Fuzzy Dijkstra Algorithm to Find the Shortest Path

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Abstract - Shortest path for crisp numbers can be found efficiently using the Dijkstra's algorithm. There are some uncertain conditions in the environment which introduces a fuzzy number for the distances between two nodes. A classical Dijkstra algorithm is modified to fuzzy Dijkstra algorithm to address the problem. Graded mean integration is used to enhance the algorithm by calculating and comparing the path length between two nodes when the arc length is fuzzy. A sample network is shown to understand the shortest path for fuzzy numbers.

Key Words: dijkstra's algorithm, fuzzy numbers, graded mean integration

1.INTRODUCTION

It is necessary to meet the customers requirements while making use of the resources and find the shortest path for any network. There are a number of algorithms which are used to find the shortest path. Shortest path could be found using classical Dijkstra algorithm. However the challenge occurs when the factors affecting the cost is fuzzy. A fuzzy number is the one whose possible values residing between 0 and 1 has some weight known as the membership function. In order to address the concept related to fuzzy logic, soft computing is used. Computational intelligence is used at times when there is no exact solution to a problem and there is no algorithm known that can give the right solution in a given time. Applied soft computing is used to deal with problems related to fuzzy logic. Hence soft computing is used to evaluate the shortest path using Dijkstra algorithm under uncertain environment like inconsistency with respect to information that exist in real world.

2. Body of Paper

Various works have been carried out by authors in the field of soft computation. For instance if a person has to travel from one city to another city by car he might take c hours. However if the road is bad and there is some problem with the car then the person might take c+3 hours to reach the other city. Such situations occur in communication as well. There real time problems can be solved by modifying the classical Dijkstra algorithm such that it deals with fuzzy sets [1]. A number of methods have been given to deal with the fuzzy numbers. Zadeh [2] describes an approach that use algorithms for fuzzy numbers with an outline of branching questionnaire. A fuzzy relation can be obtained by representing it in an algebraic form.

It is observed that the shortest path is not just confined to the geographical sense, it needs to be measured using parameters like capacity, cost and time. It leads to problem where shortest path has to be found out using the least cost in optimal time. The shortest path calculation becomes complicated with the involvement of fuzzy numbers when compared with respect to crisp numbers. In such cases defuzzification function is applied using Yager's ranking index which converts trapezoidal numbers to crisp numbers[3]. The number of parameters affecting the shortest path may vary from one network to the other.

This paper mainly concentrates on finding the shortest path between two nodes under uncertain environment. A graded mean integration method is applied which supports the fact that the addition and multiplication of a triangular fuzzy numbers yields a crisp number in a canonical representation[4]. The result can be calculated for trapezoidal fuzzy numbers as well by considering four parameters as factors that affect the path in a network. In addition to that, the efficiency of the program can be improved by using better defuzzification techniques.

Fig -1: Block Diagram

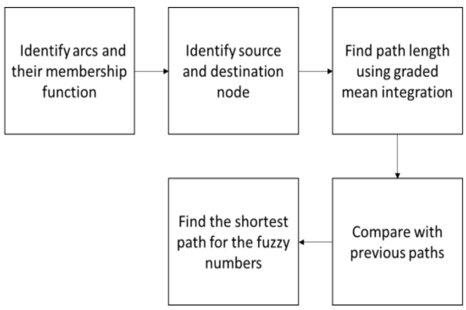

Fig.1 shows the block diagram for the method carried out to find the shortest path for fuzzy numbers. For a given network, it is necessary to identify the arcs and the membership functions associated with it based on some parameters that affect the shortest path. For a given source node and a destination, shortest path has to be determined. Since the environment is uncertain, the arcs are fuzzy numbers instead of crisp numbers. Dijkstra algorithm can be modified as fuzzy Dijkstra algorithm for finding the shortest path. Canonical representation of triangular fuzzy numbers are subjected to addition and multiplication which results in a crisp number [4].

For any set Q, the node with shortest distance from source is represented as u and v as a variant. Initially the distance from the source node is considered to be zero, distance from the source node to the inaccessible node is considered to be

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infinity and the previous distance for v is considered to be zero. If Q is not empty and there exists a path from source to node u, then remove the nodes which are not accessible from the source node. An alternate distance is calculated using the distance from source to node u and the distance between u and v by applying the graded mean integration.

If there exists an alternate distance less than the distance from source node to node v then replace the distance to v with alternate distance and store the previous distance for v to u. Finally select the shortest distance from source to v and replace it with u and repeat the same step until the destination node is met. The shortest path to each destination node from the source node is calculated. This paper can be modified by considering the trapezoidal fuzzy number between two nodes to find the shortest path in a network by applying the fuzzy Dijkstra algorithm.

Fig -2: A Network

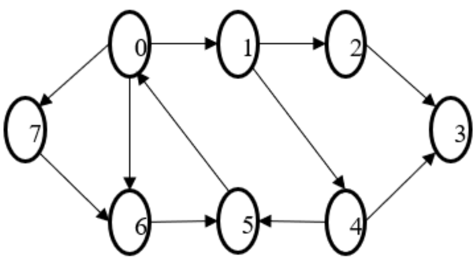

Table 1

Arc length of a network

Arc	Membership function
(0,1)	(10,13,15)
(0,7)	(8,10,12)
(0,6)	(9,12,14)
(1,2)	(8,11,13)
(1,4)	(10,12,14)
(2,3)	(7,10,13)
(4,3)	(12,14,16)
(4,5)	(11,13,15)
(5,0)	(7,9,12)
(6,5)	(11,13,15)
(7,6)	(8,10,13)

For the given directed graph that represents a network, the membership function is mentioned in table 1. Operations are performed on triangular fuzzy number to evaluate the shortest distance between each node from the source node. The membership as can be shown as below for Triangular fuzzy number Ã denoted by (a, b, c) .

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ \frac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases} \qquad (1)$$

$$P(\tilde{A}) = \frac{1}{6}(a1 + 4 \times a2 + a3) \qquad (2)$$

For node 4 from 0 there are two paths. One path could be from 0→1→4→5 and 0→6→5. For first path , the path length can be calculated as follows.

Arc(0,5)

= Arc(0,1) + Arc(1,4) + Arc(4,5)

= (10,13,15) + (10,12,14) + (11,13,15)

= $\frac{1}{6}(a1 + 4 \times a2 + a3)$

Graded mean integration representation of triangular fuzzy numbers are obtained for each arc which represents the arc length which is a fuzzy number. Fuzzy Dijkstra's algorithm is used to find the shortest path in uncertain environment.

It is analyzed that though the path between two nodes are subjected to a numbers of parameters due to uncertain conditions in the environment, it is possible to find the shortest path by using appropriate fuzzy Dijkstra algorithm. For the network the shortest path is shown in Fig 3.

```
|0.0|12.83|23.66|33.66|24.83|24.83|11.83|10.0|

Path = 1 <- 0
Path = 2 <- 1 <- 0
Path = 3 <- 2 <- 1 <- 0
Path = 4 <- 1 <- 0
Path = 5 <- 6 <- 0
Path = 6 <- 0
Path = 7 <- 0
```

Fig -3 The shortest Path from source to every node.

3. CONCLUSIONS

This paper shows a significant work on how Dijkstra algorithm can be applied to arcs in a network that is fuzzy in nature. The distance between two paths which are fuzzy numbers are compared and shortest path is shown using fuzzy Dijkstra algorithm. There can be changes in the climate or other changes which results the distance between two places to be fuzzy.

An example of a sample network is shown in the experimental design along with its arc and membership function which is then converted to fuzzy and given as input to the Dijkstra algorithm. This shows the shortest path from source to each node. This can be introduced for a large network, to find the distance between two cities .

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