

EDGE COLOURING OF A COMPLEMENT INTUITIONISTIC FUZZY GRAPH

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Abstract

In this paper, to introduce an algorithm to find the complement of intuitionistic fuzzy graph and also coloring this complement intuitionistic fuzzy graph using (α, β) cut.

Key words: Complement intuitionistic fuzzy graph, edge color, (α, β) cut of intuitionistic fuzzy graph.

1. INTRODUCTION

Intuitionistic Fuzzy graph theory was introduced by Krassimer T Atanassor in [3]. In [7], R.Parvathi, M.G. Karunambigai and K.Atanassor introduced some important operations on Intuitionistic fuzzy graphs.

The IF sets are more practical and applicable in real life situations. Intuitionistic fuzzy set deal with incomplete information that is, degree of member function, non member function but not indeterminate and inconsistent information that exists definitely in many system including belief system, decision support system etc.

2. PRELIMINARIES

2.1 Definition

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ where $\forall u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

2.2 Definition

An Intuitionistic fuzzy set A in a set X is defined as an object of the form $A = \{ \langle X, \mu_A(X), \nu_A(X) \rangle / x \in X \}$ where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ defined the degree of membership and degree of non membership of the element $x \in X$ respectively and for every $x \in X; 0 \leq \mu_A(X) + \nu_A(X) \leq 1$.

2.3 Definition

Intuitionistic fuzzy graph is of the form $G = (V, F)$ where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0,1]$ and $\nu_1 : V \rightarrow [0,1]$ denote the degrees of membership and non membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$ for every $v_i \in V (i = 1, 2, \dots, n)$
- (ii) $E \subset V \times V$ where $\mu_2 : V \times V \rightarrow [0,1]$ and $\nu_2 : V \times V \rightarrow [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$$

$$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$$

and

$$0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1 \text{ for}$$

each $(v_i, v_j) \in E$.

Here the triplet $(v_i, \mu_{1i}, \nu_{1i})$ denote the vertex, the degree of membership and non membership of the vertex v_i . The triplet $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denote the edge, the degree of membership and degree of non membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

2.4 Definition

Let A be intuitionistic fuzzy set of universe set X. Then (α, β) -cut of A is crisp set $(\alpha, \beta(A))$ of the Intuitionistic fuzzy set A is given by

$$(\alpha, \beta(A)) = \{x : x \in X \ni \mu_A(X) \geq \alpha, \nu_A(X) \leq \beta\}$$

, where $(\alpha, \beta) \in [0, 1]$ with $\alpha + \beta \leq 1$.

2.5 Definition

The Complement of an Intuitionistic fuzzy graph $G = \langle V, E \rangle$ is an Intuitionistic fuzzy graph, $\bar{G} = \langle \bar{V}, \bar{E} \rangle$ where

$$(i) \quad \bar{V} = V$$

$$(ii) \quad \overline{\mu_{1i}} = \mu_{1i} \text{ and } \overline{\nu_{1i}} = \nu_{1i} \text{ for all } i = 1, 2, \dots, n$$

$$(iii) \quad \overline{\mu_{2ij}} = \mu_{1i} \cdot \mu_{1j} - \mu_{2ij} \text{ and } \overline{\nu_{2ij}} = \nu_{1i} \cdot \nu_{1j} - \nu_{2ij} \text{ for all } i, j = 1, 2, \dots, n$$

3. COLOURING OF COMPLEMENT INTUITIONISTIC FUZZY GRAPH

For solving this problem we have done the calculation into three cases. In first case we

take an intuitionistic fuzzy graph (G) which has four vertices and five edges. All the vertices and edges have membership value. In second case we find the complement of this intuitionistic fuzzy graph (\bar{G}) . In third section we define the edge colouring function to colour the complement intuitionistic Fuzzy graph.

Case 1

Consider a intuitionistic fuzzy graph with have four vertices, v_1, v_2, v_3, v_4 and corresponding membership value 0.15, 0.2, 0.2, 0.1 and non membership value 0.75, 0.7, 0.8, 0.85 respectively. Graph consists of five edges e_1, e_2, e_3, e_4, e_5 with their corresponding membership value 0.01, 0.01, 0.04, 0.02, 0.01 and non membership value 0.5, 0.5, 0.56, 0.4, 0.6 respectively. Corresponding Intuitionistic fuzzy graph is shown in figure 1.

$$\mu_1 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.01 & 0.01 & 0.00 \\ 0.01 & 0.00 & 0.04 & 0.02 \\ 0.01 & 0.04 & 0.00 & 0.01 \\ 0.00 & 0.02 & 0.01 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 1

$$\nu_1 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.50 & 0.50 & 0.00 \\ 0.50 & 0.00 & 0.56 & 0.40 \\ 0.50 & 0.56 & 0.00 & 0.60 \\ 0.00 & 0.40 & 0.60 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 2

$$E_1 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & e_1 & e_2 & 0 \\ e_1 & 0 & e_3 & e_4 \\ e_2 & e_3 & 0 & e_5 \\ 0 & e_4 & e_5 & 0 \end{bmatrix} \end{matrix}$$

Matrix 3

$$E_2 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & e_1 & e_2 & 0 \\ e_1 & 0 & 0 & 0 \\ e_2 & 0 & 0 & e_5 \\ 0 & 0 & e_5 & 0 \end{bmatrix} \end{matrix}$$

Matrix 6

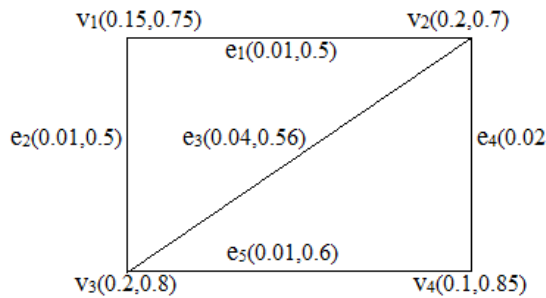


Figure 1

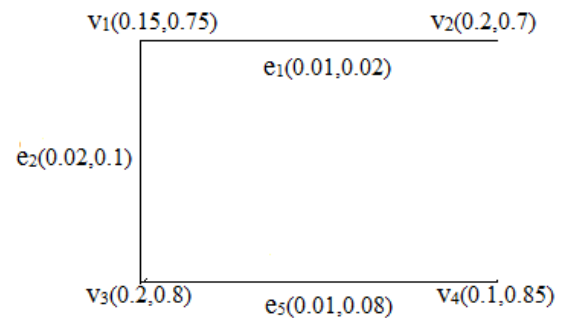


Figure 2

Case 2

We find the complement of intuitionistic fuzzy graph using [7].

$$\mu_2 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.01 & 0.02 & 0.00 \\ 0.01 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.01 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 4

$$\nu_2 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.02 & 0.10 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.08 \\ 0.00 & 0.00 & 0.08 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 5

Case 3

In this Intuitionistic fuzzy graph there are four cuts. They are four (α, β) cuts. They are

$$\{(0.01,0.08), (0.01,0.02), (0.02,0.1), (0.1,0.1)\}$$

For $(\alpha, \beta) = (0.01,0.08)$

$$\mu_3 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.01 & 0.02 & 0.00 \\ 0.01 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.01 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 7

$$v_3 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.02 & 0.10 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.08 \\ 0.00 & 0.00 & 0.08 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 8

$$E_3 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & e_1 & e_2 & 0 \\ e_1 & 0 & 0 & 0 \\ e_2 & 0 & 0 & e_5 \\ 0 & 0 & e_5 & 0 \end{bmatrix} \end{matrix}$$

Matrix 9

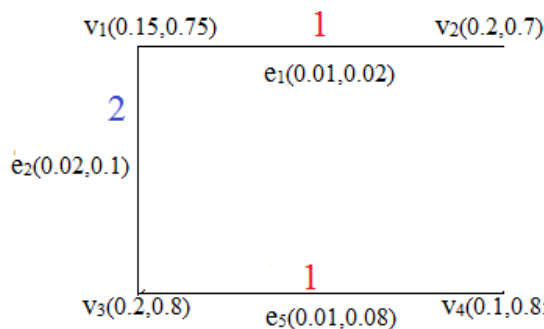


Figure 3

The chromatic number of this graph is 2.

For $(\alpha, \beta) = (0.01, 0.02)$

$$\mu_4 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.01 & 0.02 & 0.00 \\ 0.01 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 10

$$v_4 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.02 & 0.10 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{matrix}$$

Matirx 11

$$E_4 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & e_1 & e_2 & 0 \\ e_1 & 0 & 0 & 0 \\ e_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Matrix 12

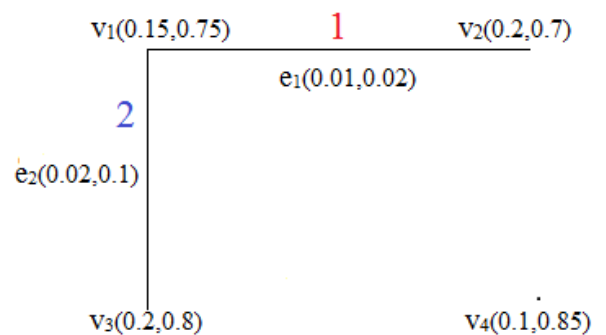


Figure 4

The chromatic number of this graph is 2.

For $(\alpha, \beta) = (0.02, 0.01)$

$$\mu_5 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.00 & 0.02 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{matrix}$$

Matirx 13

$$v_5 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.00 & 0.10 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.10 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 14

$$E_5 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 0 & e_2 & 0 \\ 0 & 0 & 0 & 0 \\ e_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Matrix 15

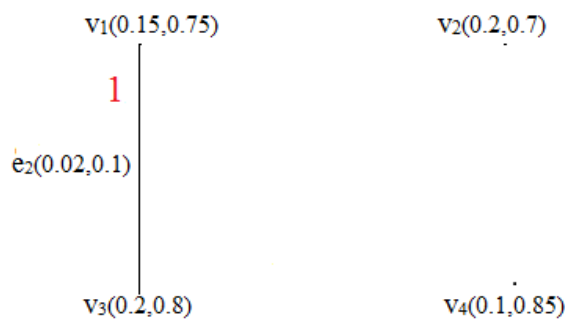


Figure 5

The chromatic number of this graph is 1.

For $(\alpha, \beta) = (0.1, 0.1)$

$$\mu_6 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 16

$$v_6 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \end{matrix}$$

Matrix 17

$$E_6 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Matirx 17

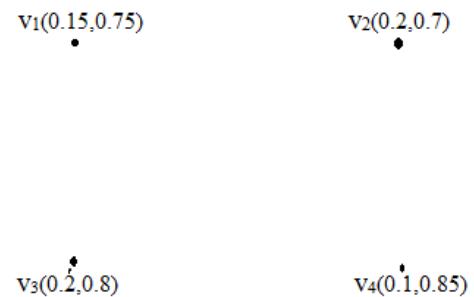


Figure 6

The chromatic number of this graph is 0.

4. CONCLUSION

In this paper, coloring all the edges in complement fuzzy graph are introduced, edge chromatic number depends on (α, β) - cut value also analysed.

REFERENCES

- [1] Arindam Dey, Anita Pal, "Vertex coloring of a fuzzy graph using alpha cut".
- [2] Arindam Dey, Dhrubajyoth Ghosh, Anita Pal, "Edge coloring of complement fuzzy graph".
- [3] Krassimir T, Atanassor, "Intuitionistic fuzzy sets, Fuzzy sets and systems".
- [4] A.Prasanna, M.A.Rifayathali, S. Ismail Mohideen, "Strong Intuitionistic Fuzzy graph coloring".
- [5] M.G.Karunambigai, R.Parvathi, R.Buvaneswari, "Arcs in Intuitionistic Fuzzy Graphs".
- [6] P.K.Sharma , " (α, β) -cut of intuitionistic fuzzy groups".
- [7] S.Thilagavathi, R. Parvathi, M,G,Karunambikai, "Operations on Intuitionistic Fuzzy Graphs II"
- [8] R.Vinitha, "Edge coloring in Intuitionistic Fuzzy
- raphs".
- G