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Developing PID Controller for Re-Entry Phase of Reusable Rocket

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Abstract – This article focuses on the mathematical models which are developed for the re-entry phase of reusable rockets. Furthermore, after deriving the necessary mathematical and equations of motion, a code is generated in python which uses a PID controller to validate how the re-entry phase of a reusable rocket is successfully achieved.

Key Words: PID, control system, differential equations, python, re-entry phase manoeuvre, reusable rockets

1.INTRODUCTION

Reusable rockets are the latest piece of technology which are capable of reducing the launch cost of spacecrafts by a huge margin. SpaceX is a great example of the cost efficiency which one can achieve using reusable rocket technology. Other companies around the world are as well aiming to achieve this technology, in order to decrease launch costs and send a greater number of satellites in the orbit.

However, it should be noted that conducting a successful re-entry maneuver for a reusable rocket requires deriving mathematical models and solving equations of motion. Once this algorithm is derived, it has to be fed to the software which can control the rocket. In order to develop this algorithm, I have used python as a coding language where I have designed a PID controller in a closed loop feedback system which generates the required results for re-entry phase of a reusable rocket. The mathematical model and the code will be discusses in the later part of this article.

2. Body of Paper

Section 2 of this paper is further divided into four parts. The first three parts help us understand the rocket equations which are responsible for guidance of the rocket, safely back to the launching pad. Fourth section, shows the results generated by the python code implemented, to generate a successful re-entry phase maneuver.

In Section 2.1, I have discussed the non-linear equation of a rocket motion. The non-linear equations of the rocket help us in understanding how the fins and placement of fins along with forces can be modelled to derive the equilibrium point of the rocket motion. The derived equations are as follows:

2.1 Modelling of the rocket motion equations:

$$D_{\text{left fin}} = D\sin(\pi - \phi_D) = D\sin\phi_D$$

$$D_{\text{right fin}} = D\cos\phi_D$$

$$T_{\text{vector}} = T\cos\big(\pi - (\theta + \phi_T)\big) = -T\cos(\theta + \phi_T)$$

$$M\ddot{x} = -T\cos(\theta + \phi_T) + F_s\cos\theta$$

$$M\ddot{x} = -T\cos(\theta + \phi_T) + D\sin\phi_D\cos\theta$$

$$\ddot{x} = -\frac{T}{M}\cos(\theta + \phi_T) - \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2)\sin\phi_D\cos\theta$$

$$T_{vector} = T\sin\big(\pi - (\theta + \phi_T)\big)$$

$$T_{vector} = T\sin(\theta + \phi_T)$$

$$M\ddot{y} = T\sin(\theta + \phi_T) + F_s\sin\theta - Mg$$

$$M\ddot{y} = T\sin(\theta + \phi_T) + 2D\sin\phi_D\sin\theta - Mg$$

$$\ddot{y} = \frac{T}{M}\sin(\theta + \phi_T) + \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2)\sin\phi_D\sin\theta - g$$

$$J\ddot{\theta} + T\sin\phi_T\frac{L}{2} = 0$$

$$J_g = \frac{ML^2}{12}$$

$$\frac{ML^2}{12}\ddot{\theta} + T\sin\phi_T\frac{L}{2} = 0$$

$$\ddot{\theta} = \frac{-T\sin\phi_T\cdot\frac{L}{2}}{\frac{ML^2}{12}}$$

$$\ddot{\theta} = \frac{-6T\sin\phi_T}{ML}$$

2.2 Achieving equilibrium point:

Section 2.2, is dedicated to achieving the equilibrium point from the rocket motion of equation. The equilibrium point is also regarded as the operating point. This means, it helps us in identifying the initial state of the rocket, which once the payload is delivered, the rocket has to achieve again upon navigating the descent phase.

$$\dot{x} = 0 \Rightarrow -\frac{T}{M}\cos(\theta + \phi_T) - \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2)\sin\phi_D\cos\theta = 0$$

$$-T\cos(\theta + \phi_T) = 2K_D(\dot{x}^2 + \dot{y}^2)\sin\phi_D\cos\theta$$

$$2K_D v^2\sin\phi_D\cos\theta + T\cos(\theta + \phi_T) = 0$$

$$2K_D v^2\sin\phi_D\cos\theta = 0$$

$$\sin\phi_D = 0 \quad \text{or} \quad \cos\theta = 0$$

$$\phi_D = k\pi, \quad k = 0,1,2,..$$

$$\theta = \frac{(2k+1)\pi}{2}, \quad k = 0,1,2,...$$

$$T\cos(\theta + \phi_T) = 0$$

$$\ddot{\theta} = 0$$

$$\frac{-6T\sin\phi_T}{ML} = 0$$

$$T = 0 \quad \text{or} \quad \sin\phi_T = 0$$

$$(T \neq 0), then(\sin\phi_T = 0)$$

This is not possible.

From the condition $\phi_D = 0$ we obtain:

$$\phi_T = k\pi, \quad k = 0,1,2,\ldots$$

$$\left[0 = -\frac{T}{M}\cos\theta - \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2)\sin\phi_D\cos\theta\right]$$

$$0 = -\cos\theta\left[T + 2K_D(\dot{x}^2 + \dot{y}^2)\sin\phi_D\right]$$

Since $(\cos\ \theta = \ 0)$,we obtain:

$\theta = \frac{(2k+1)\pi}{2}, \quad k = 0,1,2,..$

For tension (T):

$$T = -2K_D(\dot{x}^2 + \dot{y}^2)\sin\phi_D$$

Since, the tension force cannot be negative this condition is not feasible.

Now, we take into account the next equilibrium condition:

$$0 = \frac{T}{M}\sin\theta + \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2)\sin\phi_D\sin\theta - g$$

$$0 = \frac{T}{M} + \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2)\sin\phi_D - g$$

In the case that, $\phi_D = 0$:

$$0 = \frac{T}{M} - g \Rightarrow T = Mg$$

This condition is not feasible because equilibrium will not be maintained.

If $\phi_D = \frac{\pi}{2}$:

$$0 = \frac{T}{M} + \frac{2K_D}{M}(\dot{x}^2 + \dot{y}^2) - g$$

$$T + 2K_D(\dot{x}^2 + \dot{y}^2) = Mg$$

$$T = Mg - 2K_D(\dot{x}^2 + \dot{y}^2)$$

The above equation describes the feasible force for equilibrium.

2.3 Linearization of the equations and stability analysis:

$$\boldsymbol{J\ddot{\theta} = -G}$$

$$J\ddot{\theta} + T\sin\phi_t \cdot \frac{L}{2} = 0$$

$$f = J\ddot{\theta} + \frac{T^*L}{2}\delta\phi_t = 0$$

$$\left.\frac{df}{dq}\right|_{q^*} = 0\cdot\delta x + 0\cdot\delta y + 0\cdot\delta\theta + \frac{L}{2}\sin\phi_t^*\,\delta + \frac{T^*L}{2}\cos\phi_t^*\,\delta\phi_t + 0\cdot\delta\phi_0$$

$$= \frac{T^*L}{2}\delta\phi_t$$

$$\left.\frac{df}{d\dot{q}}\right|_{q^*} = 0$$

$$\left.\frac{df}{d\ddot{q}}\right|_{q^*} = J\delta\ddot{\theta}$$

$$J\delta\ddot{\theta} + \frac{T^*L}{2}\delta\phi_t = 0$$

$$M\ddot{x} = -F$$

$$M\ddot{x} + T\cos(\theta + \phi_t) + 2K_D(x^2 + y^2)\sin\phi_t\cos\theta = 0$$

$$\left.\frac{df}{dq}\right|_{q^*} = 0\cdot\delta x + 0\cdot\delta y + 0\cdot\delta + (-\sin\phi_t\cos\phi_t) - \cos\phi_t\sin\phi_t\,\delta$$

$$+\cos(\theta + \phi_t)\,\delta T + T[-\sin\phi_t\cos\theta - \cos\phi_t\sin\theta]\delta\phi_t$$
$$+2K_D(x^2 + y^2)\cos\phi_t\cos\theta - 2K_D(x^2 + y^2)\sin\phi_t\sin\theta\,\delta$$

Now, for the given conditions:

$$\phi_t^* = 0, \quad \sin\phi_t^* = 0, \quad \cos\phi_t^* = 1$$

$$\theta^* = \frac{\pi}{2}, \quad \sin\theta^* = 1, \quad \cos\theta^* = 0$$

$$\phi_0^* = \frac{\pi}{2}, \quad \sin\phi_0^* = 1, \quad \cos\phi_0^* = 0$$

Thus,

$$-T^*\delta\theta - 2K_D(x^2 + y^2)\sin\phi_t\sin\theta\,\delta\theta - T^*\delta\phi_t = 0$$

$$\left.\frac{df}{dq}\right|_{q^*} = 4K_D\dot{x}\sin\phi_t^*\cos\theta^*\,\delta + 4K_D\dot{y}\sin\phi_t^*\cos\theta^*\,\delta y = 0$$

$$\left.\frac{df}{d\ddot{q}}\right|_{q^*} = M\delta\ddot{x}$$

$$M\delta\ddot{x} - T^*\delta\theta - 2K_D(x^2 + y^2)\delta\theta - T^*\delta\phi_t = 0$$

$$M\ddot{y} = T\sin(\theta + \phi_t) - 2K_D(x^2 + y^2)\sin\phi_t\sin\theta + Mg = 0$$

$$M\ddot{y} - (\sin\theta\cos\phi_t + \cos\theta\sin\phi_t)T - 2K_D(x^2 + y^2)\sin\theta\sin\phi_t + Mg = 0$$

$$-T[\cos\theta^* \cos\phi_t^* - \sin\theta^* \sin\phi_t^*]\delta\theta - 2K_D(x^2 + y^2)\sin\phi_t^* \cos\theta^*\, \delta\theta - (\sin\theta^* \cos\phi_t^* + \cos\theta^* \sin\phi_t^*)\delta T$$
$$-T[\sin\theta^* \sin\phi_t^* + \cos\theta^* \cos\phi_t^*]\delta\phi_t - 2K_D(x^2 + y^2)\sin\phi_t^* \cos\theta^*\, \delta\phi_t + Mg = 0$$

Now,

$$\phi_t^* = 0, \quad \sin\phi_t^* = 0, \quad \cos\phi_t^* = 1$$
$$\theta^* = \frac{\pi}{2}, \quad \sin\theta^* = 1, \quad \cos\theta^* = 0$$
$$\phi_0^* = \frac{\pi}{2}, \quad \sin\phi_0^* = 1, \quad \cos\phi_0^* = 0$$

Thus,

$$M\delta\ddot{y} - 4K_D\dot{x}\delta x - 4K_D\dot{y}\delta y - \delta T + Mg - T^* - 2K_D(x^2 + y^2) + Mg = 0$$

2.4 Results generated for PID controller of a rocket re-entry phase

In this sub-section, I have discussed the results which are generated for the rocket re-entry phase. The main reason I have preferred to utilize a PID controller over any other forms of control design such as MPC and LQR composer, is because of the simple nature of the control system and the output generating capacity as compared to other control systems.

Furthermore, a PID controller runs using real-time values and keeps correcting the state of the system based on the input values in real time. This nature of the system is of essential importance so that the rocket can ignite the thrusters or actuators based on the information feedback sent in by the sensors on-board the rocket.

A PID controller stands for Proportional-Integral and Derivative controller. It is these three basic components which make up the fundamental building and working blocks of the control system.

Proportional Control: The value of proportional component is based upon the difference between the target point and process variable. The proportional gain is defined as the ratio of output response to error signal.

Integral Control: The component of integral control sums the error over a period of time. This implies that the integral gain will gradually increase over time, till the error is zero.

Derivative Control: The derivative component is in proportion to the rate of change of process variable. It means that in case the process variable i.e. the error is increasing rapidly the output decreases.

Now, that we have understood the basic components of a PID controller, let us move to the next part of this sub-section.

I have used python as the coding language to design a PID controller and generate results which help us in understanding that the control system is working correctly and the rocket re-entry phase is achieved.

The parameters of PID gains used are:

$K_P = 0.36$
$K_I = 40.0$
$K_D = 0.000809$

The above-mentioned values of the three gains of a PID controller are calculated using the **Ziegler-Nichols** method.

The maximum allowable height achieved by the rocket should be 15 units in the positive x-direction. The initial set pint coordinates of the rocket is (0,-100).
Once, the code is executed, a successful result is generated.

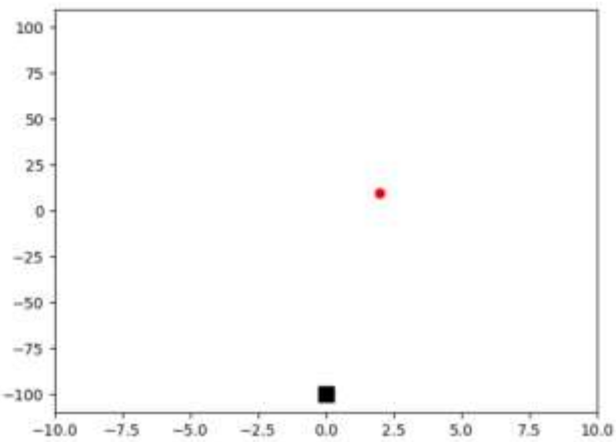

Fig -1: Final state of the rocket post re-entry phase

Fig.1 is the final result generated by the simulation post successful execution of the code. As one can see from the figure that, the final point of the rocket is at point (0, -100), which means that the rocket has landed successfully from its initial staring set point.

Hence, a successful re-entry phase is executed and the PID controller is working.

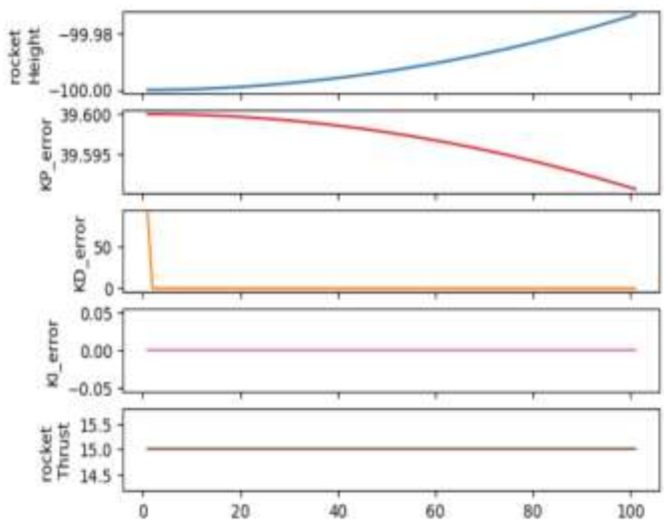

Fig -2: Plots representing the values of the three gains for closed loop system, height of the rocket and thrust by engines

In Fig. 2, the graphs generated help us in understanding the process though which the three gains of a PID controller respond for the chosen condition and how a successful re-entry phase is generated.

One of the plots in Fig. 2 represent the rocket height, it can be seen how the plot represents a steady decline in the height of the rocket, which represent the descent of the rocket back to its starting point or the launch pad.

3. CONCLUSIONS

From this particular simulation results it can be inferred that a PID controller in a closed loop system can help the engineers in creating a strong foundational base to create a reusable rocket which is guided by the principles of equations of motion for a rocket. The system can in-turn help the rocket to navigate the re-entry descent phase of the rocket. Thereby, landing the rocket back to the launching pad.

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