Impact of Permeable Lining of the Wall on the Peristaltic Flow of Rabinowitsch Fluid in a Vertical Channel

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Abstract: Rabinowitsch fluid model is a well-established model for studying the non-Newtonian nature of the fluid This study is carried out to analyze the Rabinowitsch fluid
through flexible and symmetric vertical channel with suction and injection under peristaltic motion. Considering long wavelength and low Reynolds number assumptions,. The present work is the mathematical investigation of permeable lining of the wall on the peristaltic flow of Rabinowitsch fluid in vertical channel with suction and injection . Pressure rise and the variation of various influencing parameters of the flow are studied graphically
Keywords: peristalsis,vertical channel, Rabinowitsch fluid ,porous thickness of wall, permeable wall,

1. Introduction

Peristalsis is when the muscular ducts move back and forth at regular intervals. This is caused by transverse progressive waves moving through the flexible wall. The wave that moves from low pressure to high pressure along the edges of the tract is the mechanism that makes this happen flow is a vital mechanism prompted by the progressive wave of area contraction and expansion which goes along with the walls of the distensible tube or channel. The peristalsis occurs typically in the development of bolus through the oesophagus, urine flow through the ureter, chyme advancement in the gastrointestinal tract, embryo transport inside the uterine cavity and the vasomotion of blood in vessels, This mechanism has been utilized by the researchers to outline a few modern applications, for example, in the nuclear industry, peristaltic pump, roller and finger pumps, the examination on the peristaltic transport of Non-Newtonian fluid has been of most outrageous centrality to various researchers because of its application in bioengineering and medicine. The Rabinowitsch fluid model reveals the complex rheological behaviours of biological fluids. Rabinowitsch fluid model exhibits the nonlinear association between the strain rate and shear stress.

Researchers are now focusing on transport because it has many uses in biological systems, chemical industries, physiology, and therapeutic procedures. Shapiro *et al*.[1] have looked into peristaltic motion in lubricated systems. Ramanamurthy *et al*.[2] use a curved two-dimensional peristaltic conduit to explain how a viscous liquid moves in a way that is not steady. Sreenadh et al. [3] investigated Peristaltic transport of Bingham fluid in a Channel with permeable walls. The Rabinowitsch fluid acts differently depending on the nonlinear factor, representing a Newtonian fluid, a shear thickening fluid, and a shear thinning fluid. Prasad et al.[4] Effect of variable liquid properties on peristaltic flow of a Rabinowitsch fluid in an inclined convective porous channel. Sadaf *et al*.[5] used a Rabinowitsch fluid model to study the effects of viscous dissipation and convective boundary conditions in a non-uniform peristaltic tube with different wall properties. Sing *et al*.[6] investigated the Rabinowitsch fluid model in an elastic peristaltic tube for homogeneous-heterogeneous reactions. Saravana *et al.[7]* looked at how heat flow affects the flow of a Rabinowitsch fluid in a peristaltic conduit with flexible, angled walls. Nadeem et al.[8] analyzed the Analysis of combined convective and viscous dissipation effects for peristaltic flow Rabinowitsch fluid model. S. Srinivas and R.Gayathri,[9] investigated Peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium. Sreenadh. *et al*.[10],Effect of heat transfer on the peristaltic flow of micropolar fluid in a vertical asymmetric channel with permeable walls, Kavitha, A., *et al.* [11] studied peristaltic flow of a micropolar fluid in a vertical channel with long wave length approximation, Sankad and Patil [12] calculated the pumping influences of dissimilar parameters taking place in the flow of Herschel Bulkley liquid in a non-uniform canal with a permeable lining. N.Chandrasekhara achari and Sreenadh [13] analyse erodible porous lining and peristalsis on the fluid of micropolar in a vertical channel In this paper the peristaltic flow of a Rabinowitsch fluid in a vertical channel with permeable lining on walls is investigated, under long wavelength and low Reynolds number assumptions. The fluid is injected into the channel perpendicular to the lower porous layer with constant velocity V_O and is sucked bent to the upper permeable layer with the identical velocity V_O, the pressure rise, friction forces, and velocity profiles are obtained. The results are obtained and discussed. The effects of various significant parameters are discussed graphically.

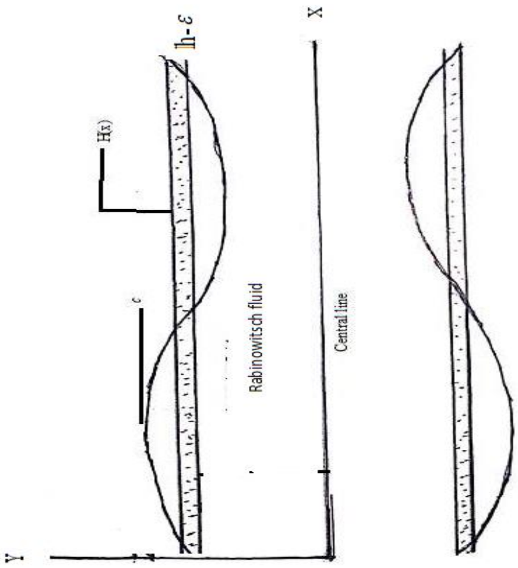
Rabinowitsch fluid model is a well-established model for studying the non-Newtonian nature of the fluid.TheshearingstressandshearingstrainfortheRabinowitschfluidmodelisconnectedbythe relation as given below:

$$\tau_{YZ} + \gamma\tau_{XY}^{3} = \mu\frac{\partial u}{\partial y} \qquad (1)$$

peristaltic flow in a curved channel for Rabinowitsch fluid. In the present paper, heat transfer and peristaltic transport of Rabinowitsch fluid flowing through a channel have been investigated.

2. Mathematical formation

Consider the flow of a non-Newtonian fluid complying Rabinowitsch fluid modelthrough a channel of uniform thickness. Sinusoidal wave proliferates on the wall of the channel and moving with speed *c*. Taking(*X*,*Y)* as rectangular coordinate in a fixed frame, the geometry of peristaltic flow is shown in Fig. 1.



The geometry of wall surface is given as

$$H(X,t') = a + b\sin\frac{2\pi}{\lambda}(x - ct) \qquad (2)$$

Where a is half channel width, b is the amplitude of the wave, t' is time, λ is the wavelength.

Where U and V are components of velocity in X and Y directions respectively in a fixed frame of reference.

The transformation between fixed frame and wave frame is given by

$$u' = U - c\,,\ v' = V\,, x' = X - ct\,,\ y' = Y \quad (3)$$

Where u', v', x', y' are axial velocity, transverse velocity, axial coordinate and transverse Coordinate respectively in wave frame

Introducing non-dimensional parameters as follows

$$u = \frac{u'}{c},\ v = \frac{v'}{c\delta},\ x = \frac{x'}{\lambda}, y = \frac{y'}{a},\ h = \frac{H}{a},\ \delta = \frac{a}{\lambda},\ p = \frac{p'a^2}{\lambda\mu c},\ \mathrm{Re} = \frac{\rho a c}{\mu},$$
$$\vec{\phi} = \frac{b}{a}\phi\ , \tau_{xx} = \frac{a\tau'_{xx}}{c\mu},\ \tau_{xy} = \frac{a\tau'_{xy}}{c\mu}, \tau_{yy} = \frac{a\tau'_{yy}}{c\mu}, \alpha = \frac{c^2\mu^2}{a}\gamma \quad (4)$$

Using Eq. (3) and Eq. (4) in Eq. (1), with the assumption of long wavelength and low Reynolds number approximation, we get

$$\tau_{YZ} + \alpha\tau_{XY}^3 = \frac{\partial u}{\partial y} \quad (5)$$

$$h = 1 + \phi \sin 2\pi x \quad (6)$$

and

$$\frac{\partial \tau_{xy}}{\partial y} - = \frac{\partial p}{\partial x} + \eta \quad (7)$$

$$\frac{\partial p}{\partial y} = 0 \quad (8)$$

The boundary conditions for equations, ar as follows

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y=0 \quad (9)$$

$$u = -1 - \beta\frac{\partial u}{\partial y} \quad \text{at} \ \ y=h-\varepsilon \quad (10)$$

where the dimensionless quantities α, ϕ are the parameters of pseudo plasticity,

amplitude ratio, ε porous thickening of the wall

Solving Eq. (6) with the boundary conditions (9) and (10), we obtain the velocity

$$u = 1 + G\left(\frac{y^2}{2} - \frac{(h-\varepsilon)^2}{2} - \beta(h-\varepsilon)\right) + \alpha G^3\left(\frac{y^4}{4} - \frac{(h-\varepsilon)^4}{4} - \beta(h-\varepsilon)^3\right) \quad (11)$$

where $G = \frac{\partial p}{\partial x} + \eta$

Where β is permeability parameter, and u is the velocity. The volume of flux q

at each cross section in a wave frame is given by

$$q = \int_0^{h-\varepsilon} u dy \tag{12}$$

Volume flow rate Q(X ,t) between the central line to channel wall in a fixed frame is

$$Q(X,t) = \int_0^H U(X,Y,t) \tag{13}$$

The time averaged volume flow rate over one period $T(=\lambda/c)$ of the peristaltic wave is

$$\vec{Q} = \frac{1}{T}\int_0^T Q dt \tag{14}$$

This can be reduced in dimensionless form as

$$Q = q + 1 \tag{15}$$

Using Eq(11) and Eq(12) we have

$$G(h-\varepsilon)^2(h-\varepsilon+3\beta)+\frac{3}{5}\alpha G^3(h-\varepsilon)^4(h-\varepsilon+5\beta)+3(q+h-\varepsilon)=0 \tag{16}$$

In the limiting case, as α→0 Eq. (16) reduces to result of Shapiro et al. (1969) $G = \frac{-3(q+h-\varepsilon)}{(h-\varepsilon)^2(h-\varepsilon+3\beta)}$

(17)

As Eq. (16) is the nonlinear equation of first order, it is difficult to find an analytic solution of pressure, however, for small values of the pseudo plasticity parameter , Eq. (16) can be perturbed as follows (α<<1), Eq. (16) can be perturbed as follows

$$P = P_0 + \alpha P_1 \tag{18}$$

so that

$$\frac{dp}{dx} = \frac{-3(q+h-\varepsilon)}{(h-\varepsilon)^2(h-\varepsilon+3\beta)} + \frac{81}{5}\alpha\frac{(q+h-\varepsilon)^3(h-\varepsilon+5\beta)}{(h-\varepsilon)^4(h-\varepsilon+5\beta)^3} - \eta \tag{19}$$

Using (14) and (18) we have

$$\frac{dp}{dx} = \frac{-3(Q-1+h-\varepsilon)}{(h-\varepsilon)^2(h-\varepsilon+3\beta)} + \frac{81}{5}\alpha\frac{(Q-1+h-\varepsilon)^3(h-\varepsilon+5\beta)}{(h-\varepsilon)^4(h-\varepsilon+5\beta)^3} - \eta \tag{20}$$

Pressure rise over one wave cycle is

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \tag{21}$$

The dimensionless frictional force F across one wave length is

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right) dx \tag{22}$$

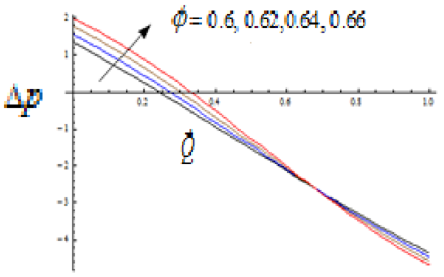


Fig 2. The variation of pressure rise Δp against time average volume flow rate $\vec{Q}$ for different values of ϕ with fixed α=0.002, β =0.03, ε =0.001, η =1.

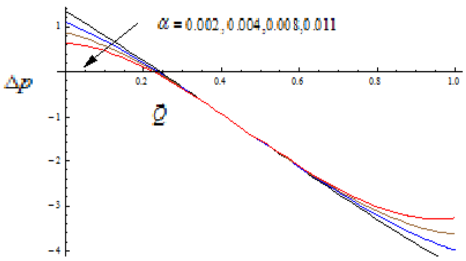


Fig 3. The variation of pressure rise Δp against time average volume flow rate $\vec{Q}$ for different values of α with fixed ϕ =0.5, β =0.03, ε =0.001, η =1.

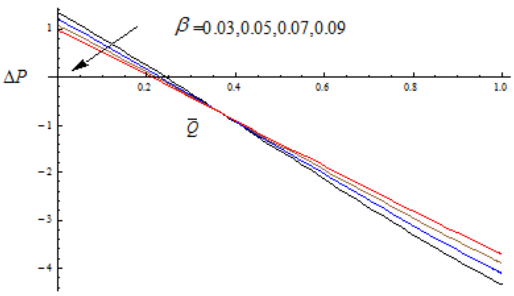


Fig 4. The variation of pressure rise Δp against time average volume flow rate $\vec{Q}$ for different values of β with fixed α=0.002, ϕ =0.5, ε =0.001, η =1.

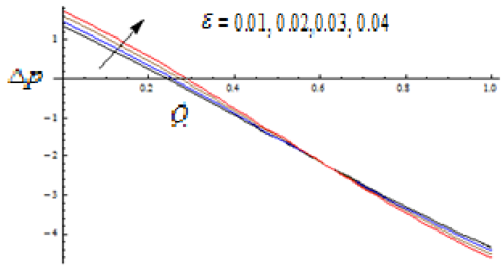


Fig 5. The variation of pressure rise Δp against time average volume flow rate $\bar{Q}$ for different values of ε with fixed α=0.002, ϕ =0.5, β =0.03, η =1.

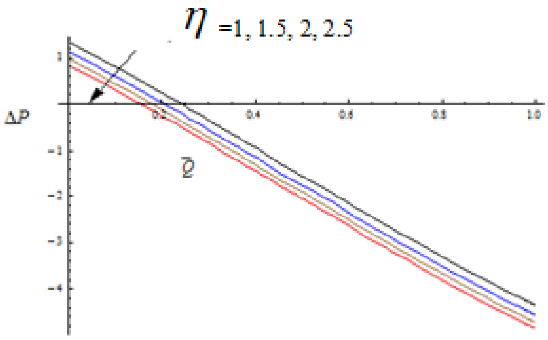


Fig 6. The variation of pressure rise Δp against time average volume flow rate $\bar{Q}$ for different values of η with fixed α=0.002, β =0.03, ϕ =0.5, ε =0.001, .

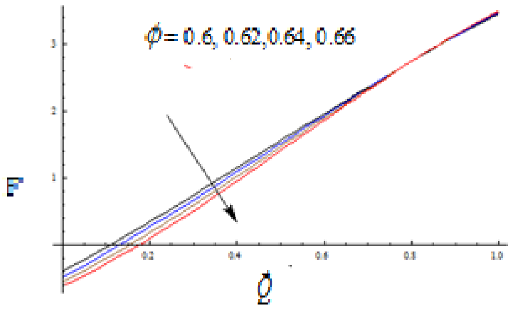


Fig 7. The variation of frictional force F against time average volume flow rate $\bar{Q}$ for different values ϕ with fixed α=0.002, β =0.03, ε =0.001, η =1.

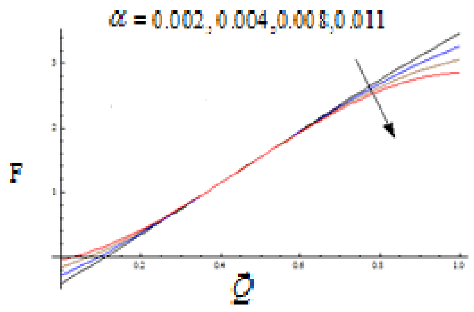


Fig 8. The variation of frictional force F against time average volume flow rate $\bar{Q}$ for different values of α with fixed ϕ=0.5, β=0.03, ε=0.001, ,η=1.

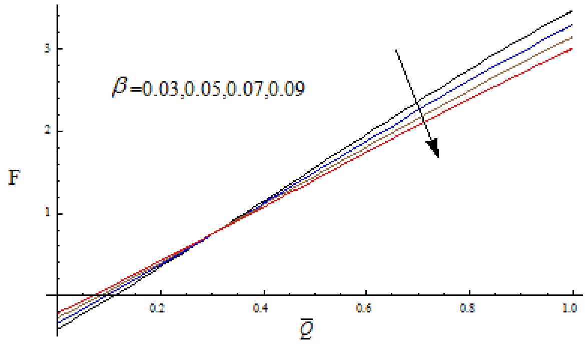


Fig 9. The variation of frictional force F against time average volume flow rate $\bar{Q}$ for different values of β with fixed α=0.002, ϕ=0.5, ε=0.001, η=1.

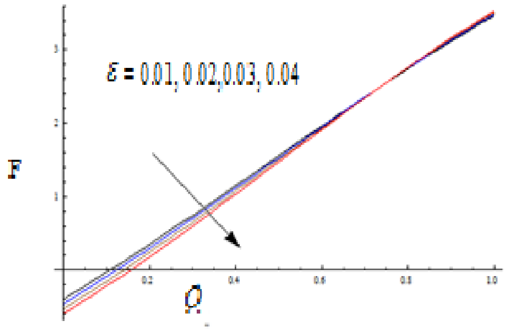


Fig 10. The variation of frictional force F against time average volume flow rate $\bar{Q}$ for different values of ε with fixed α=0.002, ϕ =0.5, β=0.03, ,η=1.

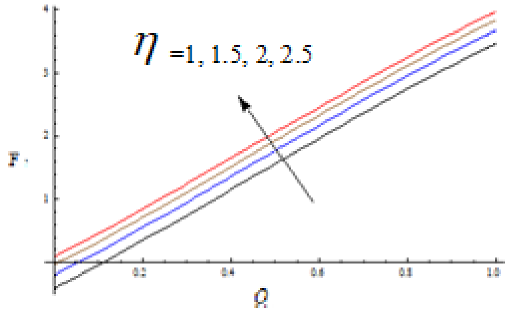
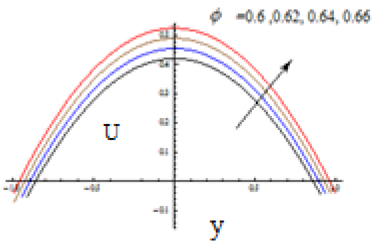


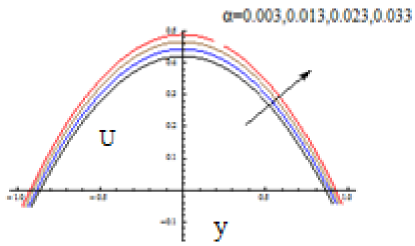
Fig 11. The variation of frictional force F against time average volume flow rate $\bar{Q}$ for different values of η with ϕ =0.5, α=0.002, β=0.03, ε=0.001,

3. Graphical Results

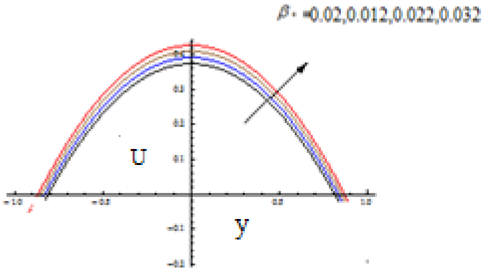
The results of influential parameters upon the pressure rise and frictional force are discussed through graphs. Fig (2-6) represents the effects of pressure rise Δp against time average volume flow rate $\bar{Q}$ for various parameters. In Fig (2) represents the behavior of pressure rise for various values of amplitude ratio ϕ by keeping remaining parameters fixed. It is observed that pressure rise increases with the increase in amplitude ratio ϕ . Fig (3) shows the variation of pressure rise for various values of pseudo plasticity α. Here it is seen that pressure rise decrease with increase in pseudo plasticity α upon fixing remaining parameters. Fig (4) depicts the behavior of pressure rise for different values of permeability parameter β. Here it is seen that pressure rise decrease with increase in permeability parameter β upon fixing remaining parameters. Fig (5) represents the behavior of pressure rise for various values of porous thickening of the wall ε by keeping remaining parameters fixed. It is observed that pressure rise increases with the increase in porous thickening of the wall ε. Fig (6) elucidate the changes in pressure rise for various values of angle of inclinations θ.It is observed that as η increases, the pressure rise behaves oppositely under fixed values of remaining parameters of interest. Fig (7-11) are drawn to study the effects of frictional force F against time average volume flow rate $\bar{Q}$ for various parameters. Fig (7) indicates the behavior of frictional force for different values of amplitude ratio ϕ. It is seen that frictional force increases with increase in amplitude ratio ϕ.Fig (8) demonstrates the effect of pseudo plasticity α on frictional force. It is noticed that frictional force increases with increase in the value of pseudo plasticity α. Fig (9) shows the influence of permeability parameter β. It is evident from the graphs that increase in the values of permeability parameter β, decreases the frictional force. Fig (10) indicates the behavior of frictional force for different values of porous thickening of the wall ε It is seen that frictional force increases with increase in porous thickening of the wall ε Fig (11) reveals the impact of angle of inclination θ on frictional force. It is noticed that growing value of angle of inclination η the frictional force rises accordingly.



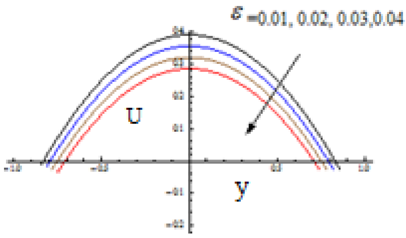
Velocity profile for different amplitude ratios



Velocity profile for different pseudo plasticity,



Velocity profile for different permeabily thickening



Velocity profile for different porous

4. Conclusions

In the present study, we have discussed the effects of peristaltic flow of Rabinowitsch fluid in a vertical channel with permeable walls. Analytical solutions for velocity, volume flux, pressure rise and frictional force are obtained and analyzed through graphs. The findings of the current model help in analyzing the flow of biological fluids under the peristaltic action.

Characteristics of the Rabinowitsch fluid depends upon the pseudoplastic parameter α. For different values of α, Rabinowitsch fluid behaves differently. If α=0 the fluid behaves like Newtonian fluid, when α<0 the fluid behaves like dilatant fluid and when α>0 the fluid behaves like pseudoplastic. The fluid is more thin when the pseudoplastic parameter α is large and the other observation is pressure rise drops quickly in region of peristaltic transport. The effects of amplitude ratio, pseudo plasticity, permeability parameter, porous thickening of the wall, angel of inclination on pressure rise Vs volume flow rate $\bar{Q}$ are studied and it is noted that with growing values of impact parameters like pseudo plasticity, permeability parameter, angle of inclination the pressure rise decreases. On the other hand we can see that frictional force increase with increased values of pseudo plasticity and angel of inclination. Where as it presents opposite behavior for amplitude ratio and porous thickening of the wall.

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