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Dynamics of Non-static Plane Symmetric Isotropic Universe with Special Form of Deceleration Parameter

Dr. Devyani. D. Raut
College of Engineering And Technology, Akola
Assistant Professor of Mathematics

Abstract: -In this context we have study the dynamics of non-static plane symmetric cosmological model with special form of deceleration parameter by utilizing Saez-Ballester Theory of Gravitation (SBTG). Also we have discussed some kinematical and geometrical parameters of the models.

Keywords: - Dynamics of Fluid, Non-Static Plane Symmetric Universe, Special Form of Deceleration Parameter, SBTG

I. INTRODUCTION

Quite possibly the most difficult issues of present day cosmology is to portray the positive late time acceleration (Perlmutter & et al., 1997) (Riess & et al., 1998) (Copeland & et al., 2006)[1-3] through a solitary self-predictable hypothetical plan. To be sure, the actual physical origin of the deliberate enormous accelerate isn't very much accounted on hypothetical grounds, without summoning the presence of an extra liquid which drives the universe elements, ultimately ruling over different species. According to the concordance world perspective on cosmology, the universe experienced an early time decelerated stage, sought after by the progression of radiation and matter directed periods, prior to going to the current, sped up stage. The sped up stage can't be adequately depicted through broad relativity standard model of molecule material science since extra degree(s) of opportunity have all the earmarks of being on a very basic level required. From one hand we may trademark these extra levels of opportunity to new, colorful types of issue, on the whole named as Dark energy (DE)[4-5] (Spergel & et al., First-Year Wilkinson Microwave Anisotropy Probe (WMAP)* Observations: Determination of Cosmological Parameters, 2003) (Spergel & et al., Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, 2007). The most un-complex comprehension for this DE is the introduction of a cosmological consistent identifying with state of state limit $\omega = -1$. Furthermore, in the assembled works dissipated from the cosmological reliable there are various candidates of DE which is identified with the energy thickness of a dynamical scalar field, for instance, substance $(\omega > -1)$ [6, 7] (Ratra & Peebles, 1988) (Wetterich, 1998). Spirit field $(\omega < -1)$ [8, 9] (Caldwell, 2002) (Wei & Tian, 2004)and Quinton (that can inverse apparition locale to center district) [10, 11] (Elizalde & et al., 2004) (Nojiri & et al., 2005)Chaplygin gas [12] (Kamenshchik & et al., 2001),k-essence [13-16] (Chiba & et al., 2000) (Armendariz-Picon & et al., 2000) (Scherrer, 2004) (Capozziello & et al., 2010), Tachyon field, Holographic and Age realistic DE.

Demianski et al. (1987)(Demianski et al., 1987) examined the elements of a 11-dimensional homogeneous cosmological model. They accepted that the t=const.hyper surfaces are results of a 3-dimensional Bianchi type-IX space and a 7-dimensional torus. Coley (2005) (Coley, 2005)studied dynamics of cosmological model in Bran-World scenario. Ananda and Bruni (2005)(Ananda& Bruni, 2006) investigated Cosmo-elements and dull energy with non-straight condition of express: a quadratic model.Li (2017)(Li, 2017) inspected the elements of two-scalar-field cosmological models. Not at all like in the circumstance of remarkable potential, they found that there are late-time attractors inwhich one scalar field rules the energy thickness of universe and the other one rot. They had additionally talked about the chance of various attractors model which is valuable to acknowledge theevolution of the universe from a scaling period to late speed increase time.Bhoyar and Chirde (2018) (Bhoyar & Chirde , 2018)examined Bianchi type-III and Kantowski Sachs cosmological model containing attractive field with variable cosmological steady in everyday hypothesis of relativity. The overall arrangements of the Einstein's field conditions for the cosmological models they had acquired under the suspicion of against solid liquid. Tandon(2018)investigated, a model in the presence of general relativity for the setting of Bianchi structure I cosmological models. For finding a deterministic model of the universe and to get the specific arrangements of the field conditions of Einstein, they guessed Hubble boundary which gives the fixed worth of deceleration boundary that gains the necessary aftereffects of the field conditions of Einstein. Pawar and Shahare(Pawar & Shahare, 2019) (2019) researched elements of shifted Bianchi type-III cosmological model in gravity hypothesis. (Pavluchenko, 2019) studied Dynamics of the cosmological models with perfect fluid in Einstein– Gauss– Bonnet gravity: low-dimensional case.Reddy et al. (2019) (Reddy, Aditya,Naidu (2018)researched a dull energy model within the sight of mass less scalar field in the edge work of locally rotationally symmetric (LRS) Bianchi type-II space-time. To track down a deterministic model of the universe they had utilized the half and half development law to settle Einstein's field conditions. They had additionally, utilize a connection between metric potential for this reason. Singh and Sonia (2020)(Singh & Sonia, 2020) studied dynamical System Perspective of Cosmological Models Minimally Combined with Scalar Field.Zhuravlev and Chervon(2020) (Zhuravlev & Chervon, 2020)introduced a subjective examination of chiral cosmological model (CCM) elements with two scalar fields in the spatially level Friedman– Robertson– Walker Universe. The asymptotic conduct of chiral models they explored dependent on the qualities of the basic places of the self-collaboration potential and zeros of the metric segments of the chiral space. The grouping of basic marks of CCMs they proposed. The part of zeros of the metric segments of the chiral space in the asymptotic elements they dissected. They had shown that such zeros lead to new basic marks of the relating dynamical frameworks.Borgade et al. (Borgade et al., 2021) (2021) committed to oneself impelling examination of elements of mass gooey string in LRS Bianchi type-I cosmological model inside the condition of elective hypothesis of gravity with Langrangian be the indiscreet perform of gravity hypothesis. Vinutha and Kavya(Vinutha & Sri Kavya, 2021) (2021) examined dynamics of cosmological model.

Several authors(Mohanty, G, Sahoo, 2002),(B. Mishra, 2003) and (Rao & Davuluri, 2013) studied the non-static plane symmetric cosmological models. With respect to these models number of Researchers (Adhav et al., 2010)(Katore & Rane, 2006)(Pawar & Solanke, 2016)(R. K. Mishra & Chand, 2020)used the Saez- Ballester theory of gravity(Sáez & Ballester, 1986) to study the cosmological Models.

Hence with this discussion we have studied dynamics of fluid in non-static plane symmetric universe in the framework of Saez-Ballester theory of gravitation utilizing special form deceleration parameter. The whole discussion for this context in mentioned in the following manner.

II. MODEL AND THE FIELD EQUATIONS

Let us consider a non-static plane symmetric space-time of the form

$$ds^2 = e^{2h}\left(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2\right), \tag{1}$$

Where, h and s are the functions of cosmic time t and r,θ,z are the general cylindrical co-ordinates.

The field equation in SBTG is

$$G_{ij} - \breve{\omega}\varphi^n\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,a}\varphi^{,a}\right) = -T_{ij} \tag{2}$$

Where the scalar field satisfying the equation

$$2\phi^m\phi^{j}_{,j} + m\phi^{m-1}\phi_{,b}\phi^{,b} = 0; \tag{3}$$

where ϕ is the Scalar field and m is constant.

The dynamics of fluid energy momentum tensor of the form is mentioned as,

$$T_i^j = (p+\rho)u_i u_j - pg_{ij} \tag{4}$$

$$T_i^j = diag[-p,-p,-p,\rho], \tag{5}$$

where p is the pressure and ρ is the energy density of the fluid respectively.

By considering the SBTG field equations from (2) & (3) with respect to energy momentum tensor (4) by utilizing model (1) we have the following set of field equations as:

$$\frac{1}{e^{2h}}\left[2\ddot{h} + \dot{h}^2 + \frac{2\dot{h}\dot{s}}{s} + \frac{\ddot{s}}{s}\right] - \frac{\breve{\omega}}{2}\phi^n\dot{\phi}^2 = -p \tag{6}$$

$$e^{-2h}\left[2\ddot{h}+\dot{h}^2\right]-\frac{\breve{\omega}}{2}\phi^n\dot{\phi}^2=-p \quad (7)$$

$$e^{-2h}\left[\frac{2\dot{h}\dot{s}}{s}+3\dot{h}^2\right]+\frac{\breve{\omega}}{2}\phi^n\dot{\phi}^2=\rho \quad (8)$$

III. SOLUTION OF THE FIELD EQUATIONS

The arrangement of Equation (6) to (8) is having with three directly autonomous conditions with five unknowns $h, s, \phi, p \,\&\, \rho$. In order to obtain its solution, we consider special form of deceleration parameter which has a significant importance in cosmology since it elegantly illustrates different cosmic evolutionary phases given by the following equations.

The scale factor withrespect to the special form of deceleration parameter is:

$$a=\left(e^{\alpha kt}-1\right)^{\frac{1}{\alpha}} \quad (9)$$

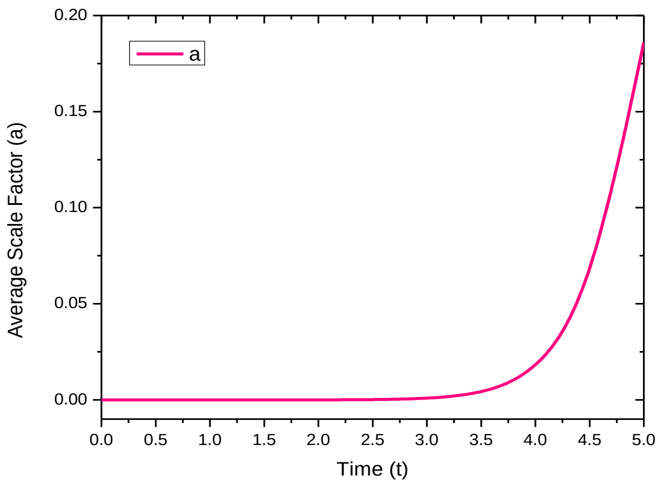

Figure 1 Plot of Scale Factor Vs Time with suitable choice of constants $k=1.1\,\&\,\alpha=0.11$.

The average scale factor of the model with respect to special form of deceleration parameter increases exponentially as shown in the above **Figure 1** and at *t=0* the average scale factor vanishes hence *t=0* is the singular point of the scale factor.

For getting the values of metric potentials; we assume that the relation between the metric potentials as:

$e^h = s^n; n>1$ (10)

Hence with this the metric potentials are found out to be,

$$s=\left(\frac{1}{\alpha}\right)^{\frac{1}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha(4n+1)}} \quad (11)$$

$$e^{h}=\left(\frac{1}{\alpha}\right)^{\frac{n}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{3n}{\alpha(4n+1)}} \tag{12}$$

Both the metric potentials as mentioned above are vanishes at *t=0* hence it is the singular point of the metric potentials.

Utilizing the metric potentials (11) and (12) the resulting pressure of the Universe is found out to be,

$$p=\frac{\alpha_1}{\alpha}e^{\alpha kt}\left(e^{\alpha kt}-1\right)^{\frac{-6n}{\alpha(4n+1)}-1}\left[6n\alpha-\frac{9n^2}{(4n+1)\left(1-e^{-\alpha kt}\right)}\right]+\frac{c_2}{\alpha\left(1-e^{-\alpha kt}\right)}\left(e^{\alpha kt}-1\right)^{\frac{6n(n+1)}{\alpha(4n+1)}} \tag{13}$$

The above expression is the expression of pressure of the universe filled with dynamics of fluid and it is vanishes at *t=0* and is the singular point of the equation of pressure. As the nature of scale factor the equation of pressure is having with exponential nature.

Utilizing the metric potentials (11) and (12) the resulting energy density of the Universe is found out to be,

$$\rho=\frac{9\alpha_1 n(3n+2)}{\alpha}\left[\frac{1}{\left(1-e^{-\alpha kt}\right)}\right]^{2}\left(e^{\alpha kt}-1\right)^{\frac{-6n}{\alpha(4n+1)}}+\frac{c_2}{\alpha\left(1-e^{-\alpha kt}\right)}\left(e^{\alpha kt}-1\right)^{\frac{6n(n+1)}{\alpha(4n+1)}}\ for\left(\alpha_1=\left(\frac{1}{\alpha}\right)^{\frac{-2n}{4n+1}}\frac{k^2}{4n+1}\right) \tag{14}$$

The energy density of dynamics of fluid is as shown in the above equation and while observing it we can conclude that the energy density of the universe is increases exponentially with respect to cosmic time and in the similar manner at *t=0* the energy density reaches at 0 hence it is the singular point of the energy density of the model of the universe.

Utilizing the metric potentials (11) and (12) the resulting Saez- Ballester scalar potential of the Universe is found out to be,

$$\phi=C_1'''\left(e^{\alpha kt}-1\right)^{\frac{6n}{\alpha(4n+1)}} \tag{15}$$

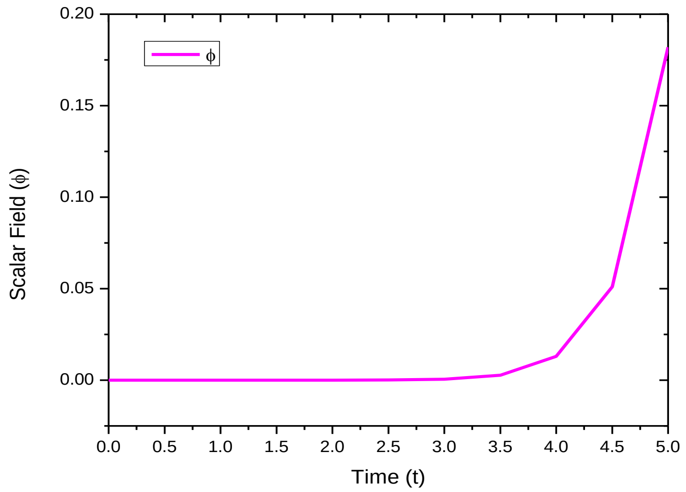

Figure 2 Plot of Scale Factor Vs Time with suitable choice of constants $n = 0.52, k = 1.1 \& \alpha = 0.11.$

The graphical behavior of the scalar field of the model is as shown in the **Figure 2;** the scalr field of the resulting universe is varying exponentially with respect to time variable and is null at *t=0*.

Utilizing the metric potentials (11) and (12) in the model (1) the resulting Cosmological model of the Universe is found out to be,

$$ds^2 = \left[\left(\frac{1}{\alpha}\right)^{\frac{2n}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{6n}{\alpha(4n+1)}}\right]\left(dt^2 - dr^2 - r^2 d\theta^2 - \left[\left(\frac{1}{\alpha}\right)^{\frac{2}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha(4n+1)}}\right]dz^2\right), \quad (16)$$

The above model (16) of the universe is the special form of deceleration parameter cosmological model and we got the potential functions of this model in terms of special form. At $t = 0$ the model shows singularity and model turn into constant. In the similar manner the potential functions (14 &15) are also vanishes for $t = 0$.

IV. KINEMATICAL PARAMETERS OF THE MODEL

The kinematical parameters of the universe and its graphical behavior are discussed in this section.

The spatial volume with respect to the average scale factor which is nothing but the special form of deceleration parameter is found out to be,

$$V = \left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}} \quad (17)$$

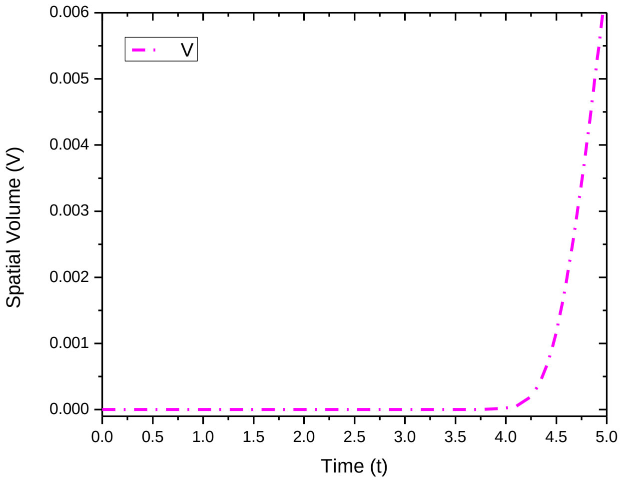

Figure 3 Plot of Spatial VolumeVs Time with suitable choice of constants $k=1.1\&\alpha=0.11$.

The Hubble parameter for model (1) is found out to be,

$$H=\frac{\dot{a}}{a}=\frac{ke^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\quad(18)$$

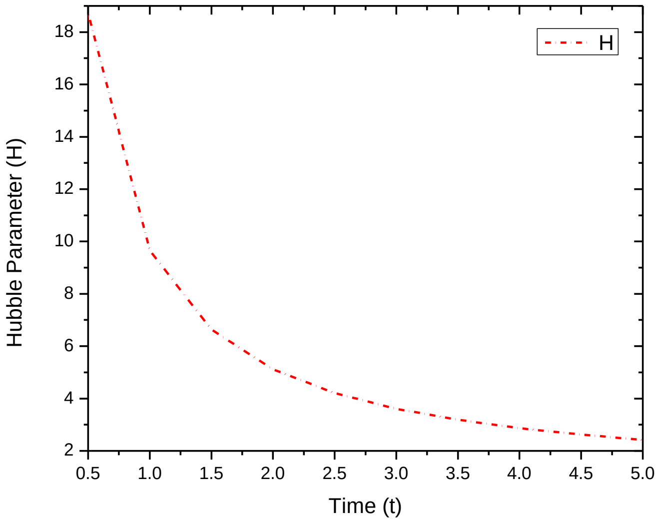

Figure 4 Plot of Hubble Parameter Vs Time with suitable choice of constants $k=1.1\&\alpha=0.11$.

The obtained expansion scalar θ is,

$$\theta=3H=\frac{3ke^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\quad(19)$$

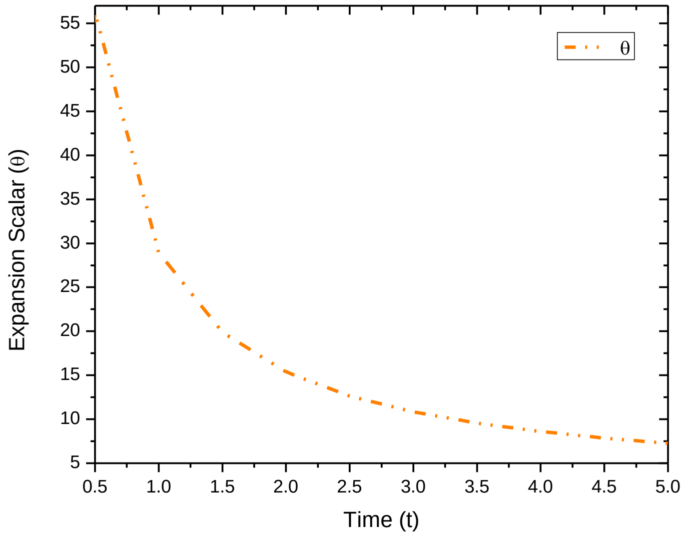

Figure 5 Plot of Expansion Scalar Vs Time with suitable choice of constants $k = 1.1 \,\&\, \alpha = 0.11$.

The mean anisotropy parameter for the given expansion of the model (1) is found out to be,

$$A_m = \frac{\left(e^{\alpha kt} - 1\right)}{k e^{\alpha kt}} \quad (20)$$

The resulting shear scalar of the given model is

$$\sigma^2 = \frac{3k}{2} \frac{e^{\alpha kt}}{\left(e^{\alpha kt} - 1\right)} \quad (21)$$

The deceleration parameter is found out to be,

$$q = \frac{\alpha}{e^{\alpha kt}} - 1 \quad (22)$$

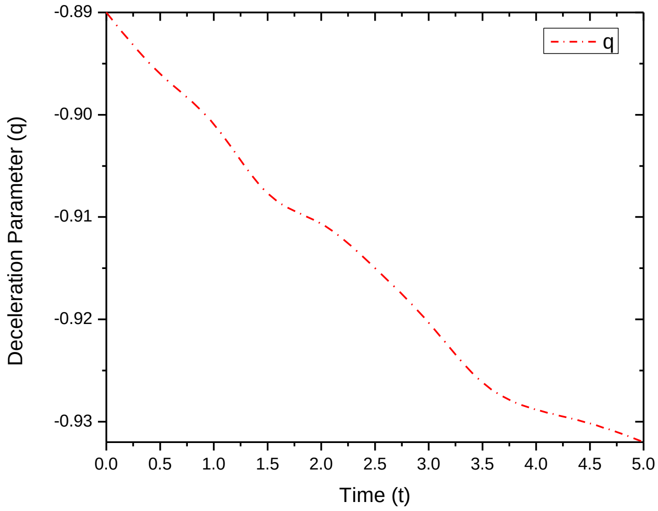

Figure 6 Plot of Deceleration Parameter Vs Time with suitable choice of constants $k = 1.1 \& \alpha = 0.11$. The nature of deceleration parameter of the model is as shown in the above graphical representation. While observing above **Figure 6**it can be concluded that the values of deceleration parameter are lies in between $-0.89 \le q \le -0.93$ and by the recent observation we can say that the universe shows the decelerated expansion of the universe.

V. CONCLUSIONS

In this present article, we have explored the dynamics of non-static plane symmetric cosmological models in the frame work of SBTG.For finding the solutions of field equations we have consider the special form of deceleration parameter. The metric potentials found for this model are the exponential function of cosmic time t.The resulting cosmological model having a singularity at, $t = 0$ and also the model is in exponent form. The parameters $p, \rho \& \psi$ is having the singular point at *t=0*. The deceleration parameter is lies in between $-0.89 \le q \le -0.93$ and by the recent observation we can conclude that the universe shows the decelerated expansion of the universe.Hence these all findings show that our current universe is experiencing an accelerated expansion. Some physical and kinematical parameters of the models have been graphically described.

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