figure, we can observe that the response of Kaiser window technique has increasing attenuation by going away from the passband and does not have equal sized ripples which gives errors for filter whereas filter designed by REMEZ exchange algorithm has equal sized ripples which has low error. Another advantage of REMEZ algorithm is the transition bandwidth is little and the response follows straight response to stopband whereas the transition bandwidth is wider for Kaiser Window and thereby reduces the functionality of the filter near stopband. For high pass filter the stop band attenuation is -36.51 dB at (N=27), -113.6 dB at (N=70)

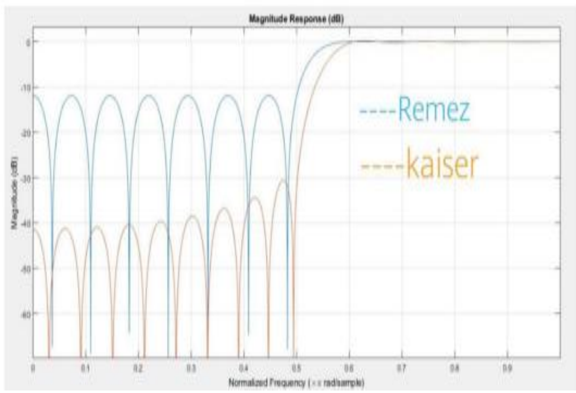

Figure 2: Frequency response (in db) of High Pass FIR Filter using REMEZ and Kaiser Algorithm

Table 1: Comparison of Pass band and stop band Ripple

S.No.	Order Of Filter	Algorithm	Pass band Ripple	Stop band Ripple
1.	M= 27	Existing Algorithm [Ref 1]	0.009	0.145
		Proposed Algorithm	0.0002	0.0019
2.	M= 70	Existing Algorithm[Ref 1]	0.000123	0.0012
		Proposed Algorithm	0.00034	0.0154

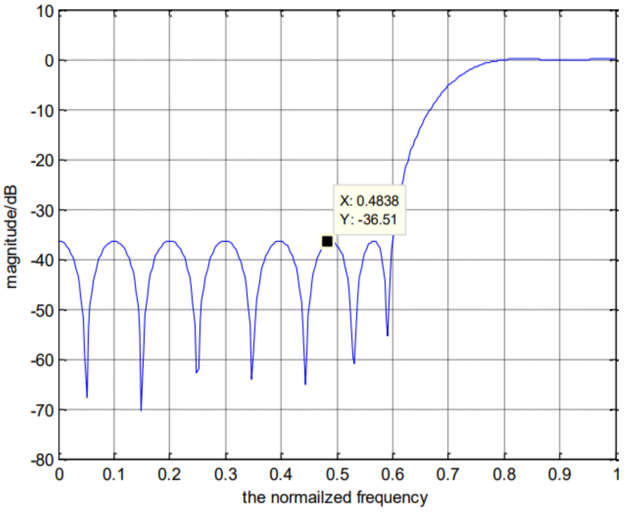

Figure 3: Frequency response of FIR HPF using Proposed method (N=27)

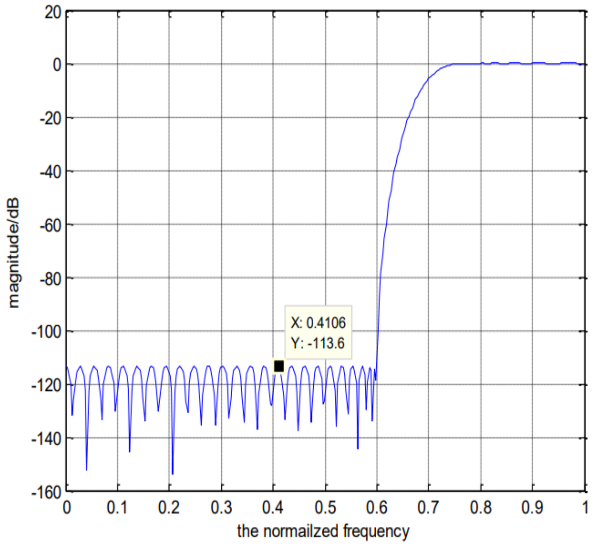

Figure 4: Frequency response of FIR HPF using Propose method (N=70)

8. CONCLUSION

REMEZ algorithm is one of the Optimal filter design technique that is used to design best filter for a given length of FIR filter using equiripple design which is highly advantageous due to its response is approximately near to the ideal frequency response with low order when compared to the window technique. Window technique is simple to design due to existence of well-defined formulas in Matlab toolbox but the disadvantage lies in higher order and more number of coefficients for the same filter design. The equiripple filter has ripples of same gain in stop band where filters designed by window techniques has increasing attenuation in stopband while going away from transition band and there will be distortion at band edge in the pass band. This result in more computations for determining coefficients in window filters which is un recommended and the approximation error cannot be influenced in different frequency ranges. It is strongly recommended to use minimax strategy for designing a filter which minimizes the error between desired and actual output using a weighted function done by REMEZ algorithm.

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