Fifth Order Butchers RK Methods in MAGDM using Square Root Fuzzy Sets

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Abstract

In the realm of fuzzy set theory, the concept of a square root fuzzy set introduces a novel approach to handling uncertainty and imprecision in data representation. Traditional fuzzy sets use membership functions to quantify the degree of truth or belonging, typically ranging from 0 to 1. The square root fuzzy set extends this framework by applying the square root transformation to the membership values, resulting in a new set of membership degrees that are more sensitive to lower values and less sensitive to higher values. The Square Root Fuzzy Matrices are used as the dataset, and the weights are calculated employing Runge-Kutta methods.The Square root Fuzzy Hybrid Geometric operator and the Square root Fuzzy Weighted Geometric operator both supply solutions that we must use in the decision-making process. This paper presents examples showing the efficiency and usefulness of the Canberra Distance calculation, which is used for ranking

Keywords: Square Root Fuzzy Set, Square Root Fuzzy Weighted Geometric, Square Root Fuzzy Hybrid Geometric, Butcher 's Fifth Order Runge-Kutta Method.

1. Introduction

Fuzzy set theory, pioneered by Zadeh [10] in 1965, has undergone significant development, especially with the introduction of Intuitionistic Fuzzy Sets (IFS) and Pythagorean Fuzzy Sets (PFS). These advancements have broadened the scope of fuzzy logic applications, particularly in decision-making processes. This review synthesizes key research in these areas, highlighting theoretical advancements and practical applications. Atanassov [5][11] introduced IFS as an extension of classical fuzzy sets to incorporate a degree of hesitation, providing a more nuanced representation of uncertainty. IFS are characterized by a membership function, a non-membership function, and a hesitation degree, offering a comprehensive framework for handling incomplete or vague information.Atanassov's work laid the foundation for various applications and extensions, including hybrid and geometric aggregation operators [6][13]. Yager [15] extended the concept of fuzzy sets to Pythagorean Fuzzy Subsets, where membership degrees are treated as Pythagorean triples. This extension allows for more precise modeling of uncertainty and has been explored in several subsequent studies [21][22]. Peng and Yuan [22] discussed the fundamental properties of Pythagorean fuzzy aggregation operators, which are crucial for integrating multiple sources of fuzzy information. Akila and Robinson [1] and Jenifer Rose and Akila [4] explored MAGDM methods using Intuitionistic Triangular Fuzzy Sets (ITFS). Their research emphasizes the ability of ITFS to improve decision-making by better handling multiple attributes and preferences. The application of numerical methods and aggregation operators to MAGDM problems has been further explored by Liang et al. [7] and Xu and Cai [20]. These studies highlight the importance of selecting appropriate aggregation techniques to address complex decision-making scenarios effectively.Xu [16] and Xu & Yager [17] have contributed significantly to the development of intuitionistic fuzzy aggregation operators, which are vital for combining different fuzzy information sources in decision-making. Liao and Xu [18] expanded on this by introducing hybrid weighted aggregation operators, enhancing the flexibility of aggregation methods.Rahman et al. [19][24][25] investigated various Pythagorean fuzzy aggregation operators, including hybrid geometric and Einstein operators. Their work demonstrates how advanced aggregation methods can be applied to solve MAGDM problems, offering new approaches to handling fuzzy information.Al-Shami et al. [3] applied Square Root Fuzzy Sets (SR-Fuzzy Sets) and their weighted aggregated operators to decision-making, showcasing how SR-Fuzzy Sets can refine the analysis of membership values and improve decision outcomes. Kozae et al. [12] demonstrated the application of Intuitionistic Fuzzy Sets in the context of COVID-19, highlighting the versatility of IFS in addressing real-world problems with significant uncertainty. Arumugham et al. [2] provided a comprehensive overview of numerical methods essential for solving problems involving fuzzy sets. Their work supports the implementation of fuzzy theory in practical applications through effective numerical solutions. Theoretical advancements by Yager [15] and Xu [16] in aggregation operators and fuzzy membership functions have significantly impacted the field, offering new ways to model and analyze fuzzy information.

2. Preliminaries:

2.1 Definition:Square Root Fuzzy Set (SR- FS)

Let F be a universal set that containsγ_R : F → [0,1] , φ_R : F → [0,1]are mapping. Then the SR- Fuzzy Set is described by the following: $R = \{\langle q, \gamma_R(q), \varphi_R(q)\rangle : q \in F\}$, where $\gamma_R(q)$is the degree of membership and $\varphi_R(q)$is the degree of non-membership of $q \epsilon F \ to \ R$,that contains$0 \leq (\gamma_R(q))^2 + \sqrt{\varphi_R(q)} \leq 1$. Then there is a degree of indeterminacy of $q \in F \ to \ R$ defined by $\pi_R(q) = 1 - \left[(\gamma_R(q))^2 + \sqrt{\varphi_R(q)}\right]$.

2.2 Definition: Let$R_1 = (\gamma_{R_1}, \varphi_{R_1})andR_2 = (\gamma_{R_2}, \varphi_{R_2})$be two SR-FSs; then

1. $R_1 = R_2$ if and only if $\gamma_{R_1} = \gamma_{R_2}$and $\varphi_{R_1} = \varphi_{R_2}$
2. $R_1 \geq R_2$ if and only if $\gamma_{R_1} \geq \gamma_{R_2}$and $\varphi_{R_1} \leq \varphi_{R_2}$

2.3 Definition: Canberra Distance Formula:

Let$R_1 = (\gamma_{R_1}, \varphi_{R_1})andR_2 = (\gamma_{R_2}, \varphi_{R_2})$be two SR-FSs. Then we used the Canberra distance between $R_1$and R_2 as follows:

$$d(R_1, R_2) = \sum \frac{|x_i - y_i|}{|x_i + y_i|}$$

2.4 SR-FWG Operator for Group Decision Making

Definition: Let $\beta_i (i = 1, 2, \ldots . m)$"be a collection of Square root number, let $SR - \mathrm{FWG}: \mathrm{R}^n \to \mathrm{R}$, if $SR - FWG\ (R_1, R_2, R_3 \ldots . R_m) = \left(\prod_{i=1}^{m} (\beta_{(i)})^{w_i}\right) = \left(\prod_{i=1}^{m} (\gamma_{R_i})^{w_i}, \prod_{i=1}^{m} (\varphi_{R_1})^{w_i}\right)$

Then SR - FWG is called Square Root Fuzzy Weighted Geometric operator of dimension m, where $w = (w_1, w_2, \ldots \ldots . w_n)^T$ is the weight vector of $\beta_i (i = 1, 2, \ldots m)$, $\sum_{i=1}^{n} w_i = 1, w_i \, \epsilon \, [0,1]$.

2.5 SR- FHG Operator for Group Decision Making

Definition: Let $\beta_i (i = 1, 2, \ldots . m)$be a collection of square root fuzzy numbers. A Square Root fuzzy hybrid geometric (SR-FHG) operator of dimension m is a mapping $SR - \mathrm{HG}: \mathrm{R}^n \to \mathrm{R}$, if SR-FHG$(\beta_1, \beta_2, \ldots \ldots . \beta_n) = \prod_{i=1}^{m} (\beta_{\sigma_{(i)}})^{w_i} = \left(\prod_{i=1}^{m} \left(\gamma_{R_{\sigma_{(i)}}}\right)^{w_i}, \prod_{i=1}^{m} \left(\varphi_{R_{\sigma_{(i)}}}\right)^{w_i}\right)$where $(\sigma_1, \sigma_2, \ldots . \sigma_m)$ is a permutation of $(1, 2, \ldots . m)$that contains$\beta_{\sigma_{(i-1)}} \geq \beta_{\sigma_{(i)}}$ for all i , &$w = (w_1, w_2, \ldots \ldots . w_m)^T$ is the weighted vector"of $\beta_i (i = 1, 2, \ldots n)$, $\sum_{i=1}^{n} w_i = 1, w_i \, \epsilon \, [0,1]$.

3. Algorithm for MAGDM Problems

Step 1: Both the SR - FWG operator and the square root fuzzy decision matrix are “provided.

$r_{ij}^{\sim(k)} = \left(\gamma_{ij}^{(k)}, \varphi_{ij}^{(k)}\right) = SR - FWG_w\left(r_{i1}^{\sim(k)}, r_{i2}^{\sim(k)}, \ldots\ldots. r_{im}^{\sim(k)}\right)$here$(i = 1, 2, \ldots. m)$; $k = 1,2, \ldots \ldots$ to derive the individual value $r_i^{\sim(k)}$.

Step 2: Apply the SR- FHG operator, to derive the overall square root fuzzy values

$r_i^{\sim}, (\, i = 1,2, \ldots n)$ of the alternative.

Step 3: Employing Canberra Distance Formula, to compute the distance between the”two values (γ_i, φ_i).

Step 4: Rank the alternatives and arrange them in ascending “order.

3.1 To Find Weight by using Runge Kutta Method:

Butcher‘s Fifth Order Runge-Kutta Method:

The Initial Value Problem is given $\frac{dy}{dx} = f(x, y), y(x_i) = y_i$

$$K_1 = f(x_n, y_n)$$

$$K_2 = f\left(x_n + \frac{1}{4}h, y_n + \frac{1}{4}K_1 h\right)$$

$$K_3 = f\left(x_n + \frac{1}{4}h, y_n + \frac{1}{8}K_1 h + \frac{1}{8}K_2 h\right)$$

$$K_4 = f\left(x_n + \frac{1}{2}h, y_n - \frac{1}{2}K_2 h + K_3 h\right)$$

$$K_5 = f\left(x_n + \frac{3}{4}h, y_n + \frac{1}{16}K_1 h + \frac{9}{16}K_4 h\right)$$

$$K_6 = f\left(x_n + h, y_n - \frac{3}{7}K_1 h + \frac{2}{7}K_2 h + \frac{12}{7}K_3 h - \frac{12}{7}K_4 h + \frac{8}{7}K_5 h\right)$$

$$y_{(n+1)} = y_n + \frac{h}{90}(7K_1 + 32K_3 + 12K_4 + 34K_5 + 7K_6)$$

Problem proposed by Decision maker 1:

For the Initial Value Problem $\acute{u} = xy, x(0) = 1\, with\, h = 0.1$ on the interval [0,0.7] use the Fifth Order Runge Kutta”Method.

Solution: Given $\acute{u} = xy, x(0) = 1\, with\, h = 0.1$

For $j = 0, x_0 = 0, y_0 = 1$

$K_1 = (0)(1) = 0$

$K_2 = 0.03 * 1 = 0.025$

$K_3 = 0.03 * 1.0003125 = 0.0250078$

$K_4 = 0.05 * 1.00125078 = 0.0500625$

$K_5 = 0.08 * 1.00281602 = 0.0752112$

$K_6 = 0.1 * 1.00501476 = 0.1005015$

The approximate value of $Y_1 = y(0.1) = 1.0051796$

Similarly, we have $Y_2 = y(0.2) = 1.0104377$

$Y_3 = y(0.3) = 1.015855541$

$Y_4 = y(0.4) = 1.021519538$

$Y_5 = y(0.5) = 1.027524929$

$Y_6 = y(0.6) = 1.03397885$

$Y_7 = y(0.7) = 1.041004426$

The "Weighting Vectors are $W = (0.1405, 0.1412, 0.1420, 0.1428, 0.1436, 0.1445, 0.1455)$

Problem proposed by Decision Maker 2:

Utilizing the Third and Fourth-Order Runge Kutta Method, solve the Initial Value" Problem

$\frac{dy}{dx} = (1 + xy), y(0) = 2 \, with \, h = 0.1.$

Runge Kutta Third Order method:

$K_1 = 0.1(1 + 0 * 2) = 0.1$

$K_2 = 0.1(1 + 0.1025) = 0.110025$

$K_3 = 0.1(1 + 0.10276) = 0.1212005$

$Y_1 = y(0.1) = 2.011021675$

Similarly, we have $Y_2 = y(0.2) = 2.048427733$

$Y_3 = y(0.3) = 2.119583225$

The Weighting Vectors are $\gamma = (0.32546, \; 0.33151, \; 0.34303)$

Runge Kutta Fourth Order Method:

$K_1 = 0.1(1 + 0.2) = 0.1$

$K_2 = 0.1(1 + 0.10025) = 0.110025$

$K_3 = 0.1(1 + 0.10276) = 0.110276$

$K_4 = 0.1(1.2110276) = 0.12110$

$Y_1 = y(0.1) = 2.110284$

Similarly, we have $Y_2 = y(0.2) = 2.22922$

$Y_3 = y(0.3) = 2.36960$

The Weighting Vectors are $w = (0.31454, \; 0.33227, \; 0.35319)$

4.Numerical Illustration

The next section will provide an example of the new strategy in a decision-making dilemma. Consider this: in order to provide for their infant, people prefer to purchase high-quality brands of baby supplies. At the outset, consumers weigh three potential options: Little's Soft Brand (A_3), Himalaya Brand (A_2), and Mama Earth Brand (A_1). The people utilize a panel of experts to assess these leading brands. According to this team of experts, the economic circumstances play a crucial role. Following thorough examination, they take into account seven potential attributes: S1: Price, S2: Excellence, S3: Construction and Materials, S4: Cleaning Ease; S5: Safety requirements; S6: Versatility; S7: Sensory Stimulation. Under the aforementioned seven qualities and the weighting vector, under the aforementioned three options of the decision makers and build, respectively, the three potential alternatives (A1, A2, & A3) are to be computedemploying the square root fuzzy numbers by weighting vector. The decision matrices (3×7) are
$W = (0.1405, 0.1412, 0.142, 0.1428, 0.1436, 0.1445, 0.1454)^T$ and the weight vector
$\gamma = (0.3255,\ 0.3315,\ 0.343) w = (0.3145,\ 0.3323,\ 0.3532)$ respectively.

$$R_1 = \left(\begin{bmatrix} (0.7,0.2)(0.4,0.5)(0.4,0.7)(0.5,0.5)(0.4,0.5)(0.7,0.1)(0.8,0.1) \\ (0.6,0.3)(0.3,0.6)(0.3,0.8)(0.6,0.4)(0.4,0.7)(0.6,0.1)(0.7,0.2) \\ (0.8,0.1)(0.5,0.4)(0.5,0.5)(0.6,0.4)(0.7,0.2)(0.8,0.1)(0.5,0.4) \end{bmatrix}\right)$$

$$R_2 = \left(\begin{bmatrix} (0.5,0.5)(0.8,0.1)(0.4,0.2)(0.4,0.7)(0.7,0.1)(0.4,0.5)(0.4,0.5) \\ (0.6,0.4)(0.7,0.2)(0.6,0.3)(0.3,0.8)(0.6,0.1)(0.3,0.6)(0.4,0.7) \\ (0.6,0.4)(0.5,0.4)(0.8,0.1)(0.5,0.5)(0.8,0.1)(0.5,0.4)(0.7,0.2) \end{bmatrix}\right)$$

$$R_3 = \left(\begin{bmatrix} (0.8,0.1)(0.4,0.7)(0.7,0.1)(0.4,0.5)(0.5,0.5)(0.7,0.2)(0.7,0.1) \\ (0.7,0.2)(0.3,0.8)(0.6,0.1)(0.3,0.6)(0.6,0.4)(0.6,0.3)(0.6,0.1) \\ (0.5,0.4)(0.5,0.5)(0.8,0.1)(0.5,0.4)(0.6,0.4)(0.8,0.1)(0.8,0.1) \end{bmatrix}\right)$$

Step 1: Apply the SR -FWG operator

$$\tilde{r_{11}} = [(0.7)^{0.1405} * (0.4)^{0.1412} * (0.4)^{0.142} * (0.5)^{0.1428} * (0.4)^{0.1436} * (0.7)^{0.1445} * (0.8)^{0.1454}, (0.2)^{0.1405} * (0.5)^{0.1412} * (0.7)^{0.142} * (0.5)^{0.1428} * (0.5)^{0.1436} * (0.1)^{0.1445} * (0.1)^{0.1454}]$$

$\tilde{r_{11}} = (0.535735076, 0.289150531)$

Similarly,

$\tilde{r_{12}} = (0.47572796, 0.359930607)$

$\tilde{r_{13}} = (0.61582666, 0.251777408)$

$\tilde{r_{21}} = (0.53405533, 0.291247509)$

$\tilde{r_{22}} = (0.47370976, 0.362357101)$

$\tilde{r_{23}} = (0.616087445, 0.251287656)$

$\tilde{r_{31}} = (0.579755416, 0.230671119)$

$\tilde{r_{32}} = (0.503583859, 0.273088301)$

$\tilde{r_{33}} = (0.628797174, 0.226821002)$

Step 2: Use the SR -FHG operator with the current Square root fuzzy matrix.

$r_1 = (2.80743403, 2.59085102)$

$r_2 = (2.76801091, 2.64937395)$

$r_3 = (2.844962551, 2.56267982)$

Step 3: Compute the Canberra distance formula between the two values of Collective matrices.

$d(x_1^{\sim}, y_1^{\sim}) = 0.040120706$

$d(x_2^{\sim}, y_2^{\sim}) = 0.021899304$

$d(x_3^{\sim}, y_3^{\sim}) = 0.052200703$

Step 4: Rank all alternative solutions, $A_i (i = 1, 2, 3)$.

$$A_2 < A_1 < A_3$$

As a result, the best option is A_2.

$A_2$is Himalaya Brand.

5. Conclusion

Numerical solutions are obtained by the use of Runge-Kutta Methods. To determine the best choice, the Multiple Attributes Group Decision Making Problem is examined. Weights are determined in this manner by applying Runge-Kutta Methods to a dataset and making use of the Square Root Fuzzy Hybrid Geometric and Square Root Fuzzy Weighted Geometric operators. The best alternatives are selected using the Canberra distance formula. We have demonstrated the feasibility of this proposed method.

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