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Existence and Uniqueness of solutions for a Fuzzy Impulsive Differential Equation with State DependentDelay

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Abstract

In this paper, we proved the existence and uniqueness of solutions for a fuzzy impulsive differential equation with state dependent delay described by the form,

$$\frac{d}{dt}[u(t) + G(t, u_t)] = A(t)u(t) + F(t, u_{\rho(t,u_t)}), t \in I = [0, \alpha]$$

$i = 1, 2, 3, \ldots, n$ and $x_0 = \psi$

$$\Delta u(t_i) = I_i(u(t_i))$$

We used Contraction theorem to get the solution.

Keywords: Impulsive differential equations, Fixed point theorem, Neutral equations, Semigroups of linear operators, Contraction principle, Fuzzy number, Uniqueness.

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1 Introduction

The variables and parameters in modelling real-life problems, have only vague, imprecise or incomplete or experimental data, applications of various operating circumstances, or maintenance induced errors. One can use the fuzzy environment to define the exact values regarding parameters, variables and initial conditions,transforming general Differential equations into Fuzzy Differential equations to subdue the insufficiency of precision. To describe the uncertainties fuzzy numbers are used as differential coefficients. Fuzzy Differential Equations applications are linked to nonlinear modelling and control. Chaotic analysis, quantum systems, and engineering issues, and in civil engineering, have all benefited from research into FDE solutions.

A Differential equation with impulse effects is known as Impulsive differential equation [2].Differential equations are widely used to represent many processes that have been studied in applied science field. But the scenario is distinct in some physical events that have an abrupt change in their states like biological systems, theoretical physics, mathematical economy, electric technology, radio physics,

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metallurgy, chemical technology, fluctuations in pendulum systems and death of population both due to an impulsive effect and so on [1]. Researchers are often shown interest in Impulsive Differential Equation like Usha V et al [12] and Sayooj et al [6, 9–11]. They have developed the concepts of Impulsive Differential Equation to obtain the semilinear Functional equation's stability result and also for nonlocal random equations in 2021.

In this work, we have combined the concepts of fuzzy and Impulsive differential equation to solve the problem under consideration.

For the below system, Bheeman Radhakrishnan et al [3] proved the existence of nonlinear fuzzy solutions,

$$\frac{du(t)}{d(t)} = a(t)u(t) + \int_0^t k(t, s, u(s))ds + f(t, u(t)), \quad t \in [0, b] \tag{1.1}$$

$$u(0) + h(u) = u_0, \tag{1.2}$$

$$\Delta u(t_k) = I_k(u(t_k^-)) \tag{1.3}$$

obtained the result by the approach of Banach fixed point theorem. Mallika Arjunan et al. [5] established the existence of solutions for impulsive neutral functional differential equations with state dependent delay by using Larey Schauder Alternative fixed-point theorem. Osmo Kaleva [8] deals with fuzzy set values mappings of a real variable whose values are in R^n. He also studied differentiability and integrability properties of such functions and gave an existence and uniqueness solution. Abdel-Rady et al. [1] established the existence and uniqueness of solution for the first order impulsive differential equation and show that these results can be applied to second order impulsive differential equation. Also, provided examples to illustrate the main results. Anguraj et al. [4] studied the existence of solutions for impulsive functional differential equation with state- dependent delay. Larey- Schauder Alternative fixed-point theorem is used to establish this result. The purpose of this paper is to establish the existence and uniqueness of solutions for a fuzzy impulsive differential equation with state dependent delay described by the form,

$$\frac{d}{d(t)}[u(t) + G(t, u_t)] = A(t)u(t) + F(t, u_{\rho(t,u_t)}) \quad t \in I = [0, \alpha] \tag{1.4}$$

$$X_0 = \psi \tag{1.5}$$

$$\Delta u(t_i) = I_i(u_{t_i}), \ i = 1, 2, ...n \tag{1.6}$$

Where

$$A : J \to [E]_N$$

is fuzzy coefficient, $[E]_N$ is the set of all fuzzy convex, upper semi-continuous, normal fuzzy numbers with bounded α level intervals. Also, $u_{\rho(t,u_t)}$ represents the State Dependent Delay, $\rho : J \times J \to [E]_N$, $G : J \times [E]_N \to [E]_N$, $F : J \times [E]_N \to [E]_N$, $\Delta u(t_i) = I_i(u_{t_i})$ are nonlinear continuous functions.

From the works described above, the main aim of this paper is to use contraction theorem to establish the existence and uniqueness solution for a fuzzy impulsive differential equation with state dependent delay.

2 Preliminaries

Consider set of all upper semi-continuous convex normal fuzzy numbers with bounded α- level internals [6] $[E]_N$. That is, if $m \in [E]_N$, the α–level set is given by

$$[m]^\alpha = \{x \in R : m(x) \geq \alpha, 0 < \alpha \geq 1\} \quad (2.1)$$

is a closed bounded interval which is denoted by

$$[m]^\alpha = [m^\alpha_q, m^\alpha_r] \quad (2.2)$$

and there exists a $t_0 \in$ R such that $m(t_0) = 1$. If $\mu_m(x) = \mu_n(x)$ for all $x \in$ R, then the fuzzy numbers m and n are said to be equa $m = n$. It follows that,

$$m = n \Leftrightarrow [m]^\alpha = [n]^\alpha \quad (2.3)$$

for all $\alpha \in (0, 1]$. A fuzzy number m be decomposed into its level sets,

$$m = \int_0^1 \alpha[m]^\alpha, \quad (2.4)$$

where $\int$ is the union of $[m]^\alpha$ where $\alpha \in$ (0,1].

Lemma 2.1[7]

If m,n $\in [E]_N$, then for $\alpha \in (0, 1], i, j = q, r$

$$[m + n]^\alpha = [m^\alpha_q + n^\alpha_q, m^\alpha_r + m^\alpha_r], \quad (2.5)$$

$$[m \times n]^\alpha = [min\{m^\alpha_i n^\alpha_j\}, min\{m^\alpha_i m^\alpha_j\}], \quad (2.6)$$

$$[m - n]^\alpha = [m^\alpha_q - n^\alpha_q, m^\alpha_r - m^\alpha_r] \quad (2.7)$$

Lemma 2.2

Let $[m^\alpha_q, m^\alpha_r], 0 < \alpha \leq 1$, [7] be a given family of nonempty intervals. Suppose

$$[m^\beta_q, m^\beta_r] \subset [m^\alpha_q, m^\alpha_r], \forall\ 0 < \alpha \leq \beta \quad (2.8)$$

$$[[\lim_{k\to+\infty} m^{\alpha k}_q, \lim_{k\to+\infty} m^{\alpha k}_r] = [m^\alpha_q, m^\alpha_r] \quad (2.9)$$

when (α_k) is a nondecreasing sequence converging to $\alpha \in$(0,1], then the family $[m^\alpha_q, m^\alpha_r]$, for all $0 < \alpha \leq 1$, are the α-level sets of a fuzzy number m$\in E_N$; conversely, if $[m^\alpha_q, m^\alpha_r]$, $0 < \alpha \leq 1$ are the α-level sets of a fuzzy number $m \in E_N$ then the conditions (2.8) and (2.9) hold true.

For a nonempty subset M contained in R^n and $x \in R^n$, the distance d(x,M) from x to M is given by

$$d(x, M) = \inf\{\|x - m\| : m \in M\} \quad (2.10)$$

The Hausdroff separation for nonempty subsets M and N contained in R^n,

$$d^*_H(M, N) = \sup d(n, M) : n \in N \tag{2.11}$$

In general, $d^*_H(M, N) \neq d^*_H(N, M)$. The Hausdroff distance between nonempty subsets M and N of R^n is,

$$d_H(M, N) = \max\{d^*_H(M, N), d^*_H(N, M)\} \tag{2.12}$$

This is now symmetric in M and N. Consequently,

(i) $d_H(M, N) \geq 0$ with $d_H(M, N) = 0$ if and only if $\bar{M} = \bar{N}$,

(ii) $d_H(M, N) = d_H(N, M)$, and

(iii) $d_H(M, N) \leq d_H(M, \rho) + d_H(\rho, N)$,

for any nonempty subsets of M,N, and P of R^n. The Hausdroff distance (2.12) is a metric, the Hausdroff metric. The supremum metric d_∞ on E^n is defined by

$$d_\infty(u_1, u_2) = \sup\{d_H([u_1]^\alpha, [u_2]^\alpha) : \alpha \in (0, 1]\}, \tag{2.13}$$

for all $u_1, u_2 \in E^n$ and is metric on E^n.
On $C(J, E^n)$, the supremum metric H_1 is defined by

$$H_1(x_1, x_2) = \sup\{d_\infty(x_1(t), x_2(t)) : t \in J\} \tag{2.14}$$

3 Existence Results

We establish the result which satisfy a global Lipschitz condition. There exist finite constants $\zeta_G, \zeta_F, \zeta_I > 0$ such that

$$\begin{aligned} d_H([G(s, u_s)]^\alpha, [G(s, v_s)]^\alpha) &= \zeta_G d_H([u_s]^\alpha, [v_s]^\alpha) \\ d_H([F(s, u_{\rho(s,u_s)})]^\alpha, [F(s, v_{\rho(s,u_s)})]^\alpha) &= \zeta_F d_H([u_{\rho(s,u_s)}]^\alpha, [v_{\rho(s,u_s)}]^\alpha) \\ d_H([I_i(u(t_i))]^\alpha, [I_i(v(t_i))]^\alpha) &= \zeta_I d_H([u_t]^\alpha, [v_t]^\alpha) \\ d_H([\rho(s, u_s]^\alpha, [\rho(s, v_s)]^\alpha) &= \zeta_\rho d_H([u_s]^\alpha, [v_s]^\alpha) \end{aligned} \tag{3.1}$$

for all u, $v \in E_N$.
Assume I be a real interval. Then, Fuzzy process is a mapping $x : I \to E_N$ denoted by

$$[x(t)]^\alpha = [x^\alpha_q(t), x^\alpha_r(t)], t \in I, 0 < \alpha \leq 1 \tag{3.2}$$

The derivative $x'(t)$ of a fuzzy process X ,

$$[x'(t)]^\alpha = [(x^\alpha_q)'(t), (x^\alpha_r)'(t)], 0 < \alpha \leq 1 \tag{3.3}$$

The fuzzy integral $\int_a^b x(t)dt$, $a, b \in I$ is stated as

$$\left[\int_a^b x(t)dt\right]^\alpha = \left[\int_a^b x^\alpha_q(t)dt, \int_a^b x^\alpha_r(t)dt\right] \tag{3.4}$$

given that on the right-hand side, Lebesgue integrals exist.

3.1 Theorem

If $M_1 > 0$, F and G satisfy a global Lipschitz condition, then the fuzzy impulsive differential equation with state dependent delay has only one solution, for every $u \in E_N$.

Proof

For each $u(t) \in E_N$, and $t \in J$, we have

$$(F_0 u(t)) = T(t)\,[\psi(0) + G(0,\psi)] - G(t,u_t) - \int_0^t AT(t-s)G(s,u_s)ds + \int_0^t T(t-s)F(s,u_{(\rho(s,u_s))})ds + \sum T(t-t_i)I_i(u(t_i)),\ t \in I \tag{3.4}$$

Where T(t) is a fuzzy number and $T(t)^\alpha = [T^\alpha_q(t), T^\alpha_r(t)]$ and $|T^\alpha_i(t)| \le \zeta_T$, for a constant $\zeta_T > 0$.

$d_H([F_0(u)(t)]^\alpha, [F_0(v)(t)]^\alpha)$

$$\begin{aligned} &= d_H\Big(\Big[T(t)[x_0 + G(0,\psi)] - G(t,u_t) - \int_0^t AT(t-s)G(s,u_s)ds + \int_0^t T(t-s)F(s,u_{\rho(s,u_s)})ds \\ &\quad + \sum_{(0<t_i<t)} T(t-t_i)I_i(u_t)\Big]^\alpha, \Big[T(t)[x_0 + G(0,\psi)] - G(t,v_t) - \int_0^t AT(t-s)G(s,v_s)ds \\ &\quad + \int_0^t T(t-s)F(s,v_{\rho(s,v_s)})ds + \sum_{(0<t_i<t)} T(t-t_i)I_i(v_t)\Big]^\alpha\Big) \end{aligned} \tag{3.5}$$

$$\begin{aligned} &\le d_H\Big([T(t)[x_0 + G(0,\psi)]]^\alpha, [T(t)[x_0 + G(0,\psi)]]^\alpha\Big)\ d_H\big([G(t,u_t)]^\alpha, [G(t,v_t)]^\alpha\big) \\ &\quad d_H\Big(\Big[\int_0^t AT(t-s)G(s,u_s)ds\Big]^\alpha, \Big[\int_0^t AT(t-s)G(s,v_s)ds\Big]^\alpha\Big) \\ &\quad + d_H\Big(\Big[\int_0^t T(t-s)F(s,u_{\rho(s,u_s)})ds\Big]^\alpha, \Big[\int_0^t T(t-s)F(s,v_{\rho(s,v_s)})ds\Big]^\alpha\Big) \\ &\quad + d_H\Big(\Big[\sum_{(0<t_i<t)} T(t-t_i)I_i(u_t)\Big]^\alpha, \Big[\sum_{(0<t_i<t)} T(t-t_i)I_i(u_t)\Big]^\alpha\Big) \end{aligned}$$

Applying equations (3.1) for the above equation we have

$d_H([F_0(u)(t)]^\alpha, [F_0(v)(t)]^\alpha)$

$$\begin{aligned} &= d_H\Big([T(t)G(0,\psi)]^\alpha, [T(t)G(0,\psi)]^\alpha\Big) - d_H\big([G(t,u_t)]^\alpha, [G(t,v_t)]^\alpha\big) \\ &\quad - d_H\Big(\Big[\int_0^t AT(t-s)G(s,u_s)ds\Big]^\alpha, \Big[\int_0^t AT(t-s)G(s,v_s)ds\Big]^\alpha\Big) \\ &\quad + d_H\Big(\Big[\int_0^t T(t-s)F(s,u_{\rho(s,u_s)})ds\Big]^\alpha, \Big[\int_0^t T(t-s)F(s,v_{\rho(s,v_s)})ds\Big]^\alpha\Big) \\ &\quad + d_H\Big(\Big[\sum_{(0<t_i<t)} T(t-t_i)I_i(u_t)\Big]^\alpha, \Big[\sum_{(0<t_i<t)} T(t-t_i)I_i(v_t)\Big]^\alpha\Big) \end{aligned}$$

From the equations (3.2), (3.3) and (3.4), we have

$$d_H([F_0(u)(t)]^\alpha, [F_0(v)(t)]^\alpha)$$

$$\le d_H\left(\left(T_q^\alpha(t)G_q^\alpha(0,\psi), T_r^\alpha(t)G_r^\alpha(0,\psi)\right), \left(T_q^\alpha(t)G_q^\alpha(0,\psi), T_r^\alpha(t)G_r^\alpha(0,\psi)\right)\right)$$

$$- d_H\left(\left(G_q^\alpha(t,u_t), G_r^\alpha(t,v_t)\right), \left(G_q^\alpha(t,v_t), G_r^\alpha(t,v_t)\right)\right)$$

$$- d_H\left(\int_0^t \left(AT_q^\alpha(t-s)G_q^\alpha(s,u_s), AT_r^\alpha(t-s)G_r^\alpha(s,u_s)\right), \left(AT_q^\alpha(t-s)G_q^\alpha(s,v_s), AT_r^\alpha(t-s)G_r^\alpha(s,v_s)\right) ds\right),$$

$$+ d_H\left(\int_0^t [T_q^\alpha(t-s)F_q^\alpha(s,u_{\rho(s,u_s)})ds, T_r^\alpha(t-s)F_r^\alpha(s,u_{\rho(s,u_s)})ds], \int_0^t T_q^\alpha(t-s)F_q^\alpha(s,v_{(\rho(s,v_s)})ds, T_r^\alpha(t-s)F_r^\alpha(s,v_{(\rho(s,v_s)})ds\right)$$

From equation (2.12), we have $d_H([F_0(u)(t)]^\alpha, [F_0(v)(t)]^\alpha)$

$$\begin{aligned}
&= \max\left\{\left|T_q^\alpha(t)G_q^\alpha(0,\psi) - T_q^\alpha(t)G_q^\alpha(0,\psi)\right|, \left|T_r^\alpha(t)G_r^\alpha(0,\psi) - T_r^\alpha(t)G_r^\alpha(0,\psi)\right|\right\} \\
&\quad - \max\left\{\left|[G_q^\alpha(t,u_t) - G_q^\alpha(t,v_t)]\right|, \left|[G_r^\alpha(t,u_t) - G_r^\alpha(t,v_t)]\right|\right\} \\
&\quad - \max\left\{\left|\int_0^t AT_q^\alpha(t-s)G_q^\alpha(s,u_s)ds - \int_0^t AT_q^\alpha(t-s)G_q^\alpha(s,v_s)ds\right|, \left|\int_0^t AT_r^\alpha(t-s)G_r^\alpha(s,u_s)ds - \int_0^t AT_r^\alpha(t-s)G_r^\alpha(s,v_s)ds\right|\right\} \\
&\quad + \max\left\{\left|\int_0^t T_q^\alpha(t-s)F_q^\alpha(s,u_{\rho(s,u_s)})ds - \int_0^t T_q^\alpha(t-s)F_r^\alpha(s,v_{\rho(s,v_s)})ds\right|, \left|\int_0^t T_r^\alpha(t-s)F_r^\alpha(s,u_{\rho(s,u_s)})ds - \int_0^t T_r^\alpha(t-s)F_r^\alpha(s,v_{\rho(s,v_s)})ds\right|\right\} \\
&\quad + \max\left\{\left|\sum_{0<t_i<t} T_q^\alpha(t-t_i)(I_i)_q^\alpha(u_t) - \sum_{0<t_i<t} T_q^\alpha(t-t_i)(I_i)_q^\alpha(v_t)\right|, \left|\sum_{0<t_i<t} T_r^\alpha(t-t_i)(I_i)_r^\alpha(u_t) - \sum_{0<t_i<t} T_r^\alpha(t-t_i)(I_i)_r^\alpha(v_t)\right|\right\} \\
&= -\max\left\{\left|G_q^\alpha(t,u_t) - G_q^\alpha(t,v_t)\right|, \left|G_r^\alpha(t,u_t) - G_r^\alpha(t,v_t)\right|\right\} \\
&\quad - \max\left\{\left|\int_0^t AT_q^\alpha(t-s)\left(G_q^\alpha(s,u_s)ds - G_q^\alpha(s,v_s)ds\right)\right|, \left|\int_0^t AT_r^\alpha(t-s)\left(G_r^\alpha(s,u_s)ds - G_r^\alpha(s,v_s)ds\right)\right|\right\} \\
&\quad + \max\left\{\left|\int_0^t T_q^\alpha(t-s)\left(F_q^\alpha(s,u_{\rho(s,u_s)})ds - F_q^\alpha(s,v_{\rho(s,v_s)})\right)ds\right|, \left|\int_0^t T_r^\alpha(t-s)\left(F_r^\alpha(s,u_{\rho(s,u_s)})ds - F_r^\alpha(s,v_{\rho(s,v_s)})ds\right)\right|\right\} \\
&\quad + \max\left\{\left|\sum_{0<t_i<t} T_q^\alpha(t-t_i)\left((I_i)_q^\alpha(u_t) - (I_i)_q^\alpha(v_t)\right)\right|, \left|\sum_{0<t_i<t} T_r^\alpha(t-t_i)\left((I_i)_r^\alpha(u_t) - (I_i)_r^\alpha(v_t)\right)\right|\right\}.
\end{aligned}$$

Further, we have

$d_H([F_0(u)(t)]^{\alpha}, [F_0(v)(t)]^{\alpha})$

$$= -\max\left\{ \left| G_q^{\alpha}(t, u_t) - G_q^{\alpha}(t, v_t) \right|, \left| G_r^{\alpha}(t, u_t) - G_r^{\alpha}(t, v_t) \right| \right\}$$

$$- A\zeta_T \max\left\{ \left| \int_0^t AT_q^{\alpha}(t-s) \left(G_q^{\alpha}(s, u_s)ds - G_q^{\alpha}(s, v_s)ds \right) \right|, \left| \int_0^t AT_r^{\alpha}(t-s) \left(G_r^{\alpha}(s, u_s)ds - G_r^{\alpha}(s, v_s)ds \right) \right| \right\}$$

$$+ \zeta_T \max\left\{ \left| \int_0^t \left(F_q^{\alpha}(s, u_{\rho(s,u_s)})ds - F_q^{\alpha}(s, v_{\rho(s,v_s)})ds \right) \right|, \left| \int_0^t F_r^{\alpha}(s, u_{\rho(s,u_s)})ds - F_r^{\alpha}(s, v_{\rho(s,v_s)})ds \right| \right\}$$

$$+ \zeta_T \max\left\{ \sum_{0<t_i<t} \left| (I_i)_q^{\alpha}(u_t) - (I_i)_q^{\alpha}(v_t) \right|, \sum_{0<t_i<t} \left| (I_i)_r^{\alpha}(u_t) - (I_i)_r^{\alpha}(v_t) \right| \right\}$$

Hence by applying (3.1),

$d_H\left([F_0(u)(t)]^{\alpha}], [F_0(v)(t)]^{\alpha}]\right)$

$$= -\zeta_G d_H([u_a]^{\alpha}, [v_s]^{\alpha}) - A\zeta_T\zeta_G \int_0^t d_H([u_s]^{\alpha}, [v_s]^{\alpha})ds$$

$$+ \zeta_T\zeta_F\zeta_\rho M_1 \int_0^t d_H([u_s]^{\alpha}, [v_s]^{\alpha})ds + \zeta_T\zeta_I d_H([u_s]^{\alpha}, [v_s]^{\alpha})$$

$$= (\zeta_T\zeta_I - \zeta_G) d_H([u_s]^{\alpha}, [v_s]^{\alpha}) + \zeta_T(\zeta_F\zeta_\rho M_1 - A\zeta_G) \int_0^t d_H([u_s]^{\alpha}, [v_s]^{\alpha})ds$$

$$= Z_1 d_H([u_s]^{\alpha}, [v_s]^{\alpha}) + Z_2 \int_0^t d_H([u_s]^{\alpha}, [v_s]^{\alpha})ds$$

Where $Z_1 = (\zeta_T\zeta_I - \zeta_G)$, $Z_2 = (\zeta_F\zeta_\rho M_1 - A\zeta_G)$

Therefore,

$d_\infty(F_0(u)(t), F_0(v)(t))$

$$= \sup_{t\in J} d_H([F_0(u)(t)]^{\alpha}, [F_0(v)(t)]^{\alpha})$$

$$\leq \sup_{t\in J} \left(Z_1 d_H([u_s]^{\alpha}, [v_s]^{\alpha}) + Z_2 \int_0^t d_H([u_s]^{\alpha}, [v_s]^{\alpha})ds \right)$$

$$= Z_1 \sup_{t\in J} d_H([u_s]^{\alpha}, [v_s]^{\alpha}) + Z_2 \sup_{t\in J} \int_0^t d_H([u_s]^{\alpha}, [v_s]^{\alpha})ds$$

$$\leq Z_1 d_\infty([u_s]^{\alpha}, [v_s]^{\alpha}) + Z_2 \int_0^t d_\infty([u_s]^{\alpha}, [v_s]^{\alpha})ds$$

Hence,

$H_1(F_0(u)(t), F_0(v)(t))$

$$= \sup_{t\in J} d_\infty(F_0(u)(t), F_0(v)(t))$$
$$= \sup_{t\in J}\{Z_1 d_\infty([u_s]^\alpha, [v_s]^\alpha) + Z_2 \int_0^t d_\infty([u_s]^\alpha, [v_s]^\alpha)ds\}$$
$$= Z_1 \sup_{t\in J} d_\infty([u_s]^\alpha, [v_s]^\alpha) + Z_2 \sup_{t\in J} \int_0^t d_\infty([u_s]^\alpha, [v_s]^\alpha)ds$$
$$\leq (Z_1 + Z_2 M_2) H_1(u_s(t), v_s(t))$$

Taking sufficiently small M_2, $(Z_1 + Z_2M_2) < 1$. This implies that, the function F_0 is a contraction mapping. According to contraction theorem, there exists only one solution for the considered system of equations. i.e.,

There exists an unique fixed point $u \in C(J, E_N)$.

4 Conclusion

We have proved the existence and uniqueness solution of Fuzzy Impulsive Differential Equation with State Dependent Delay. In future, it may serve as a prerequisite testimonial for scholars, researchers and academicians in Engineering and Science who requires to model uncertain physical problems and in real life Mathematical Modeling approach through fuzzy impulsive differential equation.

References

[1] Abdel-Rady, A. M. A. El-Sayed, S. Z. Rida and I. Ameen (2012), *On Some Impulsive Differential Equations*, Mathematical Sciences Letters, An International Journal, 1 No. 2, 105-113.

[2] Katya Dishlieva (2012), *Impulsive Differential Equation and Applications*, J Applied Computational Mathematics, volume 1, issue 6.

[3] Bheeman Radhakrishnan and Aruchamy Mohanraj (2013), *Existence of solutions for nonlinear Fuzzy Impulsive In- tegrodifferential Equation*, Malaya Journal of Matematik, S (1), 1-10.

[4] Anguraj, M.Mallika Arjunan and Eduardo Hernandez M (2007), *Existence results for an Impulsive Neutral Functional Differential Equation with State dependent delay*, Applicable Analysis: An International Journal, 86:7, 861-872.

[5] M.Mallika Arjunan and V. Kavitha, *Existence Results for impulsive Neutral Functional Differential Equations with State dependent delay*, Electronic Journal of Qualitative Theory of Differential Equations, No.26, 1-13,2009

[6] M. Mizumoto and K. Tanaka,*Some Properties of Fuzzy Numbers*, North Holland,1979.

[7] S.Seikkala (1987), *On the fuzzy initial value problem*, Fuzzy Sets and Systems, 24, 319 330,1987.

[8] Osmo Kaleva (1987), *Fuzzy Differential Equations*, Fuzzy Sets and Systems, North Holland, 24, 301- 317.

[9] Sayooj Aby Jose, Ashitha Tom, S. Abinaya and Weera Yukunthorn (2021), *Some Characterization of Results of Non-local Special Random Impulsive Differential Evolution Equation*, Journal of Applied Nonlinear Dynamics, (10)(4), 711-723,2021.

[10] Sayooj Aby Jose, Ashitha Tom, M. Syed Ali, S. Abinaya and Weerawat Sudsutad, *Existence, Uniqueness and Stability Results of Semilinear Functional Special Random Impulsive Differential Equations*, Dynamics of Continuous, Discrete and Impulsive systems, Series A: Mathematical Analysis, 28, 269- 293,2021.

[11] Sayooj Aby Jose, Weera Yukunthorn, Juan Eduardo Napoles and Hugo Leiva (2020), *Some Existence, Uniqueness and Stability Results of Nonlocal Random Impulsive Integro-Differential Equations*, Applied Mathematics - E Notes, 20,481-492,(2020).

[12] Usha. V and Sayooj Aby Jose, *Existence of solutions for Random Impulsive Differential Equations with Nonlocal conditions*, International Journal for Computer Science and Engineering, (6)(10)(2018), 549-554.

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