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Dynamic Behaviour of Functionally Graded Materials

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Abstract – The main aim of this study is to determine the natural frequency of a functionally graded material (FGM) plate using Rayleigh-Ritz method. The material properties of the plate is graded along the thickness direction. Simple power law (P-FGM), Sigmoid (S-FGM) and Exponential (E-FGM) laws are used to grade the material as per the volume fractions from bottom to top surface. A metal ceramic composition is graded throughout the thickness i.e., metal is rich on the bottom surface and ceramicis rich on the top surface of the plate. An important objective is to study the effects of metal-ceramic volume fractions and power law indices of different power laws on the natural frequencies of FGM plate. The natural frequencies are found with different boundary conditions at the edges for different power law indices. In addition, the effects of power law index using P-FGM, S-FGM and E-FGM plates on natural frequency with different boundary conditions are studied.

1. INTRODUCTION

Functionally Graded Materials (FGMs) represent a class of advanced materials whose properties vary gradually in one or more directions to suit specific functional requirements. Typically, FGMs are composed of metal and ceramic components arranged so that properties transition smoothly from the bottom to the top surface. This gradation in material composition is achieved through a continuous change in volume fractions of the constituent phases, typically represented by power law functions. Such a graded structure allows FGMs to combine the advantages of both ceramics and metals. The ceramic portion provides excellent thermal resistance due to its low thermal conductivity, while the metal portion offers enhanced toughness, reducing the likelihood of fracture under high thermal gradients. This unique combination has led to a growing interest in FGMs across industrial and academic research sectors.

The concept of material gradation in FGMs introduces multiple benefits over traditional, homogenous materials, including reduced stress concentrations and improved thermal management. FGMs eliminate sharp material interfaces, creating a gradual transition zone between metal and ceramic. This transition not only minimizes thermal stresses but also increases the durability of components subjected to extreme temperature gradients. The ability to fine-tune mechanical, thermal, and physical properties by adjusting the gradation profile has made FGMs highly desirable in fields such as aerospace, automotive, and biomedical engineering.

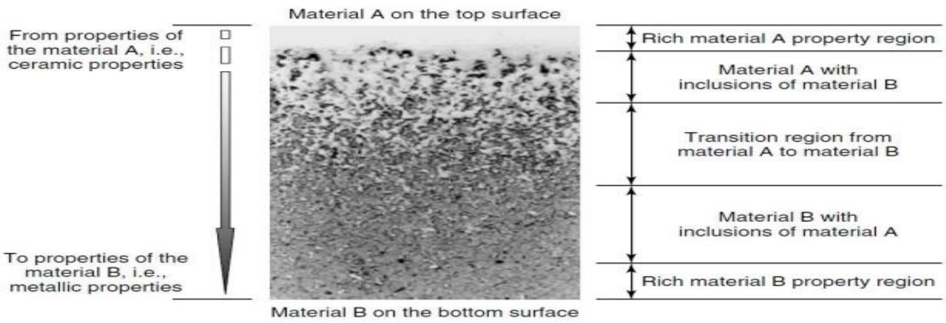

Figure 1.1 Gradation of an FGM

2. Types of Power Laws in FGMs

1.**Exponential Power Law** The variation of volume fractions according to simple power law for FGM (P-FGM) are given by volume fraction of ceramic (V_c) and volume fraction of metal (V_m) .

2.**Sigmoid Power Law**: To create a smooth gradation profile across the FGM, some researchers use two power law functions centered around the mid-thickness. These functions ensure a balanced stress distribution across the material interfaces, This type of gradation is especially beneficial in applications where reducing stress discontinuities across layers is crucial for material stability.

3.**Exponential Power Law**: This approach involves an exponential variation in properties, which provides a more drastic change in material composition than the power law approaches. The exponential function can offer unique advantages, especially in applications that require high resistance to thermal and mechanical stresses.

Materials Used in FGMs

FGMs can incorporate various materials tailored to specific applications. For this study, aluminium (metal) and ferro chrome slag (ceramic) were selected as constituent materials due to their contrasting thermal and mechanical properties. Aluminium, known for its ductility andmoderate thermal conductivity, serves as the metal component, while ferro chrome slag characterized by high thermal resistance and rigidity, acts as the ceramic component. The tablebelow summarizes the essential material properties:

Material	**Young's Modulus (E) (GPa)**	**Density(ρ) (kg/mt)**	**Poisson's Ratio (ν)**
Aluminium	70	2700	0.3
Ferro Chrome Slag	380	3800	0.3

Objectives of the Study

This study aims to explore the vibrational characteristics of FGM plates using theRayleigh-Ritz method. Specifically, the research focuses on:

1. Determining the natural frequencies of FGM plates.
2. Investigating the effect of different power laws (P-FGM, S-FGM, and E-FGM) on the frequency parameters.
3. Examining how volume fractions and power law indices influence non-dimensional frequency parameters across various boundary conditions.

METHODOLOGY:

The methodology applied in the study of free vibration of functionally graded material (FGM) plates. The method is structured around Classical Plate Theory (CPT) and mathematical formulations, including the Rayleigh-Ritz approach, to model and analyze the vibration characteristics of the FGM plate. Through the establishment of strain-displacement relationships, stress-strain relationships, and energy equations, this methodology facilitates the derivation of natural frequencies and non-dimensional frequency parameters, allowing for a comprehensive understanding of the plate's behavior under various boundary conditions.

Classical Plate Theory (CPT)

Classical Plate Theory, often referred to as Kirchhoff plate theory, provides the foundation for analyzing thin plates by simplifying the displacement field equations. In this theory, the plate deforms only due to bending and in-plane stretching, with thetransverse shear deformation neglected. This assumption is valid for thin plates, wherethe effect of transverse shear deformation is negligible compared to bending. In CPT,the displacement components of a point on the plate are defined in the x, y, and z directions as uxu_xux, uyu_yuy, and uzu_zuz, respectively.

Mathematical Formulation

To develop the mathematical model for free vibration, the methodology incorporates strain-displacement relationships, stress-strain relationships, and the calculation of strain and kinetic energy. This mathematical formulation enables the characterizationof the plate's vibrational behavior under various conditions.

Strain-Displacement Relationship

The strain-displacement relationships for an FGM plate based on CPT can be described as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_y}{\partial y} \\ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \end{Bmatrix} = \begin{Bmatrix} -z\frac{\partial^2 w}{\partial x^2} \\ -z\frac{\partial^2 w}{\partial y^2} \\ -2z\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

Stress-Strain Relationship Using Hooke's Law

The stress-strain relationship for an FGM plate follows Hooke's law, a fundamental principle in mechanics that relates stress to strain linearly. The constitutive stress- strain relationships for the FGM plate are given by:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

where σxx\sigma_{xx}σxx and σyy\sigma_{yy}σyy represent the normal stresses, τxy\tau_{xy}τxy is the shear stress, and Q11,Q12,Q_{11}, Q_{12},Q11,Q12, and Q66Q_{66}Q66 are the stiffness coefficients defined as:

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \quad Q_{12} = Q_{21} = \frac{\nu E(z)}{1-\nu^2}, \; Q_{66} = \frac{E(z)}{2(1+\nu)}$$

Here, E(z)E(z)E(z) is the Young's modulus, which varies through the plate's thicknessin an FGM plate, and vvv is Poisson's ratio. The gradual variation of E(z)E(z)E(z) inthe thickness direction reflects the material's graded nature, providing a transition from one material (such as a metal) to another (such as ceramic). This variation in material properties along the thickness has significant implications for the plate's stiffness and its response to vibrational forces.

Strain and Kinetic Energy Equations

The strain energy UUU and kinetic energy TTT of the FGM plate are critical for determining its natural frequency. The strain energy UUU represents the potential energy stored due to deformation, while the kinetic energy TTT represents the energyof motion. In Cartesian coordinates, the strain and kinetic energy can be formulated as follows:

$$U = \frac{1}{2}\iiint \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy}\right)dxdydz$$

$$T = \frac{1}{2}\iiint \rho(z)\left(\frac{\partial w}{\partial x}\right)^2 dxdydz$$

where ρ(z)\rho(z)ρ(z) denotes the density function that varies through the plate's thickness. This density variation, a characteristic of FGM plates, influences the inertiaand, consequently, the kinetic energy of the plate.

When substituted with strain-displacement and stress-strain relations, the strain and kinetic energy expressions transform to

$$U = \frac{1}{2}\int\int [D_{11}\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\} + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2]dxdy,$$

$$T = \frac{1}{2}\int\int \left(\frac{\partial w}{\partial t}\right)^2 I_0\, dxdy,$$

Where, $(D_{11}, D_{12}, D_{66}) = \int_{-h/2}^{h/2} z^2(Q_{11}, Q_{12}, Q_{66})\ dz$

Where, (D11, D12, D66) = $\int_{-h/2}^{h/2} z^2(Q_{11}, Q_{12}, Q_{66}$

) dz

are the stiffness coefficients and I0I_0I0 is the inertial coefficient derived through integration across the thickness. These energy formulations set the stage for derivingthe natural frequency of the plate.

Application of Rayleigh-Ritz Method

The Rayleigh-Ritz method provides a convenient and efficient means of approximating the plate's deflection and determining its natural frequency. In this approach, the displacement amplitude W(x,y)W(x, y)W(x,y) is expressed as a series of polynomial functions ϕi(x,y)\phi_i(x, y)ϕi(x,y) that adhere to the plate's boundary conditions.

This series expansion takes the form:

$$W(x, y) = \sum_{i=1}^{n} c_i \varphi(x, y),$$

where cic_ici are unknown constants, and ϕi(x,y)\phi_i(x, y)ϕi(x,y) are polynomial functions derived from Pascal's Triangle, selected to satisfy the boundary conditions.

Through this method, we establish a relationship between maximum strain energy andmaximum kinetic energy, resulting in the Rayleigh quotient ω2\omega^2ω2, which isused to determine the natural frequency:

$$\omega^2 = \frac{\int\int D_{11}[\{\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2\} + 2\nu \frac{\partial^2 W \partial^2 W}{\partial x^2\, \partial y^2} + 2(1-\nu)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2]dxdy}{\int\int I_0 W^2 dxdy},$$

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$$\omega^2 = \frac{\int\int D_{11}[\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\} + 2v\frac{\partial^2 w \partial^2 w}{\partial x^2\ \partial y^2} + 2(1-v)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2]dxdy}{\int\int I_0\, W^2 dxdy},$$

The resulting eigenvalue problem gives the frequency parameters, allowing for an in-depth analysis of the FGM plate's vibrational behavior across different boundary conditions and power law indices.

3.1 Non-dimensional Frequency Parameter

For further comparative analysis, a non-dimensional frequency parameter λ\lambdaλis introduced:

$$\lambda = \omega a^2 \sqrt{\frac{\rho_c h}{D_c}}$$

where D_c (flexural rigidity) = $\frac{E_c h^3}{12(1-v^2)}$ of represents the flexural rigidity of the FGM

plate. This parameter enables the comparison of plates with different material distributions and geometric properties, providing insights into how variations in material grading influence vibrational response.

4. RESULTS AND DISCUSSIONS:

Material Properties of the Constituent Materials:

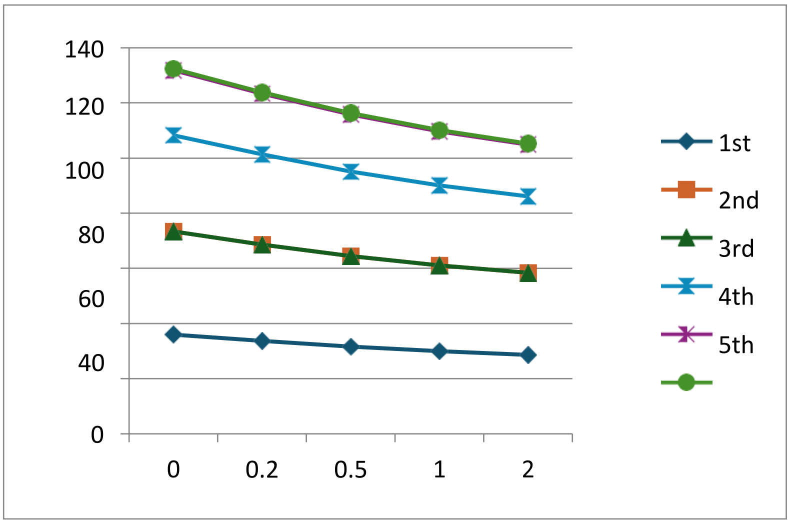
Properties	**Aluminium**	**Ferro Chrome Slag**	**Unit**
Young's Modulus(E)	70	380	GPa
Density (ρ)	2700	3800	Kg/m^3
Poisson's ratio (v)	0.3	0.3	-

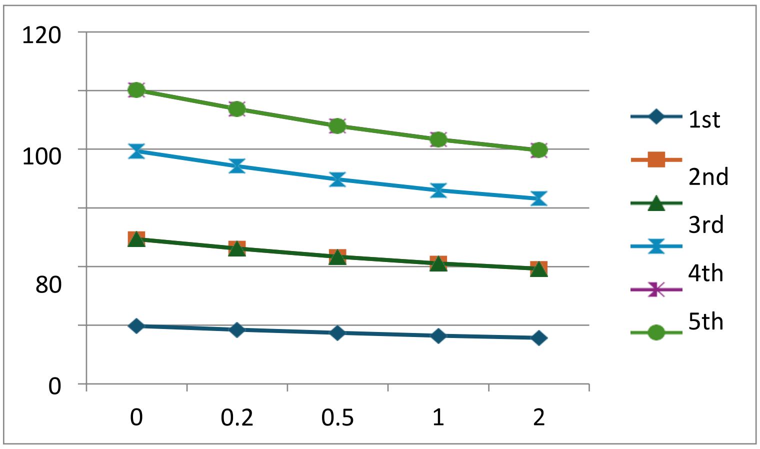
Effect of Frequency Parameters of FGM plate with different k as perP-FGM with different boundary conditions

k	λ1	λ2	λ3	λ4	λ5	λ6
0	35.9888	73.3991	73.3993	108.266	131.902	132.42
0.2	33.6715	68.6729	68.6732	101.294	123.409	123.9
0.5	31.6093	64.4671	64.4674	95.0908	115.851	116.31
1	29.9445	61.0717	61.0719	90.0824	109.749	110.18
2	28.6243	58.3792	58.3794	86.1109	104.91	105.32

Effect of frequency parameters of SSSS FGM plate withdifferent k as per P-FGM

k	λ1	λ2	λ3	λ4	λ5	λ6
0	19.7392	49.3487	49.3494	79.398	100.175	100.2
0.2	18.4682	46.1712	46.1718	74.2856	93.725	93.746
0.5	17.3372	43.3435	43.344	69.7361	87.9849	88.005
1	16.424	41.0606	41.0612	66.0631	83.3694	83.369
2	15.6999	39.2503	39.2509	63.1506	79.6761	79.694





CONCLUSIONS

Following conclusions can be drawn from the investigation carried out.

Material Gradient Influence: Different material gradations (Power Law FGM, Sigmoid FGM, Exponential FGM) significantly impact the frequency parameters of FGM plates, offering unique control over vibrational characteristics.

Power Law FGM Observations:

- Frequency parameters consistently decrease with increasing power law index kkk, dueto the transition toward a more flexible, metal-dominated material composition.
- Higher frequency modes (like 4th, 5th, and 6th) are more sensitive to kkk changes, showing greater declines compared to lower modes, making this configuration effectivefor applications that require frequency-specific damping.

Sigmoid FGM Insights:

- The Sigmoid FGM (S-FGM) produces a gradual transition in material properties, resulting in moderate declines in frequency parameters as kkk increases.
- The smoother transition offered by S-FGM reduces the risk of abrupt vibrational shifts,enhancing stability in applications where vibrational control is essential, such asaerospace components.

Exponential FGM Characteristics:

- The Exponential FGM (E-FGM) leads to a rapid transition in material properties, causing pronounced declines in frequency parameters, especially in higher modes.
- E-FGM provides enhanced flexibility and rapid adjustments in vibrational response, ideal for designs requiring abrupt changes in stiffness or damping for high-frequency applications.

Fully Clamped (CCCC) Boundary

The fully clamped (CCCC) condition exhibits the highest frequency values across all material distributions, providing maximum stiffness.

- CCCC plates are suitable for applications where structural rigidity and minimal vibration are paramount, such as machine foundations or aerospace structures.

Fully Simply-Supported (SSSS) Boundary:

- Simply-supported (SSSS) plates yield lower baseline frequencies than clamped configurations, reflecting increased flexibility and suitability for applications likebridge panels and large plates where edge rotation is allowed.

Partially Clamped (SCSC) Boundary:

- The mixed clamped-simply supported (SCSC) boundary condition results inintermediate frequency values, showing balanced rigidity and flexibility.
- The SCSC configuration is suitable for panels needing controlled flexibility along certain edges.

Three Clamped, One Free Edge (CCCF) Configuration:

- The CCCF boundary condition exhibits moderate baseline frequencies due to the presence of one free edge, which increases flexibility and decreases stiffness.
- CCCF plates are effective for cantilevered or overhanging structures where partial support is essential for dynamic load distribution.

Three Clamped, One Simply-Supported Edge (CCCS) Configuration:

- The CCCS boundary provides a stable configuration with slightly less stiffness than CCCC, yielding intermediate frequencies ideal for panels with partial support requirements.
- This setup offers both stability and some degree of flexibility along one edge, useful for structural walls and panels.

Alternating Simply-Supported and Free Edges (SFSF) Configuration:

- The SFSF setup yields the lowest frequency parameters, allowing for maximumflexibility.
- SFSF plates are suitable for lightweight structures where edge freedom is crucial, suchas flexible flooring or paneling.

Frequency Sensitivity in Higher Modes:

- Across all configurations, higher frequency modes show greater sensitivity to material composition changes, offering control over specific vibrational modes.
- This sensitivity enables targeted vibrational damping in precision applications, allowing for optimized performance under varying conditions.

Design Flexibility for Engineers:

- Engineers can select boundary and material gradients to control stiffness, flexibility, and frequency response based on the specific demands of their application.

Stability with S-FGM:

- The S-FGM's smooth material gradient supports stability across all configurations, reducing the likelihood of abrupt frequency shifts and enhancing durability under dynamic loading.

Enhanced Flexibility with E-FGM:

- The rapid composition shift in E-FGM provides greater control over higher frequencies,particularly effective for configurations requiring flexibility, such as SCSF and SSSF.

Applications of CCCC with E-FGM:

- The high frequency and stiffness of CCCC under E-FGM make it ideal for precision applications where high-frequency damping is needed to avoid resonance and fatigue.

SSSS with S-FGM:

- The SSSS boundary with S-FGM supports flexibility while maintaining predictable vibrational response, suitable for structures like bridge decks.

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