Fuzzy Sets and Convexity

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Abstract:

The purpose of this study is to look into certain convexity properties of fuzzy sets that are defined on the R. Convexity of fuzzy sets is studied using the complementary α-set.

Keywords:

Fuzzy sets, Convex Fuzzy sets , Complementary α- sets.

I. Introduction

To study properties of structures of applied sciences convexity plays a vital role.Convexity eases the study of researchers. Generalisation of convexity of crisp sets to fuzzy sets is therefore important. The terms fuzzy set, convex fuzzy set, and α-cut were initially introduced by Prof. Zadeh in 1965[1].Convex and concave fuzzy mappings were added to the notion by Yu-Ru Syau [11]. Sarkar [12] not only introduced concavo-convex fuzzy sets but also illustrated some other intriguing characteristics of this particular kind of fuzzy set. Ban constructed and thoughtfully explored convex temporal intuitionistic fuzzy sets as well as convex intuitionistic fuzzy sets [13, 14]. The generalised features of the aggregation of convex intuitionistic fuzzy sets were thoroughly analysed and characterised by Díiaz et al. [15].

Scholars Syau [5] and Xinmin Yang [2] demonstrated closed and convex fuzzy sets and investigated how they related to one another. In their study, Nadaban and Dzitac[4] discriminated between several forms of fuzzy relations and also gave examples of convex fuzzy relations. Chen-Wei-Xu[6] produced novel fuzzy relations and convexity results for fuzzy relations based on earlier work.We aim to demonstrate that if a fuzzy set B is convex, then its complementary β- set, M^{β} is not convex; $\forall\beta \in (0, 1]$. We will do this by extending the convexity of a fuzzy set with respect to complementary β- set to fuzzy relations.

II. Preliminaries

Throughout this paper, B denotes fuzzy set defined on M denotes fuzzy relation defined on R^2 Here are some definitions that will be useful in this paper.

2.1Definition[6]:

A fuzzy set B defined on R is a function; $B: R \to [0,1]$ is called as membership function and $B(x)$ is called membership grade of $B\ at\ x$.

2.2Definition[7]:

A Fuzzy relation M is a fuzzy set defined on Cartesian product of crisp sets $Y_1 \times Y_2 \times Y_3 \times \ldots\ldots \times Y_n$ where tuples $(y_1, y_2, y_3, \ldots\ldots, y_n)$ that may have varying degrees of membership value is usually represented by a real number for closed intervals[0,1] and indicate the strength of the present relation between elements of the topic. Consider $M: X \times Y \to [0,1]$ then the fuzzy relation on $X \times Y$ denoted by M or $M(x, y)$is defined as the set $M(X, Y) = \{((x, y), S(x, y))/(x, y) \in X \times Y\}$. where $M(x, y)$ is the strength of the relation in two variables called membership function. It gives the degree of membership of the ordered pair $(x, y)\ in\ X \times Y$ a real number in the interval [0,1].

2.3Definition[3]:
M be fuzzy relation on $X \times Y$. Then T is convex if and only if $M\big(\mu(x_1, y_1) + (1-\mu)(x_2, y_2)\big) \geq min[M(x_1, y_1) \wedge M(x_2, y_2)]$; $\forall (x_1, y_1), (x_2, y_2) \in X \times Y$ and $\mu \in [0,1]$.

2.4Definition[6]:
Let M be a fuzzy relation defined on $X \times Y$ and α be such that $0 < \beta \leq 1$.Then $Complementary\ \beta -$ set of M is denoted by $M_\beta-$ is defined by $M_\beta - = \{(x, y) \in X \times Y / T(x, y) \leq \beta\}$.

2.5 Definition [1]:
B be a fuzzy set defined on R and α be such that $0 < \beta \leq 1$.Then $complementary\ \beta - set$ of B, is denoted by B_β and defined by $C.B_\beta = \{x \in R / B(x) \leq \beta\}$ is a crisp set.

2.6 Definition [1]:
B be a fuzzy set defined on R. Then B is concave if and only if
$B(\mu x_1 + (1-\mu)x_2) \geq min[B(x_1), B(x_2)]$; $\forall\, x_1, x_2 \in R$ and $\mu \in (0, 1]$.

2.7 Definition[2]:
A fuzzy set B on R is said to be strongly convex fuzzy set if
$B(\mu x_1 + (1-\mu)x_2) > min[B(x_1), B(x_2)]$; $\forall\, x_1, x_2 \in R, x_1 \neq x_2$ and $\mu \in (0,1)$.

2.9 Definition [2]:
A fuzzy set B on is said to be strictly concave fuzzy set if
$B(\mu x_1 + (1-\mu)x_2) > min[B(x_1), B(x_2)]$; $B(x_1) \neq B(x_2), \forall x_1, x_2 \in R$ and $\mu \in (0,1)$.

III. Main Results

3.1 Theorem

B be a fuzzy set defined on R then B isConvex fuzzy set if and only if its complementary$\beta - set$; C. B_β is not convex; .

Proof:

Suppose M is a convex fuzzy set defined on R.
to prove that C. B_β is not convex;$\forall\ 0 < \beta \leq 1$.
Let, if possible, $C.B_\beta$ is convex for some $0 < \beta \leq 1$.
Then for any $s, t \in C.B_\beta$, such that $\beta = B(s) > B(t)$. we have $\vartheta s + (1-\vartheta)t \in C.B_\beta$.
That is $B(\vartheta s + (1-\vartheta)t) \leq \beta, 0 < \beta \leq 1$.
Therefore, $B(\vartheta s + (1-\vartheta)t) \leq \beta = B(s) = max[B(s), B(t)]$.
Implies that B is a concave fuzzy set.
A contradiction to our assumption that B is a convex fuzzy set.
Thus, C. B_β is not convex; $\forall 0 < \beta \leq 1$.
Conversely suppose that C. B_β is not convex; $\forall 0 < \beta \leq 1$.
To prove that B is a convex fuzzy set.
Suppose that B is not a convex fuzzy set.
There is $\beta' - set$ which is not convex.
Then C.$\beta' - set$ is convex.
A contradiction to that C. B_β is not convex; $\forall 0 < \beta \leq 1$.
Therefore, B is a convex fuzzy set.

3.2 Corollary:

B be a strongly (strictly) convex fuzzy set defined on R then $C.\ B_{\beta}{}^{c}$ is not convex; for all $\alpha \in (0, 1]$, where $C.\ B_{\beta}{}^{c}$ is strong $complementary\ \beta$ −cut of B.

3.3 Theorem

Convex fuzzy set B can be written as a union of all its complementary $\beta -\ sets; \forall\ \beta \in (0,1]\ i.e.$

$B = \cup_{\beta\in(0,1]}\quad C.B_{\beta}$.

Proof.

Let $x \in B$.

To prove that $x \in \cup_{\beta\in(0,1]}\quad C.\ B_{\beta}$; for some $\beta \in (0,1]$.

Take $\beta = 1\ and\ consider\ complementary\ \beta -\ set.$

Then B_1 is the largest complementary $\beta - level\ set$ containing x.

Therefore $x \in \cup_{\beta\in(0,1]}\quad C.B_{\beta}$.

Conversely, suppose that $x \in \cup_{\beta\in(0,1]}\quad C.\ B_{\beta}$.

To prove that $x \in B$.

Without loss of generality assume that $x \in\ C.B_{\chi}\ for\ some\ \chi \in (0,1]$.

Then $B(x) \leq \chi$. therefore, $(x, B(x)) \in B$.

Hence, we can write convex fuzzy set B is the a union of all its complementary $\beta -\ sets; \forall\ \beta \in (0,1]$.

3.4 Corollary

Strongly Convex fuzzy set B can be written as a union of all its complementary $Strongly\ \beta -\ sets;\ \forall\ \beta \in (0,1)\ i.e.$

$B = \cup_{\beta\in(0,1)}\quad C.B_{\beta}{}^{c}$.

3.4 Theorem

B be a convex fuzzy set defined on $R\ if\ and\ only\ if\ there\ is\ a\ sequence\ of\quad Complementary\ \beta - sets;\ C.\ B_{\beta}1, C.\ B_{\beta}2, C.\ B_{\beta}3, \ldots.\ \forall\ \beta \in (0,1]$ such that $C.\ B_{\beta}1 \subseteq C.\ B_{\beta}2 \subseteq C.\ B_{\beta}3 \subseteq, \ldots.$

Proof.

Let B be a convex fuzzy set defined on R.

Then its complementary $\beta - set;$ C. B_{β} is not convex; $\forall\ \beta \in [0,1)$.

Let us enumerate the complementary $\beta - set$ in a such manner that say $C.\ B_{\beta}1$ is the smallest complementary $\beta - set\ that\ contains\ only\ boundary\ points\ of\ a\ fuzzy\ set.$. Then enumerate for the other values of β such that $C.\ B_{\beta}1 \subseteq C.\ B_{\beta}2$.proceeds in the same manner. After successive repetition we get the required sequence of the complementary $\beta -\ set.$

Conversely suppose that $R\ if\ and\ only\ if\ there\ is\ a\ sequence\ of\quad non\ convex; complementary\ \beta - sets. C.\ B_{\beta}1, C.\ B_{\beta}2, C.\ B_{\beta}3, \ldots.\ \forall\ \beta \in (0,1]$ such that $C.\ B_{\beta}1 \subseteq C.\ B_{\beta}2 \subseteq C.\ B_{\beta}3 \subseteq, \ldots.$

To prove that B is a convex fuzzy set.

Given that $C.\ B_{\beta}1$ is the smallest complementary $\beta -\ set$ then clearly $C.C.\ B_{\beta}1 = B_{\beta}1$ is the largest $\beta -\ set$.In the same way we have, convex $\beta -\ set$ and each $\beta -\ set$ is convex;$\forall\ \beta \in [0,1)$.

Implies that B is a convex fuzzy set defined on R.

3.5 Theorem

.(i) If B is a convex fuzzy set then C.Supp(B) is a non convex set; where C.Supp(B) is a complement of Support of B .

(ii) If B is a strongly Convex fuzzy set then C.Supp(B) =ϕ .

Proof.
To prove that C.Supp(B) is a non convex set.
Let it possible C.Supp(B) is a convex set.
Let $x^1, x^2 \in C.Supp(B)$.
Then $\eta x^1 + (1-\eta)x^2 \in C.Supp(B)\ for\ some\ \eta \in [0,1)$.
Consider , $B(\eta x^1 + (1-\eta)x^2) \geq min\,[B(x^1), B(x^2)]$.
$= 0$.
Since, $x^1, x^2 \in C.Supp(B)$ implies that $B(x^1) \leq 0$ and $B(x^2) \leq 0$.
Therefore, $\eta x^1 + (1-\eta)x^2 \notin C.Supp(B)$.
A contradiction.therefore, C.Supp(B) is a non convex set.
Now to prove that if B is a strongly Convex fuzzy set then C.Supp(B) =ϕ .
Let $x, y \in C.Supp(B)\ and\ C.Supp(B) \subseteq R\ implies\ x \in R$.

$$iff\ B(x) = B[\frac{1}{2}(x-y) + \frac{1}{2}B(x+y)] \leq max(B(x-y), B(x+y)) \leq 0.$$

$B(x) = 0, thus\ x \notin C.Supp(B)$Thus such x does not exist.
Therefore, C.Supp(B) =ϕ .

IV. Conclusion

The convexity of fuzzy sets has been studied using complementary α- sets. The relationship between complementary α- sets and the convex fuzzy set was shown. Fuzzy and crisp sets are connected by the complementary α-set. A fuzzy approach to convexity's research is essential on many levels due to its extensive applications in many different fields.

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