Free Vibration Analysis of Isotropic and Orthotropic Cylindrical Shells using Finite Element Method

S. M. Shiyekar[1] and V. P. Patil[2]

1 Professor, D Y Patil College of Engineering, Akurdi, Pune

shiyekar@gmail.com

2 Former PG Student of Rajarambapu Institute of Technology, Rajaramnagar, Sangli

vishu_p4@yahoo.co.in

ABSTRACT

The understanding of fundamental natural frequencies and mode shapes is crucial for assessing the resonant behavior of structures. Numerous scholars have investigated the free vibration properties of circular cylindrical shells. These cylindrical shell configurations find applications in diverse engineering fields such as aviation, rocketry, missiles, electric motors, and locomotive engines. Within this paper, an analytical solution is derived using the "first order shear deformation theory" to determine the frequency characteristics of free vibrations in symmetric laminated circular cylindrical composite shells. The outcomes are juxtaposed against classical and higher order theories. The central aim of this research is to scrutinize the influence of various parameters on the frequency traits of laminated composite shells. Furthermore, the analysis extends to laminated composite shells employing diverse stiffening methods, with their free vibration behavior explored through the utilization of FEA software ANSYS.

Keywords: *Cylindrical shell, free vibration, frequencies, mode shapes, laminated composites.*

INTRODUCTION

In recent times, the availability of reinforced concrete has sparked interest in utilizing shells for roofing purposes. These slender shells trace their origin back to plates. While the majority of research on shell vibrations has concentrated on homogeneous isotropic shells, the issue of layered anisotropic shells has received comparatively less attention. Various theories, contingent upon the considered displacement patterns, have been utilized for the free vibration analysis of laminated composite panels. Classical lamination theory, which serves as the foundational framework for composite laminate analysis, overlooks the effects of out-of-plane stresses and strains. This limitation arises from the substantial disparity in stiffness properties between fibers and matrices in most developed composites, leading to significant influence from out-of-plane stresses and strains. Consequently, classical theory becomes inadequate for accurate analysis.

The first order shear deformation theory (FOST) and higher order shear deformation theory (HOST) incorporate the effect of transverse shear deformation, which is essential in certain scenarios. On the other hand, classical lamination plate theory (CLPT) disregards transverse shear deformation due to Kirchhoff's hypothesis, asserting that transverse normals remain straight during bending without elongation or contraction. This results in the neglect of transverse shear and normal strains in CLPT. In FOST, a first-order displacement field is assumed for transverse strain throughout the thickness, albeit actual transverse shear variations necessitate the incorporation of shear correction factors, conflicting with the zero shear stress condition at the laminate's boundaries and actual stress states through the layer thickness. Singh and Shen [1] proposed a variational full-field approach for analyzing the free vibration of open circular cylindrical laminated shells supported at specific points. They established a matrix-form differential equation based on the first order shear deformable theory of shells. Wenbin et al. [2] systematically reduced the dimensions of shell structures through the variational asymptotic method. This approach transformed two-dimensional equations into a nonlinear Reissner-Mindlin shell theory. Matsunaga [3] evaluated natural frequencies and buckling stresses of cross-ply laminated composite circular cylindrical shells, accounting for higher-order deformations including transverse shear, normal deformations, and rotatory inertia. Chandrashekhara and Nanjunda [4] presented an approximate three-dimensional elasticity solution for infinitely long cross-ply laminated circular cylindrical shell panels with simply supported boundaries, subjected to arbitrary discontinuous transverse loading. Chakravorty et al. [5] employed finite element methods to address free and forced vibration problems in isotropic and laminated composite shells with and without cutouts, utilizing eight-node isoparametric finite elements. Heyliger and Jilani [6] utilized the Ritz method to solve the weak form of motion equations for laminated anisotropic composite shells, considering various end conditions for free vibration. Nayak and Bandyopadhyay [7] explored free vibrations of laminated composite anticlastic doubly curved stiffened shells using finite element methods. Patel et al. [8] employed the finite element method for the free vibration analysis of laminated anisotropic composite conical-cylindrical shell structures. Namita and Bandyopadhyay [9] investigated nonlinear transient responses of laminated composite cylindrical and spherical shell panels with cutouts, utilizing a finite element model with eight-noded Co continuity. Viswanathan et al. [10] studied the free vibration of symmetric angle-ply laminated cylindrical shells of variable thickness, incorporating first order shear deformation theory through spline function approximation. Kant and Kommineni [11] presented higher order shear deformation theory for predicting linear and geometrically nonlinear transient responses of composite and sandwich laminated shells. Kant and coworkers [12-15] presented free vibration analysis, eliminating the use of shear correction coefficients, and accommodating isotropic, orthotropic, and layered anisotropic composite and sandwich laminates.

This paper addresses a numerical solution for the frequency characteristics governing the free vibrations of symmetric laminated composite and isotropic circular cylindrical shells. The obtained results from FEA software ANSYS [16] are compared with previously established findings from existing literature. The research delves into the impact of varying aspect ratios and different lamination schemes on the free vibration properties of shells.

NUMERICAL RESULTS AND DISCUSSION

This section involves the resolution of diverse numerical instances drawn from existing literature utilizing the Finite Element Software (ANSYS), with subsequent discussions centered around establishing the software's precision in the context of free vibration assessment of laminated composite circular cylindrical shells. The analysis uniformly adopts simply supported boundary conditions for all scenarios under examination. Modal analysis is executed through the ANSYS software, which leads to the determination of the fundamental frequency of the structure. To conduct this analysis, the Shell 181 element is employed, which integrates the utilization of the First Order Shear Deformation Theory within the ANSYS framework. The primary intent is to showcase the efficacy and adaptability inherent in the present formulations, which are substantiated through an extensive array of instances. These encompass both isotropic and laminated composite shells, all subject to simply supported boundary conditions, thereby illustrating the broad scope and effectiveness of the approach.

Example 1:

Simply supported isotropic circular cylindrical shells are taken for comparison of ANSYS results. Results from research carried out by Khalili et al. (2012) are taken for comparison of isotropic shells. Isotropic properties for analysis are E = 2.1 x 10^5, ρ = 7850 kg/m^3, ν = 0.3. Frequencies are normalized as $\bar{\omega} = {\omega h}/{\pi}\sqrt{{\rho}/{G}}$. Results are tabulated in Table 1.

Table 1 Comparison of lowest natural frequency parameters, $\bar{\omega} = {\omega h}/{\pi}\sqrt{{\rho}/{G}}$ for SS isotropic circular cylindrical shells for h/R= 0.06

L/R	Exact	HOST 12	RHOST 12a	RHOST 12b	RHOST 12c	PSDT	FOST	Present (ANSYS)
2	0.01853	0.01853	0.01853	0.01853	0.01853	0.01853	0.01853	0.01985
1	0.02781	0.02783	0.02780	0.02780	0.02780	0.02781	0.02781	0.02341
0.5	0.03691	0.03700	0.03688	0.03688	0.03688	0.03692	0.03692	0.03420

Example 2:

Results from Kant et al. (2007) for symmetric lamination scheme 0º/90º/90º/0º are compared. Orthotropic properties are taken as follows,

E_1 = 172.25 x 10^9 Pa, $E_2 = E_3$ = 6.89 x 10^9 Pa, $\nu_{12} = \nu_{23}$ =0.25 and ν_{13}= 0.01, $G_{12} = G_{13}$ = 3.445 x 10^9 Pa, G_{23} = 1.378 x 10^9 Pa, ρ = 1500 kg/m^3.

Frequency is normalized by parameter $\overline{\omega} = \frac{\omega L^2}{h}\sqrt{\rho/E_2}$.

Results are tabulated in Table 2 and are also shown graphically in Fig 3.

Table 2 Comparison of lowest natural frequency parameters, $\overline{\omega} = \frac{\omega L^2}{h}\sqrt{\rho/E_2}$ for SS Laminated circular cylindrical shells for lamination scheme 0º/90º/90º/0º (Kant et al. 2006)

R/L	h/L	HOST12	HOST11	HOST9	FOST	REDDY	Present ANSYS	Present FOST
1	0.01	66.69851	66.69851	66.69846	66.40413	66.70400	66.1256	66.5633
	0.10	12.69812	12.69959	12.69875	13.12521	13.12800	13.2154	12.6494
	0.25	7.057400	7.064030	7.073540	7.448070	--	7.5876	7.27304
2	0.01	36.85137	36.85137	36.85130	36.85796	36.85800	35.9786	36.8148
	0.10	11.99898	11.99947	11.99397	12.47130	12.47100	12.1345	12.3768
	0.25	7.149720	7.15183	7.14510	7.54375	--	7.5336	7.51297
3	0.01	27.16532	27.16532	27.16524	27.17311	27.173	27.7245	27.2012
	0.10	11.85583	11.85606	11.84944	12.33750	12.337	12.1172	12.3148
	0.25	7.17069	7.17167	7.16124	7.56461	--	7.4393	7.5543
4	0.01	22.73999	22.73999	22.73989	22.74878	22.749	22.2569	22.7678
	0.10	11.80458	11.80471	11.79767	12.28962	12.289	11.7651	12.3012
	0.25	7.17834	7.17890	7.16711	7.57217	--	7.3891	7.6047

5	0.01	20.35196	20.35196	20.35186	20.36151	20.361	20.4312	20.2718
	0.10	11.78065	11.78074	11.77350	12.26726	12.267	11.5670	12.4367
	0.25	7.18194	7.18230	7.16987	7.57571	--	7.216	7.6138
10	0.01	16.62277	16.62277	16.62264	16.63400	16.636	17.1276	16.7096
	0.10	11.74854	11.74856	11.74106	12.23726	12.236	11.1239	12.2906
	0.25	7.18679	7.18688	7.17359	7.58047	--	7.1435	7.6015
20	0.01	15.54744	15.54744	15.54731	15.55933	--	15.2198	15.6078
	0.10	11.74047	11.74048	11.73291	12.22972	--	11.0173	12.3124
	0.25	7.18801	7.18803	7.17453	7.58167	--	7.0929	7.6457

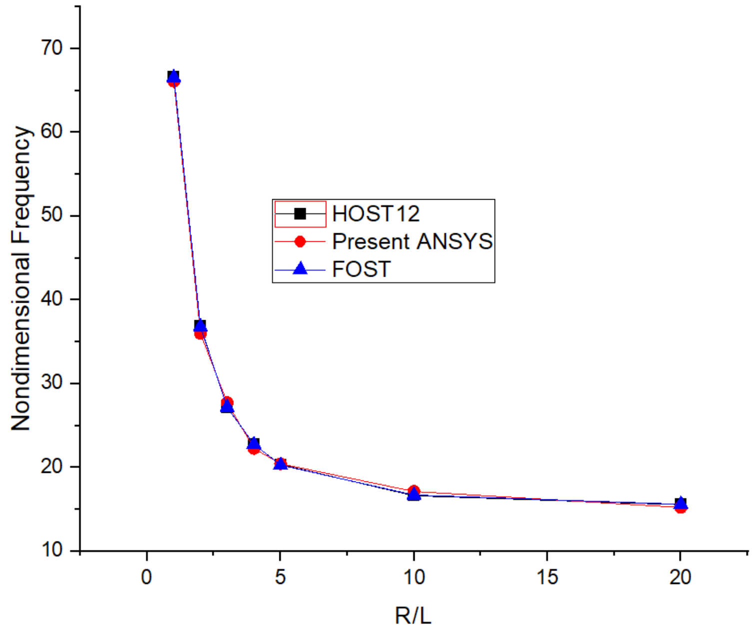

Fig 3 Graphical representation of frequency v/s R/L ratio for h/L = 0.01

Example 3:

With the same orthotropic properties from Example 2, results from Reddy (1984), are compared and tabulated in Table 3, for symmetric lamination scheme $0^o/90^o/0^o$. Results are also shown graphically in Fig 4. Frequencies are normalized to value $\varpi = \frac{\omega L^2}{h}\sqrt{\rho/E_2}$.

Table 3 Comparison of lowest natural frequency parameters, $\bar{\omega} = \frac{\omega L^2}{h}\sqrt{\rho/E_2}$ for SS Laminated circular cylindrical shells for lamination scheme 0º/90º/0º (Reddy 1984)

R/L	h/L	REDDY	Present ANSYS
1	0.01	125.99	124.7132
	0.10	16.115	17.6451
2	0.01	68.075	67.8150
	0.10	13.382	14.5671
3	0.01	47.265	46.1389
	0.10	12.731	11.8761
4	0.01	36.971	38.7600
	0.10	12.487	11.7236
5	0.01	30.993	30.1207
	0.10	12.372	11.1721
10	0.01	20.347	19.4689
	0.10	12.215	11.0108

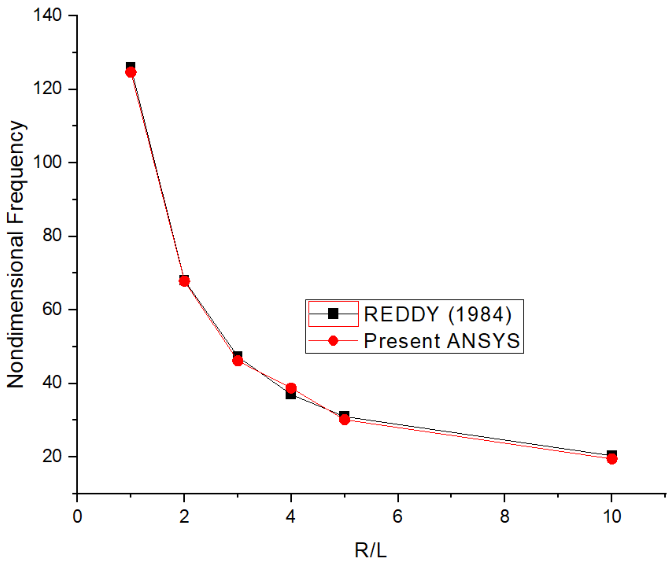

Fig 4 Graphical representation of frequency v/s R/L ratio for h/L = 0.01

CONCLUSION

This study presents ANSYS finite element method (FEM) solutions for investigating the free vibration characteristics of circular cylindrical shells. These shells can be either isotropic or laminated composites, and they are subjected to simply supported boundary conditions. The accuracy and relevance of the proposed method are established through a comprehensive comparison of results with those previously reported in the literature, encompassing a wide spectrum of thickness-to-radius and length-to-radius ratios. This involves a meticulous assessment of circular cylindrical shell responses obtained from ANSYS, which exhibits a noteworthy level of agreement with the published findings. Key findings and insights derived from the research are as follows:

For isotropic shells, a noticeable correlation is observed between an increase in the length-to-radius ratio and a subsequent decrease in frequency.

In the case of laminated shells, distinct trends emerge: when maintaining the length-to-radius ratio constant, heightened thickness-to-radius ratios result in increased frequencies. Similarly, with fixed thickness-to-radius ratios, elevating length-to-radius ratios lead to frequency reduction.

In summary, this investigation underscores the utility and accuracy of ANSYS FEM methods in analyzing the vibrational behavior of circular cylindrical shells, whether isotropic or laminated composites. The achieved congruence between the current study's results and the established literature outcomes further substantiates the validity of the presented approach.

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