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Imaginary Numbers and their Applications

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Abstract

Math is essential in our world. Without mathematics, there would be insufficient knowledge in every field, be it calculating the area of land or the monetary value of something. And our world has progressed one step at a time, proving theories and concepts. But what if we arrive at a problem that we know is correct but are unsure how to proceed? Is that the end of innovation? After encountering numerous roadblocks, brilliant mathematicians devised the concept of imaginary numbers. It sounds absurd because it essentially means the square root of a negative number, which we have all learned is insane. But without it, we would not have progressed as far or learned as much as we have. As a result, imaginary numbers are more important than previously thought. This paper delves into the history of imaginary numbers and their current impact on our universe.

Body

Since its discovery in the early days of our universe, math has been running smoothly. Discoveries were made, and knowledge was gained. Theories and concepts were created and revised. But then came an unsolvable problem. The key to solving it was to separate math from reality; to invent new numbers known as imaginary numbers, which do not exist on the number chart or in math but are a very important underlying concept. There's a reason it's called fictitious. We would not have discovered many significant things without it. Imaginary numbers, which appeared to ancient mathematicians as an excuse to achieve the answer they desired, are now very much applicable in the heart of our universal theories. "Only by abandoning math's connection to reality could we discover reality's true nature," said Derek Muller. This statement goes much deeper than it appears.

Now, if we go back in time, we need to understand how they did it. It's simpler these days, thanks to the internet and ready-made formulas. But how did our forefathers develop theories? They imagined it. They drew measurements on paper and pieced them together to complete their research. These were obviously drawn-out processes. Some men devoted their lives to proving one small detail in which they believed when the rest did not. We have such a strong foundation today because of these mathematicians. In terms of imaginary numbers in particular, well, How did the misunderstanding begin? Finding the roots of a regular quadratic equation was a popular trend among ancient mathematicians. Obviously, due to a lack of awareness and knowledge, they only considered the positive roots. What about the negative root numbers, though? Because equations had them as a solution as well. For years, mathematicians were unaware of this. And it makes sense because they were used to dealing with real-life situations. It would be absurd to imagine a square with a side length of -27. Because they didn't want to deal with negative numbers, mathematicians found ways to find only positive answers wherever possible.

They avoided a problem that would have hampered them indefinitely. But these were all variations; there was no general solution or formula because they didn't want to end up with a negative number. Mathematicians let the square root of a number be its identity decades later, observing that it cannot be considered positive or negative. They are known as complex numbers, and they include imaginary numbers. It had its controversial phases as well, with scientists and thinkers alike questioning the potential use of this imaginary number and dismissing it as gimmicky; a number invented to solve with no real meaning. The whole point of imaginary numbers is that they serve as a bridge to the final answer. The negative area does not sound right and is incorrect, but when the two solutions are added together, the negatives cancel out, resulting in the final correct answer. Rene Descartes popularised the term "imaginary," and Leonhard Euler later used the letter I to represent them. A complex number is formed by combining an imaginary number with a regular number. That's where it came from and how it came into play(*How Imaginary Numbers Were Invented*, 2021).

With the introduction of this single new non-real number — I, the imaginary unit — an entirely new mathematical world opened up for exploration. It was an odd world where squares could be negative, but the structure was similar to real numbers. This expansion to real numbers was only the beginning (Honner, 2018). Imaginary numbers have a wide range of applications in physics, engineering, and geometry. They contributed to the formation of the fundamental foundation in all of these fields, some of which are proposed as models of the mysterious relationships that underpin our physical world. We can use imaginary numbers to simulate how electronic circuits work and how electromagnetic waves travel through air and space, and simulation of fluid flow and quantum physics. But before we get there, let's dig a little deeper.

The negative root of imaginary numbers is denoted by the letter I am 1. Other properties of complex numbers formed by imaginary numbers include:

1. <u>CONJUGATE OF A COMPLEX NUMBER:</u>

- A pair of complex numbers x+iy and x-iy are said to be conjugate of each other.

2. <u>PROPERTIES OF COMPLEX NUMBERS ARE:</u>

- If x1+ iy1 = x2 + iy2 then x1- iy1 = x2 – iy2
- Two complex numbers x1+ iy1 and x2 + iy2 are said to be equal
  If R (x1 + iy1) = R (x2 + iy2)
  I (x1 + iy1) = I (x2 + iy2)

- Sum of the two complex numbers is
  (x1 + iy1) + (x2 + iy2) = (x1+ x2) + i(y1+ y2)
- Difference of two complex numbers is
  (x1 + iy1) – (x2 + iy2) = (x1-x2) + i(y1 – y2)
- Product of two complex numbers is
  (x1+ iy1) (x2 + iy2) = x1x2 – y1y2 + i(y1x2 + y2 x1)
- Division of two complex numbers is
  (x1 + iy1) (x2 + iy2) = x1x2 + y1 y2)x22+y22 + iy1x2 - y2 x1x22+y22
- Every complex number can be expressed in terms of r (cosθ + i sinθ)
  R (x+ iy) = r cosθ
  I (x+ iy) = r sinθ
  r = x2+y2 and θ = tan-1yx (*Complex Numbers and Their Applications*, 2021)

$$i^0 = 1$$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

The diagram above depicts the primary values of I when its exponents are calculated. It's natural to be curious about the next exponent, I to power 5. That is I by itself or the negative root of 1. I to the power of 6 is -1, I to the power of 7 is -I, and I to the power of 8 is 1. There is a distinct pattern here; the values repeat every four exponents. This repetitive pattern is used in a variety of equations. Imaginary numbers are also used in AC circuit equations. Obviously, the nature of the equation, its complexities, and derivation will not be discussed here because this paper is intended for laypeople only. The aim is to enlighten and motivate people to join and garner the world of mathematics.

Imagine an electronic piano. Each key produces a different tone. Volume control changes the amplitude (volume) of all the keys by the same amount. That's how real numbers affect signals.
Now, imagine a filter. It makes some keys sound louder and some keys sound softer, depending on their frequencies. That's complex numbers -- they allow an "extra dimension" of calculation (*"Where" Exactly Are Complex Numbers Used "in the Real World"?*, n.d.)

There are imaginary numbers where there is a wave. Imaginary numbers were critical in the development of wave equations; without them, there would be a large, gaping hole in the foundation of modern physics.

Erwin Schrödinger's Schrödinger equation is a well-known example. An imaginary constant was used to complete this wave equation. As previously stated, physicists were uneasy with the use of imaginary numbers in such a complex formula that governs so many things. However, this did not prevent the formulation of this equation. Many wave equations contain imaginary numbers. The real world constitutes various particles such as protons, electrons, and so on. However, according to quantum mechanics, they are not particles at all, but rather waves. In quantum mechanics, the magnitude of waves is represented by complex numbers. Think about that for a moment. We know that atoms and molecules make up all matter. Quantum mechanics demonstrates that they are waves. So pretty much everything in the real world is made up of waves. That means that every wave in our environment has complex amplitudes!

It's funny how the difficulties encountered by ancient mathematicians and their solutions became part of modern physics. It goes to show that imaginary numbers are meant to be; not just any part of a theory that can be added and removed at will. That is exactly what Mr Derek meant when he said that imaginary numbers, invented as a quirky intermediate step to get to the end, turned out to be fundamental to our understanding of reality. It also sounds like something out of a story, which it should, as this was nothing short of a miracle. Everything seemed to be falling into place, and imaginary numbers becoming a part of everyday mathematics is something to always wonder about.

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