

Factals : A Geometry of Nature

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ABSTRACT

Many structures in nature like clouds, mountains, rocks, coastlines, trees, leaves, etc. has shapes which cannot be entirely described with Euclidean geometry i.e. in terms of lines, circles, spheres, cones, triangles, etc. Benoit B. Mandelbrot (20 Nov.1924-14 Oct. 2010) a Polish born, French-American Mathematician developed a theory to measure roughness in nature, for which he gave a name as Fractals.

Key Words : fractals, Mandelbrot, Cantor set, Koch snowflake, Sierpinski gasket, Self-similarity.

"Clouds are not spheres. Mountains are not cones, Lightening does not travel in a straight line"

- Benoit Mandelbrot.

The shapes of classical geometry of Euclid are lines and planes, circles and spheres, triangles and cones. We are studying Euclid's geometry since last two millennia. But for understanding complexity of nature it has limitations. How long is the coastline of India? The answer depends on the length of your ruler (measuring unit). If a surveyor takes a set of dividers which opens to a length of one kilometer. The resulting number of kilometers is just an approximation to the true length of coastline because it skip over twists and turns smaller than one kilometer. But if the surveyor sets the dividers to a smaller length-say one meter- then he measures a greater length because the dividers will capture the more details of the coastline. If the dividers are set to one inch and the measurement is done then it is again a greater length. Mandelbrot found that as the scale of measurement becomes smaller and smaller, the measured length of coastline rises without limit - bays and sub bays, peninsulas and sub peninsulas - at least down to atomic scales. Thus any coastline is - in a sense - infinitely long.

Euclidean measurements - lengths, depth, thickness - failed to capture the essence of irregular shapes. Mandelbrot used the idea of Dimension.

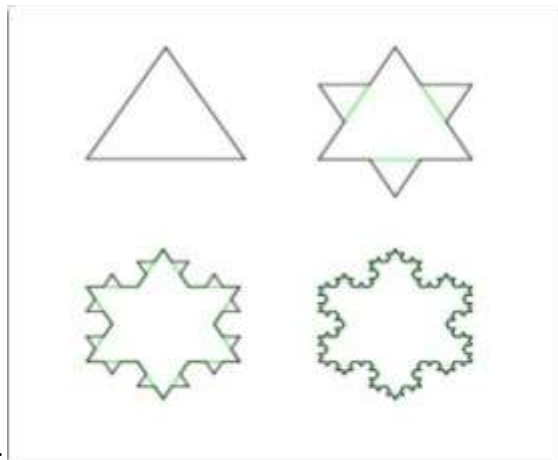
Benoit Mandelbrot - the father of fractal geometry - was born in 1924 in Warsaw. He was working in pure research wing of the International Business Machines Corporation, (IBM). The idea of fractal arose in his mind in 1960, when he was studying the distribution of large and small incomes in an economy. In his time at IBM, he came across a practical problem of noise in telephone lines used to transmit information from computer to computer. Mandelbrot discovered a consistent geometric relationship between the bursts of errors and the spaces of clean transmission. On scales of an hour or a second, the proportion of error-free periods to error-ridden periods remained constant. Mandelbrot was duplicating an abstract construction known as the Cantor set, named after mathematician George Cantor.



Figure 1: Cantors Set

To make a Cantor set, start with the interval of numbers from zero to one, represented by a line segment. Then remove the middle third. That leaves two segments; remove the middle third of each. That leaves four segments, remove the middle third of each, and continue this process to infinity. There remains a strange 'dust' of points, arranged in clusters, infinitely many yet infinitely scattered. Mandelbrot thought transmission errors as a Cantor set arranged in time. Mandelbrot studied other data. Egyptians have kept the records of the height of the river Nile for millennia. The

Nile suffers usually great variation, flooding heavily in some years and subsiding in others. Mandelbrot classified the variation in terms of two kinds of effects, which he called the Noah and Joseph Effects, which are common in Economics. The Noah effect means discontinuity: when a quantity changes it can change almost arbitrarily fast. For example prices in stock market changes suddenly from Rs.1000 to Rs. 100 due to a breaking news. The Joseph effect means persistence. There came seven years of great plenty throughout the land of Egypt and after that there was seven years of famine. The Noah and Joseph effects have different directions but they together imply that 'trends in nature are real, but they can vanish as quickly as they come'. Mandelbrot moved beyond dimensions 0, 1, 2, 3, ... to a seeming impossibility : fractional dimensions. Fractional dimensions is a way of measuring the degree of roughness or brokenness or irregularity in an object. For example twisting and zigzag coastline has a certain degree of roughness. Mandelbrot studied irregular patterns in nature and specified ways of calculating the fractional dimensions of real objects. He claimed that the degree of irregularity remains constant over different scales. His claim turns out to be true, the world displays a regular irregularity. Mandelbrot decided the name for his shapes, his dimensions, and his geometry from his school boy's dictionary. He thought fracture and fraction seemed appropriate. Hence Mandelbrot



created the word fractal. **The Koch Snowflake :-**

Figure 2: Koch Snowflake

Imagine a triangle each of its side one foot long. Imagine a certain repeated transformation as

- take the middle one-third of each side and attach a new triangle, identical in shape but one-third the size. The result is a star, the outline of this shape is twelve segments of four inch. Instead of three vertices there are six. Now take each of the twelve sides and repeat the transformation, attaching a smaller triangle onto the middle third. Continue this process onto infinity. The outline becomes more and more detailed like an ideal snowflake. It is known as Koch curve, after Swedish mathematician Helge Von Koch. The Interesting features of Koch Curve are : It is a continuous loop, never intersecting itself, because the new triangles form on each side. Each transformation adds a little area to the inside of the curve, but the total area remains finite, not much bigger than the original triangle. If a circle is drawn around the original triangle, the Koch curve would never extend beyond it. Yet the curve is infinitely long as a Euclidean straight line extending in both directions. The first transformation replaces a one-foot segment with four four-inch segments. Therefore every transformation multiplies the total length by fourth-thirds. Thus we get an infinite length in a finite space.

Self-Similarity :-

Mandelbrot's studies of irregular patterns in natural processes and of infinity complex shapes gave him an idea of self-similarity. Fractal are shapes which neither tend to vanish nor fluctuate up and down but remains essentially unchanged as one zooms in continually. Hence, the structure of every piece holds the key to the whole structure. An alternate term of this pattern is self-similar. Above all, fractal meant self-similar. **Sierpinski Gasket :-**



Figure 3 Sierpinski Gasket

Divide a given (black) equilateral triangle of side 1 into four sub triangles and then erase (whiten) the middle fourth. Then repeat the same procedure infinitely many times.

We get a pattern of self-similar triangles.

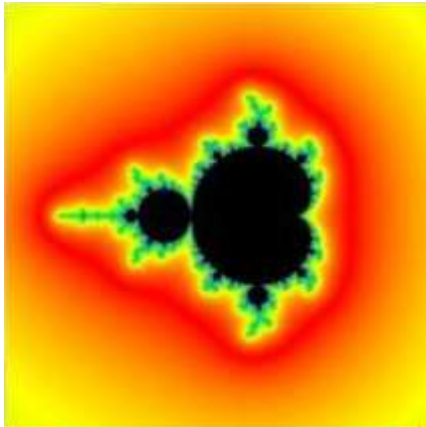


Figure 4 : Mandelbrot Set - a self similar structure Figure 5: Self-similarity in branches of a tree Self-similarity is symmetric across scale. It implies recursion, pattern inside of pattern.

Monstrous shapes like Koch curve display self-similarity because it look exactly the same even under high magnification. Self-similarity is an easily recognizable quality. It's images are everywhere in the culture like the infinitely deep reflection of a person standing between two mirrors.



Figure 6: Self-similarity in clouds

Figure 7: Self-similarity in cotton

Applications of Fractals :-

Fractal descriptions have applications to the problems of surfaces in contact with one another. For example, the contact between tire threads and road surface, contact in machine joints, contacts in electric circuits. One simple but powerful consequence of the fractal geometry of surfaces is that surfaces in contact do not touch everywhere. At some sufficiently small scale it becomes clear that gaps remain.

The physics of earthquake behavior is mostly independent of scale. A large earthquake is just a scaled-up version of a small earthquake. Clouds, like earthquakes, have characteristics irregularity: describable in terms of fractal dimension - does not change at all as they observed on different scales. Analysis of satellite pictures has shown an invariant fractal dimension in clouds observed from hundreds of miles away.

The equations of fluid flow are in many contexts dimensionless i.e. without regard to scales. Airplane wings and ship propellers can be tested in wind tunnels and laboratory basins. With some limitations, small storms act like large storms.

Blood vessels in our body branch and divide and branch again until they become so narrow to pass a single blood cell. The nature of their branching is fractal. They are like the Koch curve, a line of infinite length into a small area. The circulatory system squeezes a huge surface area into a limited volume. Yet the vessels and blood take up little space, no more than about five percent of the body, we can't take a milligram of flesh without spilling blood.

The lungs need to pack the greatest possible surface into the smallest space. An animal's ability to absorb oxygen is

proportional to the surface area of its lungs. Typical human lungs pack in a surface bigger than a tennis court. The bronchial branching is fractal. The urinary collecting system proved fractal. The biliary duct in the liver, the network of special fibers in the heart that carry pulses of electric current to the contracting muscles are fractal.

Conclusion :-

How did nature manage to evolve such complicated architecture? Mandelbrot's view is that the complications exist only in the context of traditional Euclidean geometry. As fractals, branching structures can be described with transparent simplicity. Mandelbrot compared pulmonary and vascular trees to real botanical trees, trees that need to capture sun and resist wind, with fractal branches and fractal leaves. Theoretical biologists realized that fractal scaling is not just common but universal in morphogenesis. They argued that understanding how such patterns were encoded and processed is a major challenges to biology.

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