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Harmonic Analysis with Fourier Series

Lenora Martin,

S.Y. BSc (Computer Science),

Ashoka Center for Business and Computer Studies,

Vrushali Deshpande,

Asst. Prof. Dept of Computer Science,

Ashoka Center for Business and Computer Studies.

Rahul Sonawane,

Asst. Prof. Dept of Computer Science,

Ashoka Center for Business and Computer Studies.

Nashik, India.

Abstract: This paper presents a systematic study of the analytic aspects of Fourier series and convolute them. Many different aspects are investigated of the present theory in assorted cases. Fourier series was discovered by a French mathematician Joseph Fourier (1768-1830)

Keywords: Harmonic analysis, Fourier series, Fourier integrals, Fourier expansion, even and odd function.

I. INTRODUCTION

Harmonic analysis involves representation of functions by trigonometric sums/integrals. These representations mainly use Fourier series or Fourier integrals for analyzing. It is a vast subject to discuss and is applied in areas like quantum physics, tidal analysis, neuroscience, number theory and signal processing. In this paper we will discuss the applicable prospects enveloped for using these expansions.

II. HARMONIC ANALYSIS

The term "Harmonics" originates because the Hellenic word *Harmonikos,* which means "skilled in music". Harmonic analysis is a mathematical procedure for describing and studying phenomena of a periodic reoccurring nature. Physical events like soundwaves, electromagnetic waves, tides and vibrations may be periodic in character. These movements can be measured with many consecutive values of the independent variable. By using the time period and the data (values) one can plot a curve. This will represent a function of that independent variable and generally find a mathematical expression. With the help of this implied function the sum of the numbers is expressed in sine and cosine terms. This is called Fourier series.

III. FOURIER SERIES

A Fourier series is a method of representing a periodic function as a total of trigonometric functions i.e., sine and cosine functions. It is similar to Taylor series, which represents functions as possibly infinite sums of monomial terms.

A. History of Fourier series:

The Fourier series is named after in honor of Jean-Baptiste Joseph Fourier (1768-1830). Fourier introduced this for the purpose of solving the heat equation in a metal plate, publishing its results in his 1807 *Memoire sur la propagation de la chaleur dans les corps solides* (treatise on the propagation of heat in solid bodies), also published his Theorie analytique de la chaleur (Analytical theory of heat) in the year 1822.

The heat equation may be a partial equation. Before Fourier's work, no resolution to the heat equation was known within the general case, through explicit solutions were known is the heat supply was a sine or cosine wave, these easy solutions area unit currently generally refused to as eigen solutions. Fourier's plan was to model an advanced heat supply as a superposition of simple sine and cosine waves, and to jot down the answer as a superposition or linear combination is named the Fourier series.

B. Let'f'be a valued function defined on the interval [-π,π]. Then the Fourier series expansion of *f* is given by

$$f(x) = \frac{1}{2}a_0 + a_1 \cos x + a_2 \cos 2x + \ldots$$

$$+ b_1 \sin x + b_2 \sin 2x + \ldots. \qquad (1.1)$$

were,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx \qquad (1.2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx \qquad (1.3)$$

The function sin n x and cos n x are orthogonal in the sense that

$$\int_{-\pi}^{\pi} \sin m\, x \sin n\, x \, dx = 0 \qquad m \neq n$$

$$\int_{-\pi}^{\pi} \sin m\, x \cos n\, x \, dx = 0 \quad \text{for all } m, n$$

$$\int_{-\pi}^{\pi} \cos m\, x \cos n\, x \, dx = 0 \qquad m \neq n.$$

At a point x where there is a jump discontinuity, the series in equation (1.1) converges to $[(f(x+0) + f(x-0))]/2$. *Here* $F(x + 0)$ *and* $f(x - 0)$ are the limits of $f(u)$ as u approaches x from above and below respectively. I still find it remarkable that such a wide variety of functions can be represented on an interval by Fourier series.

C. Exponential Form of Fourier Series:

The Fourier expansion of '*f*' can be written in the form

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i n x} \qquad (1.4)$$

were,

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-i n x} dx. \qquad (1.5)$$

The coefficients c_n are generally complex. The functions $e^{i n x}$ are orthogonal in the sense that

$$\int_{-\pi}^{\pi} e^{i n x} e^{-i n x} dx = 0 \qquad m \neq n.$$

D. *Fourier series expansion on Interval [a, b]:*

Iff (x) is defined on the interval [a, b], then the Fourier series representation is considered by using the same formula,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L})$$

$$\text{where } L = \frac{b-a}{2}$$

$Fourier\ coefficients\ are\ calculated\ as\ follows$:

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx,$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx,$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx,$$

$$n = 1,2,3,4, \ldots .$$

E. Even and Odd Function:

The function *f* can be expressed as,

$$f(x) = g(x) + h(x) \qquad (1.6)$$

were,

$$g(x) = \frac{1}{2}[f(x) + f(-x)] \qquad (1.7)$$

$$h(x) = \frac{1}{2}[f(x) - f(-x)]. \qquad (1.8)$$

The function g is an even function in the sense that $g(-x) = g(x)$. the function h is an odd function in the sense that $h(-x) = -h(x)$. Moreover, if *f* has a fourier expansion like that in equation (1.1), then

$$g(x) = \frac{1}{2}a_0 + a_1 \cos x + a_2 \cos 2x + \ldots.. \qquad (1.9)$$

$$h(x) = b_1 \sin x + b_2 \sin 2x + \ldots.. \qquad (1.10)$$

and

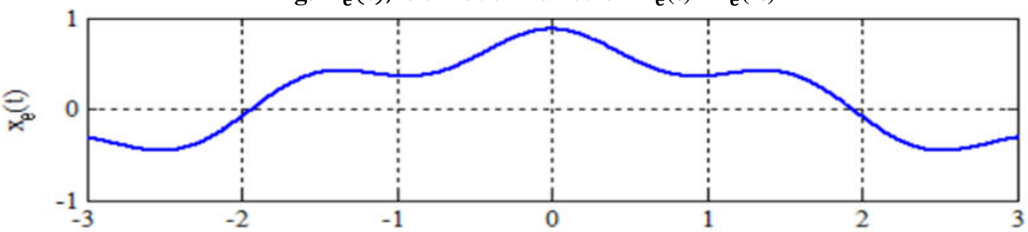
$$a_n = \frac{2}{\pi}\int_0^{\pi} g(x)\cos n\,x\,dx \qquad (1.11)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} h(x)\sin n\,x\,dx. \qquad (1.12)$$

If the function *f* is defined on the interval $[0, \pi]$, then it can be extended to the interval $[-\pi, \pi]$ as either an odd function or as an even function. Thus, *f* can be represented on $[0, \pi]$ in terms of a cosine series or a sine series.

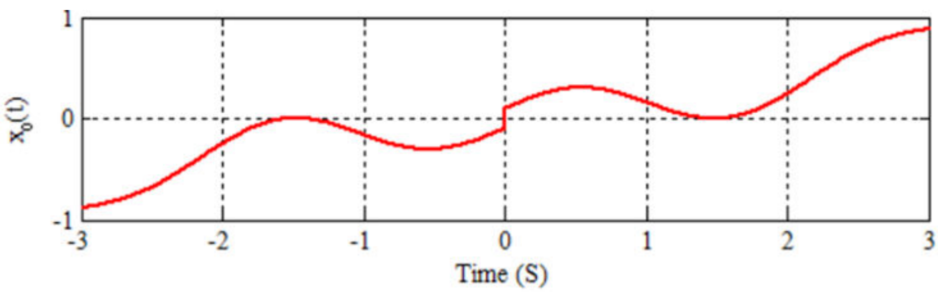
Here's a brief review of even and odd functions,

Fig.1 $x_e(t)$, is an even function $x_e(t) = x_e(-t)$



The even function, $x_e(t)$, is symmetric about t=0, so $x_e(t) = x_e(-t)$.

Fig.2 x_0 (t), an odd function x_0 (t)= -x_0(-t)



The odd function, $x_0(t)$, is asymmetric to t=0, so $x_0(t)$= -$x_0(-t)$.

IV. FOURIER INTEGRAL

The Fourier integral represents a function *h by,*

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{i\,2\pi ft}df \qquad (2.1)$$

where,

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-i\,2\pi ft}\,dt. \qquad (2.2)$$

The functions *h* and *H* are called a fourier transformation pair. The Fourier integral exists if '*h'* is completely integrable on the real number line and is of bounded variation on every finite subinterval.

A. *Relation between Fourier Transform to Fourier Series*

The Fourier transform of operate'*h'* is given by,

$$H(f) = \int_{-\infty}^{\infty} h(t)\,e^{-i\,2\pi ft}\,dt. \qquad (2.3)$$

On the interval [-L, L], *h* has the Fourier series expansion

$$h(f) = \sum_{n=-\infty}^{\infty} a_n\ e^{i\,n\,\pi\,t/L} \qquad (2.4)$$

where,

$$a_n = \frac{1}{L}\int_{-L}^{L} h(t)e^{-i\,\pi\,n\,t/L}dt. \qquad (2.5)$$

Compare equation (2.3) and (2.5),

$$La_n = H\left(\frac{n}{2L}\right) - \int_{L}^{\infty} h(t)e^{-i\,\pi\,n\,t/L}dt - \int_{-\infty}^{-L} h(t)e^{-i\,\pi\,n\,t/L}dt. \qquad (2.6)$$

If h is absolutely integrable, the integrals on the right-hand-side of equation (2.6) can be made unpredictable?>, small for sufficient large L. It is in this sense that the Fourier series coefficients a_n can be used to approximate the Fourier transform at the frequencies $\frac{n}{2L}$.

B. Limitations of Fourier series

The Fourier series converges,

1. if $\int_T x_T(t)\,dt < \infty$, i.e.,as long as the function is not infinite over a finite interval,
2. if $x_T$has a finite number of discontinuities in one period,
3. if x_T has a finite number of maxima and minima in one period,
4. excluding at discontinuities, where it converges to the midpoint of the discontinuity.

V. THE SPECTRUM ANALYZER

This instrument provides Fourier coefficient in terms of sin and cosine of Fourier expansion reading a signal. The electronical industries use spectrum analyzer for observing or inspects frequency spectra of radio frequencies and audio signals.

Here are some of the best spectrum analyzers:

A. Siglent SSA3021X digital spectrum analyzer.
B. The Rigol DSA815/TG tracking generator spectrum analyzer.
C. The RSA306B USB spectrum analyzer.

VI. DIGITAL SIGNAL PROCESSING

Signal processing concentrates on analysis, modification and synthesis of signals like sounds, images, etc. Digital signal processing converts signals from actual sources (usually analog form) into digital data and then analyze it. DSP is used in vast areas of technologies like RADAR, speech processing, SONAR, voice recognition and audio signals. Convolution is also an important technique in DSP. These techniques also have a finite range to low bandwidth.

VII.CONCLUSION

Here we discussed Fourier series and their aspects, the Fourier series is useful in many applications ranging from experimental instruments to rigorous mathematical analysis techniques. Thanks to modern developments in digital electronics, coupled with numerical algorithm, the fourier series has become one of the most widely used and useful mathematical tools available to any scientist. The spectrum analyzer and the digital signal processer are some major applications of Fourier Series. Fourier transform isn’t just limited for simple lab examples but also have a major use in astronomy. We can have much advance uses of Fourier series in future finding.

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