FINDING EIGEN VALUES FOR BUTTERFLY GRAPHS

R. Malathi

Asst.Prof.of Mathematics, SCSVMV, Kanchipuram.

---***---

Abstract - In the mathematical field of graph theory the butterfly graphs (also called bowtie graphs and hourglass graphs) are planar undirected graphs with 5 vertices and 6 edges. The n – dimensional butterfly graphs, denoted by BF(n) has vertex set v = {(x; i): x ϵ V(Q_n), $0 \leq i \leq n$}. Two vertices (x; i) and (y; j) are liked by an edge in BF(n) if and only if j = i+1 and either x = y, or x differ from y in precisely the j^{th} bit for x = y, the edge is said to be straight edge, otherwise the edge is a cross edge, for fixed i, the vertex (x, i) is a vertex on level i. the n- dimensional butterfly graphs has $2^n(n+1)$ vertices and $2^{n+1}n$ edges. Butterfly graphs and domination are very important ideas in computer architecture and communication technique. In this paper, we present results about Eigen values of butterfly graphs BF(n). We also show the Eigen value through adjacency matrix of butterfly graphs BF(n) are related to each other as a (BF(n))= (BF(n)) + 1. If n = 2, 3, 4 k.a (BF(n)) = (BF(n)) if n6 = 4k.

Key Words: Graphs, planar graphs, butterfly graphs, butterfly network, Edge, Vertex, Eigen values.

1. INTRODUCTION

Graph theory itself is typically dated as beginning with Leonhard Euler's 1736 work on the seven bridges of Konigsberge. However drawing of another most popular bounded degree derivate network of the hypercube is what is called a butterfly network.

Butterfly network have been extensively studied in the literature because of their many application in computer architectures. In the undirected case, aspect that have been considered include, for example the study of the cycle structure and hamiltonicily[1,4,11] and the development of communication and routing algorithms[10,12,6-8]. Some of these results have been recently extended also the directed case[2,3]. The knowledge of this spectrum will facilitate a new approach for a better characterization of this important category of network. Throughout this paper we consider finite, simple and undirected graph G = (V(G),E(G)) with p vertices and q edges. G is also called a(p,q) graph. We follow the basic notations and terminologies of graph theory as in [5].

2. EIGENVALUES OF GRAPHS

The Eigen value problems appear in various contexts throughout mathematics and engineering and refer to determining all possible list of Eigen value(spectra) for matrices fitting some description. Graphs often describe relationship in a physical setting, such as control of a system and the Eigen value of associated matrices govern the behavior of the system. The Eigen value problem of a given graph G is to determine all possible spectra of real symmetric matrices whose off-diagonal entries are governed by the adjacencies in G. It was thought by many researchers in the field that atleast for a tree T, determining the ordered multiplicity list of T would suffices to determine the spectra of matrices describe by T. when it was show in [9] that this was not the case, the focus of much of the research in the area shifted to the narrower question of maximum Eigen value multiplicity or equivalently maximum nullity or minimum rank of matrices described by the graphs . The Eigen values of the graphs are closely tied to the Eigen values of a graph. while you do not always need to find the Eigen values to find Eigen values in some cases using the structures of a graphs we can construct the eigenvector and thus find the Eigen values.

A famous question in spectral graph theory is "can you hear the shape of a graphs? That is given the Eigen values can you determine the graphs that produced them.

SPECTRAL GRAPH THEORY

One of the most useful ways of doing this has been by studying the various spectra of matrices (that is the Eigen values of the matrices) that can be associated with the graph. By facing at these Eigen values it is possible to get information about the graphs that might otherwise be difficult to obtain the study of the relation between Eigen values is the heart of spectral graphs theory thus someone interested in using spectral graphs theory needs to be familiar both with graph theory and the basic tools of liner algebra including Eigen values ,Eigen vectors, determinants , the courant-fischer theorem, the perron-frobenius theorem and so on…

we will introduce the most common matrices associated with graphs namely the adjacency matrix and given some simple example for each about how the Eigen values can be used to give some information about the graphs.

International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 04 Issue: 03 | Mar -2020 ISSN: 2582-3930

THE ADJACENCY MATRIX

Given a graph G we can form a matrix A called the adjacency matrix, by letting the vertices index the columns and rows and then letting

$$A_{ij} = \begin{cases} 1 & \text{If } i \text{ is adjacent to } j \\ 0 & \text{if } i \text{ is not adjacent to } j \end{cases}$$

Let G be a simple graph of order n and let A(G) be its adjacency matrix . The Eigen values $\lambda_1, \lambda_2, \ldots \lambda_n$ of A(G) are referred to as the Eigen

values of the graph G and form the spectrum of G [19][7].

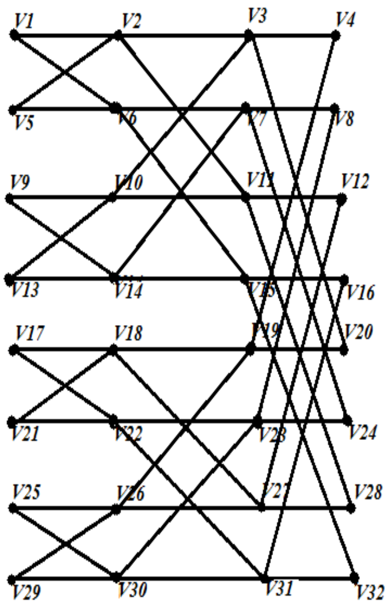
BUTTERFLY NETWORK

The **n-dimensional** butterfly network denoted by BF(n), has vertex set $V=\{(x;i):X \in V(Q_n), 0 \le i < n\}$.

The vertices (x;i) and (y;j) are linked by an edge in BF(n) if and only if j = i + 1 and either

i) X= Y
ii) X differ from y in precise the j^{th} bit.

For x= y, the edge is said to be straight edge, otherwise the edge is a cross edge for fixed I, the vertex (x,i) is vertex on level i. the graphs is show in BF(3)



BF(3)

The **n-dimensional** butterfly graphs has $\mathbf{2^n(n+1)}$ vertices and $\mathbf{2^{n+1}.n}$ edges.

The butterfly network BF(n) has $2^n(n+1)$ vertices because BF(n) has n+1 levels and there are 2^n vertices in every level . each vertex on level 0 and n is of degree 2 , otherwise, every is of degree 4. It is clear that
BF(n) is eulerian since it is connected and has no odd degree vertices[6].

BUTTERFLY NETWORK BF(r)

The definition of the butterfly network BF(r) is as follows: The r-dimensional butterfly has $(r+1)2^r$ nodes and $r.2^{r+1}$edges. The set V of nodes of an r-dimensional butterfly correspond to pairs [w,i], where i is the level of a node $(0 \le i \le r)$ and w is an r-bit binary number that denotes the row of the node. Two nodes $< w, i >$ and $< w_o, i_o >$ are linked by an edge if and only if $i_o = i + 1$ and either w and w_o are identical, or w and $w_o$differ in precisely the ith bit. An r-dimensional butterfly is denoted by BF(r).

Theorem : Let G be an r-dimensional butterfly network. Then $\beta(G) = \left\lceil \frac{r+1}{2} \right\rceil .2^r$

Proof : Let G be an r-dimensional butterfly network.

Case(i): If r is even then the number of levels which is r + 1 is an odd number. Therefore we require exactly $\left\lceil \frac{r+1}{2} \right\rceil .2^r$ vertices to cover all edges.

Case(ii): If r is odd then the number of levels which is r + 1 is an even number. So we need exactly $\left\lceil \frac{r+1}{2} \right\rceil .2^r$ vertices to cover all the edges.

Combining both the cases, $\beta(G) = \left\lceil \frac{r+1}{2} \right\rceil .2^r$

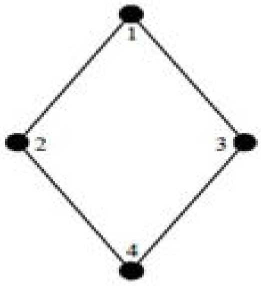
Corollary : Let G be an r-dimensional butterfly network. Then (i) If r is odd, then $\beta(G) = \frac{|V|}{2}$. (ii) If r is even, then $\beta(G) < \frac{|V|}{2}$.

Theorem: Let G be an r-dimensional butterfly network. Then $\beta(\overline{G}) = (r+1).2^r - 2.$

EIGEN VALUE WITH BUTTERFLY GRAPHS

BUTTERFLY GRAPH(1)

$$\textbf{Adjacent matrix}=\begin{bmatrix}0&1&1&0\\1&0&0&1\\1&0&0&1\\0&1&1&0\end{bmatrix}$$



$$\det(\lambda I-A)=\det\left[\lambda\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}-\begin{pmatrix}0&1&1&0\\1&0&0&1\\1&0&0&1\\0&1&1&0\end{pmatrix}\right]$$

$$\det(\lambda I-A)=\det\left[\lambda\begin{pmatrix}\lambda&0&0&0\\0&\lambda&0&0\\0&0&\lambda&0\\0&0&0&\lambda\end{pmatrix}-\begin{pmatrix}0&1&1&0\\1&0&0&1\\1&0&0&1\\0&1&1&0\end{pmatrix}\right]$$

$$\det(\lambda I-A)=\det\begin{bmatrix}\lambda&-1&-1&0\\-1&\lambda&0&-1\\-1&0&\lambda&-1\\0&-1&-1&\lambda\end{bmatrix}$$

$$\det(\lambda I-A)=\lambda\begin{vmatrix}\lambda&0&-1\\0&\lambda&-1\\-1&-1&\lambda\end{vmatrix}-1\begin{vmatrix}-1&0&-1\\-1&\lambda&-1\\0&-1&\lambda\end{vmatrix}-1\begin{vmatrix}-1&\lambda&-1\\-1&0&-1\\0&-1&\lambda\end{vmatrix}+0\begin{vmatrix}-1&\lambda&0\\-1&0&\lambda\\0&-1&-1\end{vmatrix}$$

$$\det(\lambda I-A)=\lambda\left[\lambda(\lambda+1)-1(0-\lambda)\right]-1\left[-1(\lambda+1)-1(1)\right]-1\left[-1(1)-\lambda(-\lambda-0)-1(1-0)\right]$$

$$\det(\lambda I-A)=\lambda\left[\lambda^2+\lambda+\lambda\right]-1\left[-\lambda-1-1\right]-1\left[-1+\lambda^2-1\right]$$

$$\det(\lambda I-A)=\lambda\left[\lambda^2+\lambda-\lambda+\lambda+1-0+1-1+\lambda^2+1\right]$$

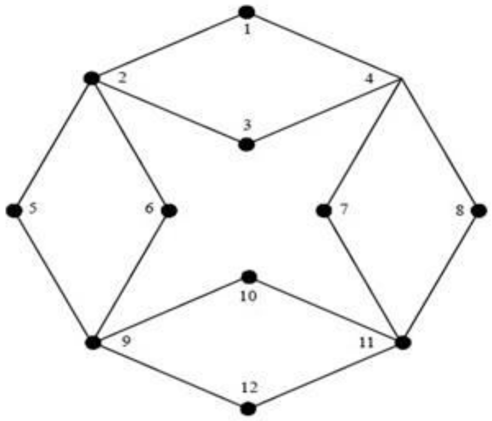
$$\det(\lambda I-A)=\lambda\left[2\lambda^2+\lambda+2\right]$$

$$\lambda=-2,0,0,2$$

Eigen value λ: -2, 0, 0, 2
Maximum value: 2

Using matlab we can find Eigen values for **n vertex**
Now we can find Eigen value for **4 vertex**

BUTTERFLY GRAPH(2)



Adjacent matrix =

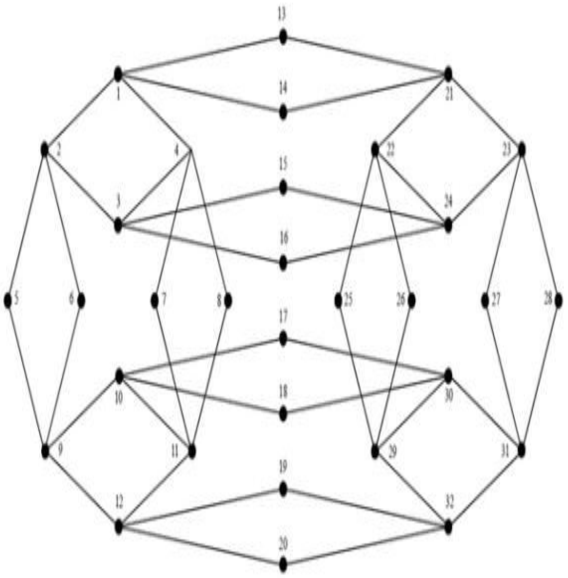
$$\begin{bmatrix}0&1&0&1&0&0&0&0&0&0&0&0\\1&0&1&0&1&1&0&0&0&0&0&0\\0&1&0&1&0&0&0&0&0&0&0&0\\1&0&1&0&0&0&1&1&0&0&0&0\\0&1&0&0&0&0&0&0&1&0&0&0\\0&1&0&0&0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0&0&0&1&0\\0&0&0&1&0&0&0&0&0&0&1&0\\0&0&0&0&1&1&0&0&0&1&0&1\\0&0&0&0&0&0&0&0&1&0&1&0\\0&0&0&0&0&0&1&1&0&1&0&1\\0&0&0&0&0&0&0&0&1&0&1&0\end{bmatrix}$$

Eigen value λ: 2, -2, -2, 0, 0, 0, 0, 0, 0, 2, 2, 2
Maximum value λ: 2

We can find Eigen values for **12 vertex**

Eigen values for **12 vertex using matlab**

BUTTERFLY GRAPHS (3)



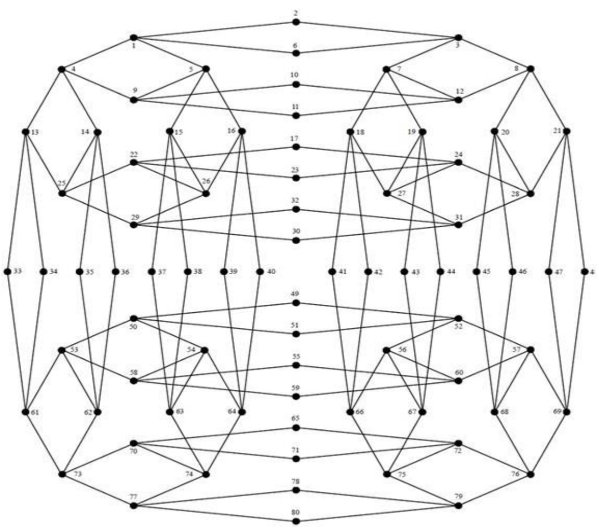
Find Eigen value for **32 vertex**

we could not able to find 32*32 matrix ,so we can using matlab to find adjacent matrix to calculate the Eigen value

Eigen value λ: -3, -2, -2, -2, -2, -2, -2, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 2, 3

Maximum value λ: 3

BUTTERFLY GRAPHS(4)



Find Eigen value for **80 vertex**

we could not able to find 80*80 matrix ,so we can using matlab to find adjacent matrix to calculate the Eigen value.

Eigen value λ: -3,- 3, -3, -3, -2, -2, -2, -2, -2, -2, -2, -2, -2, -2, -2, -1, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3.

Maximum value λ : 3

3. CONCLUSIONS

In the mathematical fields of graph theory, The **Butterfly graphs $B_{n,m}$** is a graph obtained from two cycles of the same order sharing a common vertex with an arbitrary number of pendant edges attached at the common vertex here n, m are any two positive integer. We found that Eigen values for butterfly graph BF(n) is **centre zeros becomes n/2** for n vertex. Further more, we have computed generalized an Eigen values for many graphs such that crown graphs, star graphs ,cocktail party graphs, friendship graphs,.etc,,…This Butterfly useful in networking system.

ACKNOWLEDGEMENT

I express my deep sense of gratitude to Dr.T.Venugopal, Professor of Mathematics, SCSVMV, Enathur, Kanchipuram

REFERENCES

1. D. Barth and A. Raspaud , Two edge- disjoint Hamiltonian cycles in the butterfly graphs , inform process left 51(1994),175-179.
2. J.C.Bermond , E.Darrot o. Delmas and s. Pernnes Hamilton circuits in the directed wrapped butterfly network Discr appl math 84(1-3)(1998),21-42.
3. J.C.Bermond , E.Darrot o. Delmas and s. Pernnes Hamilton cycle decomposition of the butterfly network,parallel process left 8(1998),371-385.
4. S.C.Hwang and G-H .chen, cycles in butterfly graphs , Network 35(2000),161-171.
5. F.Hararay ,Graph theory, Addison Wesley.Massachusetts,1972.
6. R.Klasing B.Monien R.peine, and E.A Stohr, Broad- casting in butterfly and de Bruijn network, discr appl math 53(1994) ,183-197.
7. D.Cvetkovic .P.Rowlinson.S.K Simic, An introduction to the theory of graphs spectra Cambridge unir.press, Cambridge 2010.
8. B.M.Maggs and B.Vocking improved routing and sorting on multi butterflies. Algorithmica 28(2000),438-437.
9. F.Barioli and S.M Fallat on two conjectures regarding an inverse Eigen values problems for acyclic symmetric matrices .Elec.j.LIN.alg.11:41-50,2004.
10. R.cole,B.Maggs and R.sitaraman,routing on butterfly network with random faults,pro IEEE annu SYMP on foundation of computer sciences ,1995,pp.558-570

11. S.A Wong,Hamilton cycles and path in butterfly graphs network 26(1995),145-150.
12. F.Comells and G.Gimenz Genetic programming to design communication algorithms for parallel architectures,parallel process left 8(1998),549-560.
13. D.Crctkoric ,M.Doob H.sachs, spectra of graphs- theory and applications, academic press, new York,1980.
14. R.Diekmann. A.Frommer and B.Monier ,Efficient schemes for nearest neighbor load balaincing parallel compaut 25(1999),789-812.
15. W.Barrett,S.M Fallat H.T.Hall,L.Hogbon J.C.HLin B.Shader. Generalaiztion of the strong Arnold property and the minimum number of distinct Eigen values of a graph. Electron J.combin,24:p2.40 2017.
16. L.Hager and A Kahng Fast spectral methods for ratio cut partitioning and clustering int conf on computer –aided design ,1991,ICCAD-91,IEEE Dig techn psp.1991,pp 10-13.
17. L.Hager and A Kahng . New spectral methods for ratio cut partitioning and clustering, IEEE trans CAD\ICS 11(1992), 1074-1085.
18. D.Crctkoric ,M.Doob H.sachs, spectra of graphs- theory and applications, academic press, new York,1980.
19. S.C.Hwang and G-H .chen, cycles in butterfly graphs , Network 35(2000),161-171.

BIOGRAPHY

Dr.R.Malathi, M.Sc., M.Phil., Ph.D in Mathematics. Published 23 papers and Presented 25 papers in both National and International Level. Editorial Board Member in IJESIRD, JECET and GJRF. Have acted as Book Editor, Associate Editor, Judge, Chairperson, Deputy Warden, Resource Person, Organizing Secretary, etc. Membership in AMTI, ISIAM and IAENG. Have published 3 books with ISBN and 5 books without ISBN in National and International level. Have received 7 awards in which 2 are International. Research Work entitled, “Design of knowledge discovery and decision support system using Jaina logic”. Guided 28+ M.Phil and M.Sc students. Participated in Various Activities actively like thesis scrutinizer, reviewer of International Conference, ICACM, Answer Script Evaluator for PG/UG, Research co-ordinator, Anti-Ragging and hospitality committee, ICC, etc.