HOMOMORPHISM OF IMAF NORMAL SUBGROUP

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ABSTRACT In this article an attempt has been made to study some new algebraic structures of intuitionistic multi-anti fuzzy normal subgroups under homomorphism are discussed.

Keywords Intuitionistic multi-fuzzy set (IMFS), Intuitionistic multi-anti fuzzy subgroup (IMAFSG), Intuitionistic multi-anti fuzzy normal subgroup (IMAFNSG).

Mathematics Subject Classification 20N25, 03E72, 08A72, 03F55, 06F35, 03G25, 08A05

1. INTRODUCTION

After the introduction of fuzzy set by Prof. Zadeh [11] several researches were conducted on the generalization of fuzzy set. The concept of Fuzzy group was introduced by Rosenfeld A. [6] in 1971. The concept Fuzzy groups and its level subgroups was introduced by P.S.Das [2] in 1981. Mukharjee N.P. and Bhattacharya P. [3] was introduced the concept of Fuzzy normal subgroups and fuzzy cosets in 1984. Sabu S. and Ramakrishnan T.V. [7, 8] was introduced the concept Multi-fuzzy subgroups and it was continuously developed by Muthuraj.R and Balamurugan.S [4, 5]. The idea of intuitionistic fuzzy set was introduced by K.T. Atanassov [1]. In 2011, P.K.Sharma [9] initiated the concept of intuitionistic fuzzy groups. T.K.Shinoj and Sunil Jacob John [10] was introduced the concept of intuitionistic multi-fuzzy set in the year of 2013. R.Muthuraj and S.Balamurugan [5] introduced the algebraic structure Intuitionistic multi fuzzy subgroup in 2014. In this paper we study the new algebraic structure intuitionistic multi-anti fuzzy normal subgroup under homomorphisms are discussed.

2. PRELIMINARIES

In this section, we site the basic or fundamental definitions that will be used in the sequel:

2.1 Definition [11]

Let X be a non-empty set. Then a **fuzzy set** $\mu : X \to [0,1]$.

2.2 Definition [5, 10]

Let $A = \{<x, \mu_A(x), \nu_A(x)>: x \in G\}$, where $\mu_A(x) = (\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \ldots \mu_{A_k}(x))$ and $\nu_A(x) = (\nu_{A_1}(x), \nu_{A_2}(x), \nu_{A_3}(x) \ldots \nu_{A_k}(x))$ such that $0 \le \mu_{A_1}(x) + \nu_{A_1}(x) \le 1$, $\forall x \in G$, $\mu_{A_1} : G \to [0,1]$ and $\nu_{A_1} : G \to [0,1]$ for all i = 1, 2, …, k. Here, $\mu_{A_1}(x) \ge \mu_{A_2}(x) \ge \mu_{A_3}(x) \ge \ldots \ge \mu_{A_k}(x)$, for all $x \in G$. That is, μ_{A_1}'s are decreasingly ordered sequence. Then the set A is said to be an **intuitionistic multi-fuzzy set (IMFS)** with dimension k of G.

2.3 Remark

Note that since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.4 Definition [4, 5]

A mapping f from a group G_1 into a group G_2 is said to be a **homomorphism** if for all a, $b \in G_1$, f(ab) = f(a)f(b).

2.5 Theorem [5]

Let f: $G_1 \to G_2$ be an onto, homomorphism of groups G_1 and G_2. If $A = \{<x, \mu_A(x), \nu_A(x)>: x \in G_1\}$ is an intuitionistic multi-anti fuzzy subgroup of G_1, then $f(A) = \{< y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / \, y \in G_2, \text{where } y = f(x)\}$ is also an intuitionistic multi-anti fuzzy subgroup of G_2, if μ_A has inf property; ν_A has sup property and μ_A, ν_A are f-invariants.

2.6 Theorem [5]

Let G_1 and G_2 be any two groups. Let f : $G_1 \to G_2$ be a homomorphism of groups. If $B = \{< y, \mu_B(y), \nu_B(y) > : y \in G_2\}$ is an IMAFSG of G_2, then $f^{-1}(B) = \{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) > : x \in G_1 \}$ is also an IMAFSG of G_1.

3. Properties of an intuitionistic multi-anti fuzzy normal subgroup of a group under homomorphism

In this section, we discuss the properties of an intuitionistic multi-anti fuzzy normal subgroup of a group under homomorphism.

3.1 Theorem

Let f: $G_1 \to G_2$ be onto, homomorphism of groups. If A = { < x , $\mu_A(x)$, $\nu_A(x)$ > : $x \in G_1$ } is an IMAFNSG of G_1, then $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / y \in G_2, \text{where } y = f(x) \}$ is also an IMAFNSG of G_2 if μ_A has inf property; ν_A has sup property and μ_A , ν_A are f-invariants.

Proof: Clearly, f(A) is an IMAFSG of G_2.

Let A be an IMAFNSG of group G_1.

Let $y_1, y_2 \in G_2$.

Since f is onto, there exist elements $x_1, x_2 \in G_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$.

Since A is an IMAFNSG of G_1, $\mu_A(x_1x_2) = \mu_A(x_2x_1)$ and $\nu_A(x_1x_2) = \nu_A(x_2x_1)$.

Also, $y_2y_1 = f(x_2)f(x_1) = f(x_2x_1)$, since f is a homomorphism.

Now, $\mu_{f(A)}(y_1y_2) = \mu_{f(A)}(f(x_1)f(x_2))$

$= \mu_{f(A)}(f(x_1x_2))$, since f is a homomorphism.

$= \mu_A(x_1x_2)$,

$\leq \max\{ \mu_A(x_1), \mu_A(x_2) \}$

$= \max\{ \mu_{f(A)}f(x_1) , \mu_{f(A)}f(x_2) \}$

$= \mu_{f(A)}f(x_2x_1)$

$= \mu_{f(A)}(y_2y_1)$, since f is a homomorphism.

That is, $\mu_{f(A)}(y_1y_2) = \mu_{f(A)}(y_2y_1), \forall\, y_1, y_2 \in G_2$.

Also, $\nu_{f(A)}(y_1y_2) = \nu_{f(A)}(f(x_1)f(x_2))$

$= \nu_{f(A)}(f(x_1x_2))$, since f is a homomorphism.

$= \nu_A(x_1x_2)$

$\geq \min\{\nu_A(x_1), \nu_A(x_2)\}$

$= \min\{\nu_{f(A)}f(x_1), \nu_{f(A)}f(x_2)\}$

$= \nu_{f(A)}f(x_2x_1)$

$= \nu_{f(A)}(y_2y_1)$, since f is a homomorphism.

That is, $\nu_{f(A)}(y_1y_2) = \nu_{f(A)}(y_2y_1)$, $\forall\, y_1, y_2 \in G_2$.

Hence, f(A) is an IMAFNSG of G_2.

3.2 Theorem

Let G_1 and G_2 be any two groups. Let $f : G_1 \to G_2$ be a homomorphism of groups. If B = { < y, $\mu_B(y)$, $\nu_B(y)$ > : $y \in G_2$ } is an IMAFNSG of G_2, then $f^{-1}(B)$ = { < x, $\mu_{f^{-1}(B)}(x)$, $\nu_{f^{-1}(B)}(x)$> : $x \in G_1$ } is also an IMAFNSG of G_1.

Proof: Clearly, $f^{-1}(B)$ is an IMAFSG of G_1.

Let B be an IMAFNSG of G_2.

For any x, $y \in G_1$,

$\mu_{f^{-1}(B)}(xy) = \mu_B(f(xy))$

$= \mu_B(f(x)f(y))$, since f is a homomorphism.

$= \mu_B(f(y)f(x))$, since B is an IMAFNSG of G_2.

$= \mu_B(f(yx))$, since f is a homomorphism.

Therefore, $\mu_{f^{-1}(B)}(xy) = \mu_{f^{-1}(B)}(yx)$, $\forall\, x, y \in G_1$.

For any x, $y \in G_1$,

$\nu_{f^{-1}(B)}(xy) = \nu_B(f(xy))$

$= \nu_B(f(x)f(y))$, since f is a homomorphism.

$= \nu_B(f(y)f(x))$, since B is an IMAFNSG of G_2.

$= \nu_B(f(yx))$, since f is a homomorphism.

Therefore, $\nu_{f^{-1}(B)}(xy) = \nu_{f^{-1}(B)}(yx), \forall x, y \in G_1$.

Hence, $f^{-1}(B)$ is an IMAFNSG of G_1.

3.3 Theorem

Let G_i(for i = 1, 2, 3, 4) be groups. Let f: $G_1 \times G_2 \to G_3 \times G_4$ be an onto homomorphism of groups. Let A and B be any two IMAFNSG's of G_1 and G_2 respectively. Let f_1: $G_1 \to G_3$ and f_2: $G_2 \to G_4$ be onto homomorphisms of groups. If A×B is an IMAFNSG of $G_1 \times G_2$, then f(A×B) is also an IMAFNSG of $G_3 \times G_4$ if A×B have inf property and also A×B is f-invariant.

Proof: It is clear.

3.4 Theorem

Let G_i(for i = 1, 2, 3, 4) be groups. Let f: $G_1 \times G_2 \to G_3 \times G_4$ be a homomorphism of groups. Let C and D be any two IMAFNSG's of G_3 and G_4 respectively. Let f_1: $G_1 \to G_3$ and f_2: $G_2 \to G_4$ be homomorphisms of groups. If C×D is an IMAFNSG of $G_3 \times G_4$, then $f^{-1}(C \times D)$ is also an IMAFNSG of $G_1 \times G_2$.

Proof: It is clear.

3.5 Theorem

Let G_i(for i = 1, 2, 3, 4) be groups. Let A and B be any two IMAFNSG's of G_1 and G_2 respectively. Let f_1: $G_1 \to G_3$ and f_2: $G_2 \to G_4$ be onto homomorphisms of groups. Let f: $G_1 \times G_2 \to G_3 \times G_4$ be an onto homomorphism of groups such that f((u, v)) = $(f_1(u), f_2(v))$. If A×B is an IMAFNSG of $G_1 \times G_2$, then f(A×B) = $f_1(A) \times f_2(B)$ if A×B have inf property and also A×B is f-invariant.

Proof: Let A×B be an IMAFNSG of $G_1 \times G_2$.

Let $(u, v) \in G_1 \times G_2$. Then $u \in G_1$ and $v \in G_2$. It implies that $f_1(u) \in G_3$ and $f_2(v) \in G_4$.

Therefore, $(u, v) \in G_1 \times G_2 \Rightarrow f((u, v)) = (f_1(u), f_2(v)) \in G_3 \times G_4$. Then

$$\begin{aligned}
\mu_{f(A\times B)}(f_1(u), f_2(v)) &= \mu_{f(A\times B)}(f(u, v)) \\
&= \mu_{A\times B}(u, v) \\
&= \max\{\mu_A(u), \mu_B(v)\} \\
&= \max\{\mu_{f_1(A)}(f_1(u)), \mu_{f_2(B)}(f_2(v))\} \\
&= \mu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))
\end{aligned}$$

Therefore, $\mu_{f(A\times B)}(f_1(u), f_2(v)) = \mu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))$, for all $(f_1(u), f_2(v)) \in G_3\times G_4$.

$$\begin{aligned}\nu_{f(A\times B)}(f_1(u), f_2(v)) &= \nu_{f(A\times B)}(f(u, v)) \\ &= \nu_{A\times B}(u, v) \\ &= \min\{\nu_A(u), \nu_B(v)\} \\ &= \min\{\nu_{f_1(A)}(f_1(u)), \nu_{f_2(B)}(f_2(v))\} \\ &= \nu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))\end{aligned}$$

Therefore, $\nu_{f(A\times B)}(f_1(u), f_2(v)) = \nu_{f_1(A)\times f_2(B)}(f_1(u), f_2(v))$, for all $(f_1(u), f_2(v)) \in G_3\times G_4$.

Hence, $f(A\times B) = f_1(A)\times f_2(B)$.

3.6 Theorem

Let G_i(for i = 1, 2, 3, 4) be groups. Let C and D be any two IMAFNSG's of G_3 and G_4 respectively. Let $f_1: G_1 \to G_3$ and $f_2: G_2 \to G_4$ be homomorphisms of groups. Let $f: G_1\times G_2 \to G_3\times G_4$ be a homomorphism such that $f((u, v)) = (f_1(u), f_2(v))$. If $C\times D$ is an IMAFNSG of $G_3\times G_4$, then $f^{-1}(C\times D) = f_1^{-1}(C)\times f_2^{-1}(D)$.

Proof: Let $C\times D$ be an IMAFNSG of $G_3\times G_4$.

Let $(u,v)\in G_1\times G_2$. Then $u\in G_1$ and $v\in G_2$. It implies that $f_1(u)\in G_3$ and $f_2(v)\in G_4$.

Therefore, $(u, v)\in G_1\times G_2$.

$\Rightarrow f((u, v)) = (f_1(u), f_2(v))\in G_3\times G_4$, since f is a homomorphism.

$$\begin{aligned}\text{Then } \mu_{f^{-1}(C\times D)}(u, v) &= \mu_{C\times D} f((u, v)) \\ &= \mu_{C\times D}(f_1(u), f_2(v)) \\ &= \max\{\mu_C(f_1(u)), \mu_D(f_2(v))\} \\ &= \max\{\mu_{f_1^{-1}(C)}(u), \mu_{f_2^{-1}(D)}(v)\} \\ &= \mu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u, v)\end{aligned}$$

Therefore, $\mu_{f^{-1}(C\times D)}(u, v) = \mu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u, v)$, for all $(u, v)\in G_1\times G_2$.

And $\nu_{f^{-1}(C\times D)}(u, v) = \nu_{C\times D} f((u, v))$

$$= \nu_{C\times D}(f_1(u), f_2(v))$$

$$= \min\{\nu_C(f_1(u)), \nu_D(f_2(v))\}$$

$$= \min\{\nu_{f_1^{-1}(C)}(u), \nu_{f_2^{-1}(D)}(v)\}$$

$$= \nu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u, v)$$

Therefore, $\nu_{f^{-1}(C\times D)}(u, v) = \nu_{f_1^{-1}(C)\times f_2^{-1}(D)}(u, v)$, for all $(u, v)\in G_1\times G_2$.

Hence, $f^{-1}(C\times D) = f_1^{-1}(C) \times f_2^{-1}(D)$.

4. CONCLUSION

The intuitionistic multi-fuzzy sets are plays an important role for the development of the theory of intuitionistic multi-anti fuzzy normal subgroups. In this article an attempt has been made to study some new algebraic structures of intuitionistic multi-anti fuzzy normal subgroups under homomorphism were discussed.

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