International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 07 Issue: 06 | June - 2023 SJIF Rating: 8.176 ISSN: 2582-3930

gb – Chromatic Number of a Bull Graph and Some Related Graphs

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Abstract -For a connected graph G, a set S_{bc} is referred to as the gb-chromatic set of a vertex in G since it is a geodetic and b-chromatic set. The smallest cardinality of a gb - chromatic set of G is known as the gb - chromatic number, and it is represented by $\varphi_{gb}(G)$. We have investigated the gb – chromatic number of a bull graph and for some related graphs.

***Key Words*:**Geodetic number, chromatic number, b – chromatic number, gb – chromatic number

1.INTRODUCTION

All of the graphs used in this study are undirected, simple, and finite. The order and size of G are p and q for any graph G with edge set E(G) and vertex set V(G) [2]. The shortest distance between two vertices in V(G), x_1, and x_2, determines the minimum size of $x_1 - x_2$ pathways in G. The term "geodesic of G" refers to an x_1 - x_2 path of size $d_g(x_1, x_2)$. We defined $I_G(x_1, x_2)$ as the collection of geodesics of G's $x_1 - x_2$ vertices. The vertex is said to reside on the $x_1 - x_2$ geodesic if c is an inner vertex of P. The vertices of the limited interval $I(x_1, x_2)$ are all situated on a geodesic of G between x_1 and x_2. Consider a non – empty set $I(S) = \bigcup_{x_1, x_2 \in S} I(x_1, x_2)$. If G is connected graph, thus S is a geodetic get g(S) such that I(S) = V(G) [1]. The geodetic number g(G) defines the least cardinality S of G.

The graph's b-chromatic number $\varphi_b(G)$ is the largest positive integer k such that G permits an appropriate k-coloring in which each color class has a representation adjacent to at least one vertex in each of the other color classes. A b – coloring is a sort of coloring. Irving and Manlove introduced the b – coloring number in [3] by considering proper coloring that is the minimum in terms of a partial order defined on the set of all V partitions (G). a set S_{bc} is referred to as the gb-chromatic set of a vertex in G since it is a geodetic and b-chromatic set. The smallest cardinality of a gb - chromatic set of G is known as the gb - chromatic number, and it is represented by $\varphi_{gb}(G)$[4]. E.W.Weisstein introduced the Bull Graph [7]. In this paper, we have investigated gb – chromatic number for bull graph and its related graph [5, 6].

2. Main Result

Theorem 2.1. For a bull graph G, then the gb – chromatic number of G is 5., ie., $\varphi_{gb}(G) = 5$.

Proof: Consider G be the bull graph, where G is a planer graph with the vertex set $\{v_i\}$ $1 \le i \le 5$. Here S = $\{v_1, v_3, v_5\}$be the minimum geodic set and also b – colorable. Here since G is 3 – colorable. Here S = $S_{bc}(G)$. Therefore $\varphi_{gb}(G) = 5$.

Corollary 2.2: For a bull graph G then $\varphi_{gb}(\overline{G}) = 3$.

Theorem 2.3. For a graph G then the middle graph of gb - chromatic number is V(G)+2 ie., $\varphi_{gb}(G) = V(G) + 2$.

Proof: Consider the graph G be the bull graph, whereas middle graph M(G) of a graph G. Let the vertices of a M(G) be $\{v_1, v_2, v_3, v_4, v_5\}$and edge set is $\{e_1, e_2, e_3, e_4, e_5\}$. Hence the geodetic number of the M(G) is vertex set V(G). S = $\{v_1, v_2, v_3, v_4, v_5\}$is the minimum but not a b – chromatic coloring. Since S receives same b – color class. Then by adding different b – color class to the set S which is not in S. Here $\varphi_{gb}(G) = V(G) + 2$.

Theorem 2.4. For a graph G then the total graph of gb – chromatic number is 5

Proof: Let G be the bull graph, where $\{v_i\}$ $1 \le i \le 5$and $\{e_j\}$ $1 \le j \le 5$be the vertices set of the total graph T(G), S = $\{v_1, e_3, v_5\}$ in the minimum geodic number of T(G), but which is not b – colorable . Since S receives same b – colorable. Thus we add the remaining color which is not in S. Then the total graph of gb – chromatic number in 5.

Theorem 2.5. For the Splitting graph of bull graph, then the gb – chromatic number is $\{v_i'\}$ for $1 \le i \le 5$.

Proof: The Bull graph S(G) of a graph G in obtained by adding a new vertex v_i' for $1 \le i \le 5$corresponding to each vertex V(G) such that N(V) = N(v') when N(V) and N(v') are neighbourhood set of V and V' respectively. For the graph S(G) , S = v_i' $1 \le i \le 5$ be the minimum geodic set and b – colorable , which satisfies the condition of gb – chromatic number. Hence $\varphi_{gb}(S(G)) = v_i'$ for$1 \le i \le 5$.

Theorem 2.6. For the graph G then the shadow graph $\varphi_{gb}(D_2(G)) = 5$.

Proof : The graph $D_2(G)$ in constructed by taking 2 copies of G say G' and G'', joining each vertex V' in G' to all the adjacent vertices of the corresponding vertex V''in G''. Let V_i' andV_i'', $1 \le i \le 5$be the vertex set of $D_2(G)$. Therefore S = $\{v_1', v_1'', v_4', v_4''\}$be the minimum geodetic set but not b – colorable. Hence, we adding remaining color to the set S, we get $\varphi_{gb}(D_2(G)) = 5$.

3. CONCLUSIONS

In this paper, we obtain the gb – chromatic number $\varphi_{gb}(G)$ has been derived for bull graph and for some related graph. This concept can be extended to several other graphs and also for the products of graphs.

ACKNOWLEDGEMENT

The authors would like to thank the referees for their helpful suggestions and valuable comments.

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