

Influence of Magnetic Field on Radial Vibration Characteristics of Elastic Hollow Cylinders

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Abstract

This study investigates the radial vibration behavior of an isotropic, elastic, hollow cylinder subjected to a uniform axial magnetic field. Using the principles of magneto-elasticity and classical elasticity theory, the governing equations are derived. Analytical solutions using Bessel functions are presented under stress-free boundary conditions. The impact of the magnetic field on the natural frequencies is discussed, demonstrating that the magnetic field significantly affects the vibrational characteristics of the structure.

Keywords

Radial vibration, hollow cylinder, magneto-elasticity, Bessel functions, magnetic field, natural frequencies

1. Introduction

The study of vibrational behavior in elastic structures plays a vital role in engineering, geophysics, and materials science. Among these structures, hollow cylinders are of particular interest due to their widespread application in pipelines, pressure vessels, aerospace components, and geotechnical systems. Understanding their vibrational characteristics is essential for ensuring structural integrity, predicting resonant behavior, and designing vibration-resistant systems. In recent years, the influence of external fields such as magnetic or electric fields on elastic bodies has garnered significant attention. This has led to the development of magneto-elasticity, a field that investigates the interaction between elastic deformation and magnetic fields. The behavior of materials under the combined influence of mechanical and magnetic forces is especially relevant in smart materials, sensors, and magneto-sensitive devices. While many studies have addressed axial and torsional vibrations in elastic cylinders, radial vibrations particularly in the presence of a magnetic field have not been explored as extensively. Radial vibrations are fundamental to understanding the dynamic response of cylindrical shells and are especially sensitive to external influences like magnetic fields. Dynamics vibrations and stresses in elastic cylinders and spheres are studied in [1]. A Magneto-elastic radial vibration of a transversely isotropic hollow cylinder is investigated in [2]. In the said paper that waves in a solid body propagating under the influence of a superimposed magnetic field can differ significantly from those propagating in the absence of a magnetic field. Also, the circular frequency increases with decreasing wall thickness of the cylindrical shell for all modes and increase with the higher modes of motion. Finally, it is noticed that the effect of the magnetic field on the mode frequency becomes less significant for the higher modes. In [3] the authors investigated free vibrations of thick hollow circular from three dimensional analyses. In this paper they developed a three-dimensional (3-D) method for the free vibration frequencies of hollow circular cylinders of elastic material. The method is based upon local coordinates whose origin is attached to the center of cylindrical wall. A vibration of circular cylinders of a perfectly conducting elastic material is examined in [4]. In this paper they studied radial vibrations of a long circular solid cylinder with a transverse magnetic field and rotary vibrations of a hollow cylinder with radial magnetic field are solved. Magneto-elastic longitudinal wave propagation in a transversely isotropic circular cylinder is presented in [5]. In this paper they investigated the longitudinal wave propagation in a perfectly conducting elastic circular cylinder in the presence of an axial initial magnetic field. A forced vibration of solid elastic cylinders is studied in [6]. Wave propagation in non-homogeneous magneto-electro

elastic hollow cylinders is investigated in [7]. In this paper they studied the Legendre orthogonal polynomial series expansion is employed to determine the wave_propagating characteristics in the hollow cylinders. The dispersion curves of the in homogeneous piezoelectric–piezomagnetic hollow cylinder and the corresponding non-piezoelectric and non-piezomagnetic hollow cylinders are calculated to show the influence of the piezoelectricity and piezomagnetism. Effect of magnetic field and non-homogeneity on the radial vibrations in hollow rotating elastic cylinder is presented in [8]. In the said paper, the displacement and stresses components has been obtained in analytical form involving Bessel function of first and second kind and of order n . The determination is concerned with the eigenvalues of the natural frequencies of the radial vibrations for different boundary conditions in the case of harmonic vibrations. Radial vibrations in an elastic hollow cylinder with rotation are studied in [9]. In this paper the authors studied the fundamental equations of elasto-dynamic in terms of radial displacement. The equation of elasto-dynamic is solved in terms of radial displacement. Effect of rotation magnetic field and a periodic loading on radial vibrations thermo-viscoelastic non homogeneous media is investigated in [10]. In this paper they studied the distribution of displacements, temperature, and stresses in the non-homogeneous medium in the context of generalized thermo-elasticity using GL (Green-Lindsay) theory in analytical form. Radial vibration of an elastic infinite cylinder with magnetic field is examined in [11]. A natural frequency of vibration of magnetic field under large deformation is presented in [12]. Radial vibrations on an elastic medium subjected to rotation and magnetic field are studied in [13]. Effect of rotation on a non-homogeneous infinite cylinder of orthotropic material with external magnetic field is investigated in [14]. Influence of non-homogeneity rotation magnetic field and initial stress on radial vibrations in thermo-viscoelastic isotropic media is studied in [15]. Rotation impact on the radial vibration of frequency equation of waves in a magnetized poroelastic medium is investigated in [16]. A longitudinal radial vibration of the elastic cylindrical shell filled with a viscous compressible fluid is examined in [17]. Eddy current loss behavior of hollow circular cylinders due to time varying electromagnetic field is presented in [18]. On the problem of radial vibrations in non-homogeneity isotropic cylinder under influence of initial stress and magnetic field is discussed in [19]. Study on the radial vibration and acoustic field of an isotropic circular ring radiator is investigated in [20]. Effect of the rotation on radial vibrations in a non-homogeneous orthotropic hollow cylinder is studied in [21]. In this paper, we investigate the radial vibrations of an elastic hollow cylinder subjected to a uniform magnetic field. The equations of motion are derived based on the principles of linear magneto-elastic theory. The effect of the magnetic field on the natural frequencies and mode shapes is analyzed to highlight its role in modifying the vibrational characteristics of the system. This study aims to provide deeper insights into the coupled magneto-elastic behavior of cylindrical structures and contribute to the design of advanced engineering systems exposed to magnetic environments.

2. Governing equations and solution of the problem

Let (r, θ, z) be the cylindrical polar coordinates. Consider a homogenous isotropic hollow elastic cylinder with inner and outer radii a and b , respectively, whose axis is in the direction of z -axis. The equations of motion are given in [1]:

$$\begin{aligned} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + F_r &= \rho \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} + F_\theta &= \rho \frac{\partial^2 v}{\partial t^2}, \\ \frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} + F_z &= \rho \frac{\partial^2 w}{\partial t^2}. \end{aligned}$$

(1)

Where ρ is the density of the cylinder. $\vec{u}(u, v, w)$ are the solid displacements. $F = (F_r, F_\theta, F_z)$ is the Lorentz force per unit volume due to the axial magnetic field is given by [22]

$$F = J \times B,$$

(2)

The stress components are given in [9]

$$\sigma_{rr} = (\lambda + 2\mu) \frac{\partial u}{\partial r} + \lambda \frac{u}{r},$$

$$\sigma_{\theta\theta} = \lambda \frac{\partial u}{\partial r} + (\lambda + 2\mu) \frac{u}{r}.$$

(3)

In eq. (3), λ, μ are all elastic constants $\sigma_{rr}, \sigma_{\theta\theta}$ solid stresses. In the case of radial vibrations, the equations of motion eq. (1) reduced to the following equations:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + F_r = \rho \frac{\partial^2 u}{\partial t^2}$$

(4)

Assume that the wave solution takes the following form

$$u(r, t) = u(r) e^{i\omega t}.$$

(5)

In eq. (5) ω is the frequency, and t is the time. Substituting eq. (2) and eq. (5) in eq. (4), we obtain

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} + \left(q^2 - \frac{1}{r^2} \right) u = 0.$$

(6)

The general solution of eq. (6) takes the following form

$$u(r) = AJ_1(qr) + BY_1(qr).$$

(7)

Where $q = \frac{\rho\omega^2}{\lambda + 2\mu + \mu_0 H_0^2}$. A, B are the arbitrary constants and $J_1(qr), Y_1(qr)$ are Bessel's functions of first kind.

Substituting equation (7) into equation (3), the components of stresses σ_{rr} given by

$$\alpha_{rr} = Aq[(\lambda + 2\mu)(\frac{1}{qr}J_1(qr) - J_2(qr)) + \frac{\lambda}{qr}J_1(qr)] + Bq[(\lambda + 2\mu)(\frac{1}{qr}Y_1(qr) - Y_2(qr)) + \frac{\lambda}{qr}Y_1(qr)].$$

(8)

3. Boundary conditions and frequency equation

In this case we study the frequency equation for different boundary conditions of hollow cylinder.

3.1 Free surface: In this case we obtain frequency equation for the boundary conditions which specify that the inner surface and outer surface are free

$$\sigma_{rr} = 0 \quad \text{at} \quad r = r_1,$$

$$\sigma_{rr} = 0 \quad \text{at} \quad r = r_2.$$

(9)

Substituting eq. (7) into eq. (9) we get two homogeneous linear equations

$$\begin{aligned} Aq[(\lambda + 2\mu)(\frac{1}{qr_1}J_1(qr_1) - J_2(qr_1)) + \frac{\lambda}{qr_1}J_1(qr_1)] + Bq[(\lambda + 2\mu)(\frac{1}{qr_1}Y_1(qr_1) - Y_2(qr_1)) + \frac{\lambda}{qr_1}Y_1(qr_1)] &= 0, \\ Aq[(\lambda + 2\mu)(\frac{1}{qr_2}J_1(qr_2) - J_2(qr_2)) + \frac{\lambda}{qr_2}J_1(qr_2)] + Bq[(\lambda + 2\mu)(\frac{1}{qr_2}Y_1(qr_2) - Y_2(qr_2)) + \frac{\lambda}{qr_2}Y_1(qr_2)] &= 0. \end{aligned} \quad (10)$$

Eliminating the constants A and B, we obtain the frequency equation in the form of

$$\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} = 0$$

(11)

Where

$$\begin{aligned} A_{11} &= q[(\lambda + 2\mu)(\frac{1}{qr_1}J_1(qr_1) - J_2(qr_1)) + \frac{\lambda}{qr_1}J_1(qr_1)], \\ A_{12} &= q[(\lambda + 2\mu)(\frac{1}{qr_1}Y_1(qr_1) - Y_2(qr_1)) + \frac{\lambda}{qr_1}Y_1(qr_1)], \\ A_{21} &= q[(\lambda + 2\mu)(\frac{1}{qr_2}J_1(qr_2) - J_2(qr_2)) + \frac{\lambda}{qr_2}J_1(qr_2)], \\ A_{22} &= q[(\lambda + 2\mu)(\frac{1}{qr_2}Y_1(qr_2) - Y_2(qr_2)) + \frac{\lambda}{qr_2}Y_1(qr_2)]. \end{aligned} \quad (12)$$

3.2 Fixed surface: In this case we obtain frequency equation for the boundary conditions which specify that the inner surface and outer surface are fixed

$$u(r) = 0 \quad \text{at} \quad r = r_1,$$

$$u(r) = 0 \quad \text{at} \quad r = r_2.$$

(13)

From eq. (7) and eq. (13), we obtain two homogeneous linear equations in A and B

$$AJ_1(qr_1) + BY_1(qr_1) = 0,$$
$$AJ_1(qr_2) + BY_1(qr_2) = 0.$$

(14)

Eliminating the constants A and B, we obtain the frequency equation in the form of

$$\begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix} = 0$$

(15)

Where

$$B_{11} = J_1(qr_1),$$

$$B_{12} = Y_1(qr_1),$$

$$B_{21} = J_1(qr_2),$$

$$B_{22} = Y_1(qr_2).$$

(16)

3.3 Inner surface fixed and outer free: In this we obtain frequency equation for the boundary conditions which specify that the inner surface and outer surface free

$$u(r) = 0 \quad \text{at} \quad r = r_1,$$

$$\sigma_{rr} = 0 \quad \text{at} \quad r = r_2.$$

(17)

From eq. (7), eq. (8) and eq. (17), we obtain two homogeneous linear equations in A and B

$$AJ_1(qr_1) + BY_1(qr_1) = 0,$$

$$Aq[(\lambda + 2\mu)(\frac{1}{qr_2}J_1(qr_2) - J_2(qr_2)) + \frac{\lambda}{qr_2}J_1(qr_2)] + Bq[(\lambda + 2\mu)(\frac{1}{qr_2}Y_1(qr_2) - Y_2(qr_2)) + \frac{\lambda}{qr_2}Y_1(qr_2)] = 0.$$

(18)

Eliminating the constants A and B, we obtain the frequency equation in the form of

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = 0$$

(19)

Where

$$\begin{aligned}
 C_{11} &= J_1(qr_1), \\
 C_{12} &= Y_1(qr_1), \\
 C_{13} &= q[(\lambda + 2\mu)(\frac{1}{qr_2}J_1(qr_2) - J_2(qr_2)) + \frac{\lambda}{qr_2}J_1(qr_2)], \\
 C_{14} &= q[(\lambda + 2\mu)(\frac{1}{qr_2}Y_1(qr_2) - Y_2(qr_2)) + \frac{\lambda}{qr_2}Y_1(qr_2)]. \\
 \end{aligned} \tag{20}$$

3.4 Inner surface free and outer fixed: In this we obtain frequency equation for the boundary conditions which specify that the inner surface free and outer surface fixed

$$\begin{aligned}
 \sigma_{rr} &= 0 \quad \text{at} \quad r = r_1, \\
 u(r) &= 0 \quad \text{at} \quad r = r_2. \\
 \end{aligned} \tag{21}$$

From eq. (7), eq. (8) and eq. (21), we obtain two homogeneous linear equations in A and B

$$\begin{aligned}
 Aq[(\lambda + 2\mu)(\frac{1}{qr_1}J_1(qr_1) - J_2(qr_1)) + \frac{\lambda}{qr_1}J_1(qr_1)] + Bq[(\lambda + 2\mu)(\frac{1}{qr_1}Y_1(qr_1) - Y_2(qr_1)) + \frac{\lambda}{qr_1}Y_1(qr_1)] &= 0 \\
 AJ_1(qr_2) + BY_1(qr_2) &= 0. \\
 \end{aligned} \tag{22}$$

Eliminating the constants A and B, we obtain the frequency equation in the form of

$$\begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix} = 0 \tag{23}$$

Where

$$\begin{aligned}
 D_{11} &= q[(\lambda + 2\mu)(\frac{1}{qr_1}J_1(qr_1) - J_2(qr_1)) + \frac{\lambda}{qr_1}J_1(qr_1)], \\
 D_{12} &= q[(\lambda + 2\mu)(\frac{1}{qr_1}Y_1(qr_1) - Y_2(qr_1)) + \frac{\lambda}{qr_1}Y_1(qr_1)], \\
 D_{21} &= J_1(qr_2), \\
 D_{22} &= Y_1(qr_2). \\
 \end{aligned} \tag{24}$$

4. Numerical results

In the above eq. (11), (15), (19), (23) we have calculated frequency. In order to illustrate theoretical results obtained the proceeding sections; we now present some numerical results. Materials chosen for this purpose are Magnesium, Copper is given in [23, 24]. The physical data are given below:

Magnesium

$$\lambda = 9.4 \times 10^{10} \text{ Nm}^{-2}, \mu = 4.0 \times 10^{10} \text{ Nm}^{-2}, \rho = 1.74 \times 10^3 \text{ Kgm}^{-3}, c_v = 1.04 \times 10^3 \text{ JKg}^{-1} \text{ deg}^{-3},$$

$$T_0 = 298 \text{ K}, K = 1.7 \times 10^2 \text{ Wm}^{-1} \text{ K}^{-1}, \alpha_t = 2.68 \times 10^6 \text{ Nm}^{-2} \text{ deg}^{-1}, \mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}, H_0 = \frac{10^7}{4\pi} \text{ Am}^{-1}.$$

Copper

$$\lambda = 7.76 \times 10^{10} \text{ Nm}^{-2}, \mu = 3.86 \times 10^{10} \text{ Nm}^{-2}, \rho = 8954 \times 10^3 \text{ Kgm}^{-3}, c_v = 383.1 \text{ JKg}^{-1} \text{ deg}^{-3},$$

$$T_0 = 293 \text{ K}, K = 386 \text{ Wm}^{-1} \text{ K}^{-1}, \alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}, \mu_0 = 4\pi \times 10^{-3} \text{ Hm}^{-1}, H_0 = 9 \times 10^5 \text{ Am}^{-1}.$$

Applying these parameter values in eq. (11), (15), (19), (23) the implicit relation between the frequency and ratio is obtained. Material -1 is magnesium and material-2 is copper and are used to show the results graphically. The frequency equation is obtained when the boundaries are free, fixed, mixed boundary conditions are calculated numerically. The values are computed in MS- Excel, and the results are depicted in figures 1-4. Figure.1-4 shows the plots of frequency against the ratio in the case of magnesium, copper. From the figure 1, as the ratio increases the frequency decreases in the case of free surface boundary. From the figure 2, frequency is in periodic in nature and frequency of material-1 values are greater than material-2. In general material-1 values are greater than material-2. From the figure 3, frequency is periodic in nature and after 0.4 the frequency decreases. The frequency of material-2 values are greater than material-1. From the figure4, frequency of material-1 increases and frequency of material-2 decreases.

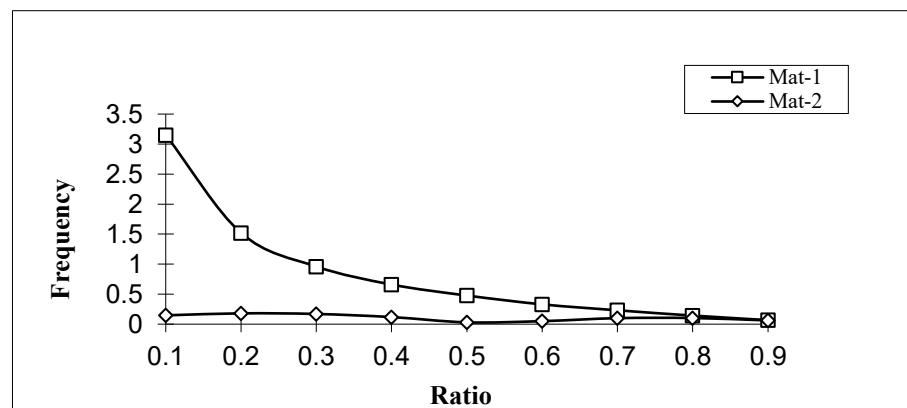


Fig-1: Variation of frequency against the ratio in the case of free traction surface

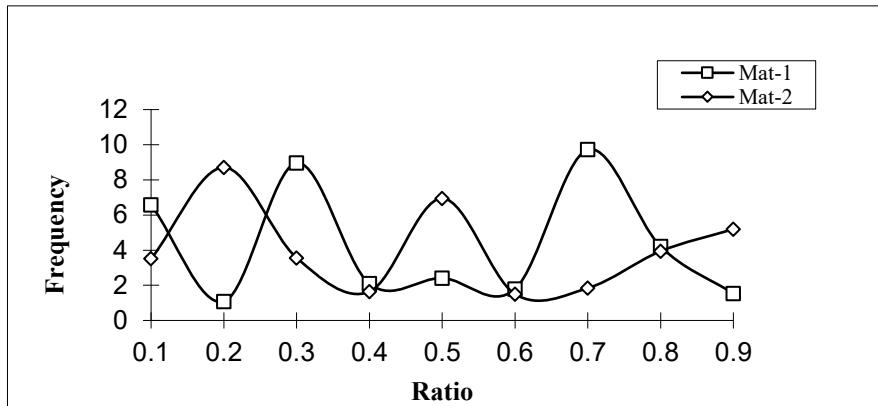


Fig-2: Variation of frequency against the ratio in the case of fixed surface

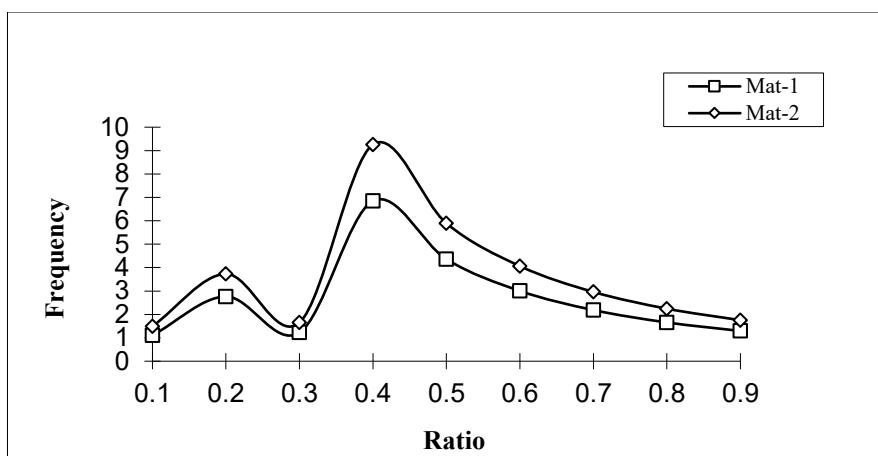


Fig-3: Variation of frequency against the ratio in the case of inner surface fixed and outer surface free.

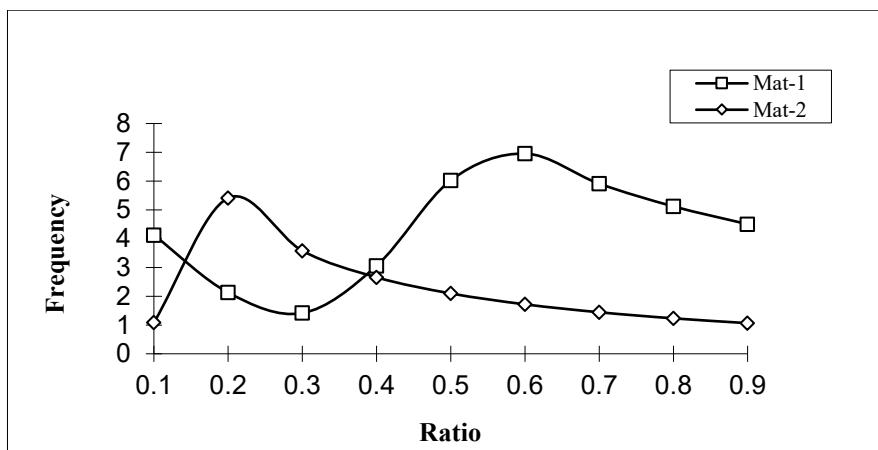


Fig-4: Variation of frequency against the ratio in the case of inner surface free and outer surface fixed.

Conclusion: In this study, we analyzed the radial vibrations of an elastic hollow cylinder subjected to a magnetic field, taking into account the coupled effects of elasticity and magnetism. The governing equations of motion were derived using the theory of magneto-elasticity and solved under appropriate boundary conditions. The results demonstrate that the presence of a magnetic field significantly alters the vibrational characteristics of the cylinder, leading to changes in both the natural frequencies and displacement profiles. Specifically, it was observed that an increase in magnetic field intensity leads to a noticeable stiffening effect, thereby increasing the frequencies of radial modes. These findings have important

implications in the design and analysis of magneto-sensitive structures in engineering and geophysical applications. Future work may extend this analysis to include temperature effects, material anisotropy, or dynamic loading conditions for a more comprehensive understanding of such systems. The frequency increases with magnetic parameter, indicating stiffening of the system due to the magnetic field. Magnesium shows higher frequencies than Copper, owing to its lower density and higher wave speed. Radial modes are more sensitive to magnetic influence for materials with lower mass density.

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