

Integrated Serviceability and Embodied Carbon Optimization of Bonded Post Tensioned Concrete Beams Under Time Dependent Prestress Loss

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Abstract Prestressed concrete design is often governed by service stress and long term deformation before ultimate flexural resistance becomes critical. This paper presents an integrated analytical framework for bonded post tensioned rectangular beams that couples staged prestress loss, transfer and service stress checks, cracking resistance, long term deflection, flexural and shear resistance, cradle to gate embodied carbon, multiobjective selection, and an exploratory reliability screen. A discrete study of 48,600 combinations covering 12 m, 15 m, and 18 m spans was evaluated. Of these, 13,859 satisfied all screening constraints. The loss model separates friction and anchorage, elastic shortening, concrete shrinkage, concrete creep, and low relaxation steel effects; environmental and low carbon material scenarios are introduced through explicit, auditable modifiers rather than hidden calibration. For the selected 15 m Pareto knee design, the predicted 50 year effective prestress loss was 16.3 percent, the service bottom fiber stress was 4.64 MPa with compression taken as positive, the long term midspan deflection was -1.1 mm, and the flexural demand to resistance ratio was 0.41. Relative to a feasible conventional baseline, the selected solution reduced the calculated cradle to gate carbon indicator by 24.9 percent while maintaining a governing utilization index of 0.53. The results show that increasing prestress force alone is not an efficient design strategy because transfer tension, concrete time effects, and steel related carbon can become controlling. A balanced combination of section depth, tendon eccentricity, and moderate strand area gives a more robust response. The framework is intended for transparent preliminary design and research comparison; project design still requires code specific detailing, durability assessment, and calibrated material data.

Index Terms bonded post tensioning, embodied carbon, flexural behavior, multiobjective optimization, prestress loss, reliability, serviceability, time dependent deformation

TABLE I
PRINCIPAL NOMENCLATURE

Symbol	Meaning
A, I, Z	section area, second moment, section modulus
A _p , A _s	prestressing and passive steel areas
b, h, L	beam width, depth, and span
c, a	neutral axis depth and equivalent stress block depth
E _c , E _p	elastic moduli of concrete and prestressing steel
e, d _p	tendon eccentricity and effective depth
f _{ck} , f _{ci}	28 day and transfer concrete strengths
f _j , f _{pe} , f _{ps}	jacking, effective, and ultimate tendon stresses
M _{cr} , M _n , M _u	cracking, nominal resistance, and factored demand
P _e	effective prestressing force
U	maximum normalized utilization index
G	cradle to gate embodied carbon indicator
phi	creep coefficient or strength reduction factor by context
beta	reliability index

I INTRODUCTION

Prestressing changes the governing design question for a concrete beam. In a reinforced concrete member the designer generally accepts flexural cracking and provides steel to control its consequences. In a prestressed member an intentionally applied compressive force shifts the stress field before service loading occurs. The section can therefore remain uncracked over a larger portion of its working range, use a smaller depth for a given span, and develop lower deflection and crack width. These advantages have made pretensioned and post tensioned systems central to bridges, parking structures, transfer girders, long span floors, tanks, and precast construction.

The same mechanism also creates a demanding sequence of checks. Concrete stresses must be acceptable at transfer, during handling, under sustained service action, and at ultimate resistance. The tendon force used in those checks is not constant because anchorage set, friction, elastic shortening, creep, shrinkage, and steel relaxation act over different time scales.

A technically sound prestressed design is consequently not obtained by maximizing tendon force. A larger force or greater eccentricity improves decompression and cracking resistance under positive bending, but it can cause excessive top fiber tension at transfer, excessive compression near the tendon level, higher camber, greater anchorage demand, and a larger quantity of high embodied carbon steel. A deeper section improves service stiffness and may allow a smaller prestressing force, yet increases self weight and concrete volume. Higher concrete strength can reduce creep and improve transfer capacity, but it may require more binder and can raise the carbon intensity of each cubic metre. The variables are coupled, and a sequential design process can conceal these interactions.

Current structural codes provide mature rules for the safety and serviceability of prestressed members. IS 1343, ACI 318, Eurocode 2, AASHTO LRFD, fib guidance, and PCI practice all require explicit consideration of prestress loss and staged stress conditions [1]-[7]. The numerical values, classifications, and permitted stresses differ, but the common engineering principle is clear: the effective force must be estimated at the relevant age and combined with the actions present at that stage. Research based on monitored full scale girders further shows that loss prediction contains meaningful uncertainty and that

excessive precision in an empirical estimate should not be confused with accuracy [8]-[13].

A second design pressure is now equally important. Cement production and steel manufacture contribute materially to the environmental burden of concrete structures, while prestressed systems concentrate both high strength concrete and high strength steel. Low clinker binders, supplementary cementitious materials, optimized geometry, and longer service life can reduce impact, but material substitution cannot be evaluated independently of shrinkage, creep, early strength, and durability [14]-[18]. A low carbon mixture that meets 28 day strength may still change transfer scheduling, loss development, or camber. Conversely, a prestressed geometry that uses less concrete may use more strand and may not minimize the whole member carbon indicator.

This study addresses these linked decisions through a transparent analytical and computational framework. It does not report a laboratory programme and does not present calculated values as measured data. Instead, a broad discrete parameter space is screened with closed form section mechanics, time dependent loss functions, flexural strain compatibility, a shear reinforcement check, and a cradle to gate material inventory. Feasible solutions are then evaluated by Pareto dominance using embodied carbon and governing structural utilization as competing objectives. An exploratory Monte Carlo analysis quantifies how uncertainty in concrete strength, effective prestress, resistance modelling, and loading changes the margin of the selected design.

The main contribution is integration. Previous work has established sophisticated procedures for prestress losses, nonlinear response, reliability, and bridge optimization, but preliminary member design is still commonly divided into separate spreadsheets or checks. The present framework places transfer stress, final service stress, cracking, deflection, ultimate resistance, shear, and carbon in one auditable chain. This makes it possible to identify why a design is infeasible, which variable governs, and how a carbon saving affects structural reserve. The resulting charts and tables are intended to support research development, teaching, and early design decisions rather than replace code compliant project calculations.

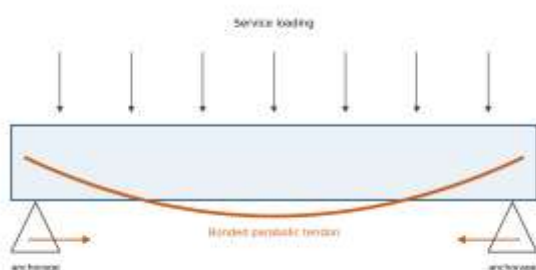


Fig. 1. Idealized simply supported bonded post tensioned beam with a parabolic tendon and equivalent prestress action.

II BACKGROUND AND LITERATURE REVIEW

A Mechanics and structural role of prestressing

The structural efficiency of prestressing follows from stress superposition and compatibility. An eccentric compressive force produces a uniform compression and a bending stress that opposes the tensile stress caused by gravity loading. When the tendon is bonded, strain changes at a cracked section are transferred locally to the tendon, so tendon stress can increase substantially as the section approaches ultimate curvature. An unbonded tendon responds to average deformation over its length and requires a different ultimate stress model. The present work is limited to bonded internal tendons because this assumption supports section based strain compatibility and isolates the interaction between force, eccentricity, time dependent loss, and section geometry.

Tendon profile is a structural variable, not merely a drafting choice. A parabolic profile approximates the moment diagram of a simply supported beam under distributed loading and can be represented by an equivalent upward load. External prestressing tests by Tan and Ng showed that deviators and tendon configuration change service deflection, cracking, tendon stress increase, strength, and ductility [19]. Later experimental and analytical studies have confirmed that increased eccentricity can improve the resisting moment but can also amplify secondary effects or reduce the available eccentricity as deflection develops in external systems [20]-[22]. Internal bonded tendons avoid some external tendon effects, although local duct curvature, friction, grouting, and anchorage zone behavior remain important.

B Prestress loss and long term response

Prestress loss is usually divided into immediate and time dependent components. Immediate effects include friction, wobble, anchorage seating, and the elastic response of concrete during force transfer or sequential stressing. Long term effects result from concrete shrinkage, concrete creep, and relaxation of prestressing steel. The components interact because a reduction in steel force changes concrete stress and therefore changes subsequent creep. Detailed time step methods capture this interaction more faithfully than a single lump sum percentage, especially when construction stages, composite decks, or changing environmental exposure are important.

NCHRP Report 496 extended loss estimation for high strength pretensioned bridge girders and emphasized realistic concrete modulus, creep, and shrinkage [8]. Barr, Kukay, and Halling compared measured losses in high performance concrete bridge girders with AASHTO and NCHRP procedures, illustrating that model bias can change with member and material conditions [9]. Swartz, Scanlon, and Schokker explained the mechanics behind the AASHTO time dependent procedure [10]. Garber

and coauthors assembled full scale data, investigated loss sensitivity, and proposed a procedure balancing usability and conservatism [11]-[13]. Collectively, these studies support component based modelling and discourage unqualified use of a fixed percentage.

Concrete creep and shrinkage are particularly uncertain over multi decade periods. ACI 209.2R compares established prediction approaches, while Model B3 and Model B4 connect long term deformation with age, humidity, geometry, composition, and drying [23]-[25]. The RILEM B4 recommendation explicitly targets multi decade application [25]. The present research does not reproduce those comprehensive formulations. It uses bounded functions calibrated to plausible design ranges and retains humidity, notional size, strength, and mixture modifiers as visible inputs. This level of model is appropriate for comparative screening, but final design should use a recognized model calibrated to project concrete when deformation sensitivity is high.

Long term deflection is not governed by loss alone. Creep changes curvature under sustained gravity action and under the eccentric prestress moment, while progressive loss reduces the upward action. Naaman's work on time dependent deflection and subsequent PCI guidance showed why separate multipliers or time step calculations are needed for camber and downward deflection [26]. Because these actions can oppose one another, apparently small modelling differences may reverse the sign of the predicted long term movement. A design that merely meets an instantaneous limit may therefore show unacceptable long term sag or excessive residual camber.

C Optimization reliability and environmental performance

Optimization of prestressed concrete has a long history. Lounis and Cohn formulated multiobjective approaches for prestressed structures, and Barakat, Bani Hani, and Taha combined multiple objectives with reliability constraints [27], [28]. Al Harthy and Frangopol treated loads, material properties, prestressing force, and response models as random variables in reliability based beam design [29]. More recent bridge research has incorporated cost, carbon dioxide emissions, safety, durability, and lifetime reliability in heuristic or surrogate assisted optimization [30]-[34]. These studies demonstrate that a mathematically optimal design depends on the chosen objectives, boundary conditions, and reliability targets; a single least weight solution is rarely sufficient.

Environmental optimization has likewise moved from material substitution to whole member and whole bridge decisions. Penades Pla and coauthors integrated optimization with life cycle assessment for a prestressed precast bridge [31]. Garcia Segura and Yepes investigated cost, emissions, and safety in post tensioned

box girders [30], while Yepes Bellver and colleagues used surrogate modelling for post tensioned slab bridge decks [33]. Jayasinghe and coauthors showed that geometric optimization can materially reduce embodied carbon in reinforced concrete beams [35]. These results motivate a member level framework in which geometry, prestress, passive reinforcement, and concrete scenario are evaluated together.

Carbon values are inherently regional and time dependent. Electricity mix, clinker factor, transport distance, recycled steel content, curing energy, plant efficiency, and environmental product declaration boundaries can all change the result. Habert and coauthors summarized multiple decarbonization pathways for cement and concrete, and Scrivener, John, and Gartner examined technically and economically viable low carbon cements [14], [15]. Smilauer, Dohnalova, and Martauz reported that a low clinker hybrid cement can exhibit shrinkage and creep within the dispersion of established structural concrete models [18]. This is encouraging, but it also shows why a universal penalty or benefit should not be assumed. The three mixture scenarios used here are therefore sensitivity cases, not product declarations.

TABLE II
REPRESENTATIVE LITERATURE AND RELEVANCE TO THE PRESENT FRAMEWORK

Study	Method	System	Key relevance
Tadros et al. [8]	Loss model	High strength girders	Concrete properties and staged loss
Barr et al. [9]	Field monitoring	HPC bridge girders	Observed model bias and regional effects
Garber et al. [11]-[13]	Database and model	Full scale girders	Balance accuracy and usability
Tan and Ng [19]	Beam tests	External tendons	Profile and deviator effects
Al Harthy and Frangopol [29]	Reliability design	Prestressed beams	Random loads, force, and models
Garcia Segura and Yepes [30]	Multiobjective	Post tensioned bridges	Cost, carbon, and safety
Penades Pla et al. [31]	Optimization LCA	Prestressed bridge	Whole life environmental comparison
Smilauer et al. [18]	Material tests	Low clinker cement	Creep and shrinkage remain mix dependent



Fig. 2. Integrated analysis sequence used to connect structural response, embodied carbon, and uncertainty.

D Research gap

The literature identifies three gaps relevant to preliminary member design. First, loss models are often

applied after a section and tendon layout have already been selected, even though the selection changes concrete stress and hence creep loss. Second, carbon optimization may treat strength class and binder scenario as inventory variables without carrying their serviceability implications into the stress and deflection checks. Third, deterministic feasible designs are frequently ranked by a single objective even when the closest service limit has little reserve under ordinary uncertainty. The present study responds with a coupled, fully disclosed parameter study and a separate probabilistic screen of the selected compromise design.

III RESEARCH SCOPE AND CONTRIBUTIONS

The investigation considers a simply supported prismatic rectangular beam with a bonded parabolic post tensioning tendon. This idealization was selected because it retains the essential mechanics of force, eccentricity, cracking, camber, and flexural resistance while allowing thousands of designs to be evaluated transparently. The beam carries self weight, an 8 kN per metre superimposed dead load, and a 20 kN per metre live load. Spans of 12 m, 15 m, and 18 m represent building transfer members and short to medium bridge or industrial framing applications. The numerical study is not tied to a proprietary tendon system.

Three contributions distinguish the framework. The first is stage consistency: the force at transfer is used with self weight, while the final effective force is used with full service loading. The second is explicit coupling of structural and environmental quantities. The concrete scenario modifies the cradle to gate factor and, conservatively, the shrinkage and creep multipliers, so an environmental choice can alter feasibility. The third is decision transparency. Every constraint is normalized into a utilization ratio, and the maximum ratio identifies the governing limit. Feasible designs are not compressed into one opaque score; the Pareto front preserves the tradeoff between calculated carbon and reserve.

The model deliberately separates verification from validation. Algebraic verification checks units, equilibrium, limiting cases, and independent hand calculations. Literature benchmarking checks whether predicted loss magnitudes and parameter trends are credible. Experimental validation would require measured concrete modulus, shrinkage, creep, tendon force, camber, and load deflection response for the selected materials and geometry. Because those measurements were not available, the word validation is not used for the calculated database. This distinction is essential for responsible use of analytical research.

TABLE III
RESEARCH QUESTIONS AND EVALUATED EVIDENCE

Research question	Primary evidence	Decision output
How do loss components evolve	Component model and RH cases	Retained force over 50 years
Which variables govern service response	Stress, cracking, and deflection grids	Constraint utilization
Can carbon fall without losing reserve	Feasible set and Pareto front	Minimum and knee designs
How stable is the selected design	Monte Carlo screening	Failure probability and beta

IV ANALYTICAL FORMULATION

A Section stress at transfer and service

Linear elastic stress superposition is used while the section remains uncracked. Compression is defined as positive. The tendon lies below the section centroid, so its eccentric force increases compression at the bottom fiber and may create tension at the top fiber. Positive gravity moment has the opposite effect. For a symmetric section, the top and bottom stresses are obtained from the direct stress, prestress moment, and external moment terms shown in (1). At transfer, the force after friction and elastic shortening is combined with beam self weight. At final service, the effective force after all modelled losses is combined with self weight, superimposed dead load, and live load.

$$\sigma_{t,b} = \frac{P}{A} \mp \frac{Pe}{Z} \pm \frac{M}{Z} \quad (1)$$

The transfer compression screen is 0.60 times the transfer concrete strength. The transfer tensile stress screen is 0.50 times the square root of that strength. At final service, the top compression screen is 0.45 times the 28 day concrete strength and the bottom tension screen is 0.45 times its square root. These values are consistent with the order of limits used in prestressed concrete practice but are defined here as study constraints, not as a substitute for a selected code exposure class or service classification. A project application must replace them with the exact governing provisions, including any requirement for bonded reinforcement in a tensile zone.

The use of gross section properties is conditional on the cracking check. A design is screened out when the applied service moment exceeds the calculated cracking moment. This maintains internal consistency between the uncracked stress and deflection models. A partially prestressed design could remain acceptable after cracking, but it would require a cracked section analysis, tension stiffening model, crack width calculation, and a different long term stiffness treatment. Those extensions are outside the current scope.

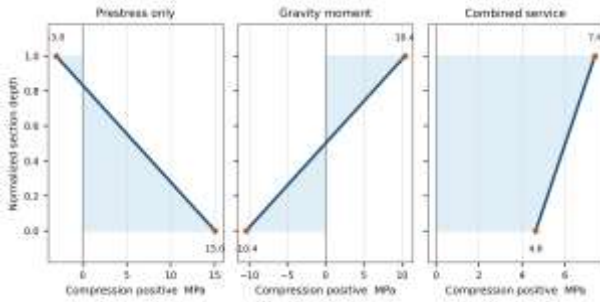


Fig. 3. Stress superposition for the selected design. Compression is positive and numerical values are in MPa.

B Prestress loss components

Total tendon stress loss is represented by five components in (2). Friction and anchorage effects are grouped because the parameter study uses one symmetric effective profile rather than a detailed jacking sequence. Elastic shortening is retained even for post tensioning because sequential stressing and force transfer alter concrete strain; a sequence coefficient of 0.50 is used. Shrinkage and creep are related to relative humidity, notional member size, concrete strength, and mixture scenario. Relaxation is based on low relaxation strand and is represented as a bounded long term fraction of jacking stress.

$$\Delta f_{p,T} = \Delta f_{p,F} + \Delta f_{p,ES} + \Delta f_{p,SH} + \Delta f_{p,CR} + \Delta f_{p,R} \quad (2)$$

$$\Delta f_{p,ES} = \eta_s \frac{E_p}{E_{ci}} \sigma_{cgp} \quad (3)$$

In (3), the concrete stress at the tendon centroid is evaluated from the jacking force and self weight. The coefficient for sequential stressing prevents the full pretensioned elastic shortening expression from being imposed on a post tensioned operation. The friction and anchorage component is 2.5 percent of jacking stress in the comparative model. Actual values depend on duct curvature, wobble coefficient, tendon length, jack direction, wedge seating, and system tolerances; those quantities must be calculated from shop drawings and stressing records in a final design.

The ultimate shrinkage strain is based on a reference value of 380 microstrain and modifiers for humidity, notional size, strength, and mixture scenario. Only the compatible fraction of free shrinkage is converted to tendon stress through the coefficient in (4). The formulation is intentionally simple enough to audit. It is not a substitute for ACI 209, fib Model Code, Eurocode 2, B3, or B4 when long term deformation is a critical design action [23]-[25].

$$\Delta f_{p,SH} = \eta_{sh} E_p \epsilon_{sh,\infty} \quad (4)$$

Creep loss in (5) is proportional to the modular ratio, an ultimate creep coefficient, and the initial concrete stress at the tendon level. A compatibility coefficient of 0.60 accounts approximately for the declining concrete stress as prestress is lost. The creep coefficient rises as relative humidity falls, decreases with concrete strength, and

varies with notional size. The low carbon scenarios apply factors of 1.07 and 1.15 to creep and 1.05 and 1.12 to shrinkage. These are deliberately conservative scenario modifiers used for sensitivity analysis. They do not claim that every low clinker concrete has higher time dependent deformation; published hybrid cement data demonstrate that low carbon systems can remain within conventional model dispersion [18].

$$\Delta f_{p,CR} = \eta_{cr} \frac{E_p}{E_c} \phi_{\infty} \sigma_{cgp} \quad (5)$$

Time development is reconstructed by multiplying each final loss by a dimensionless age function. Immediate losses are active at the start, shrinkage follows a square root exponential function, creep follows a slower one third power function, and relaxation develops logarithmically. Equation (6) defines retained jacking stress, while (7) lists the shrinkage and creep functions with time in days. The functions approach the final component values at long duration and allow results to be plotted from stressing to 50 years without discontinuity.

$$r_p(t) = 1 - \frac{\sum_i \Delta f_{p,i} g_i(t)}{f_{pj}} \quad (6)$$

$$g_{sh} = 1 - e^{-(t/180)^{0.50}}, \quad g_{cr} = 1 - e^{-(t/365)^{0.33}} \quad (7)$$

The model treats environmental humidity as constant. Real structures experience seasonal humidity and temperature, local sealing, differential drying, and construction stage changes. A time step analysis can incorporate those effects by updating concrete compliance and shrinkage for each interval. The present functions are best interpreted as a comparative index of loss severity. Their principal research value is that the same environmental assumption passes consistently into effective force, service stress, cracking resistance, camber, deflection, and reliability.

C Cracking and deflection

The cracking moment follows from the external moment required to reduce bottom fiber compression to the negative modulus of rupture. Equation (8) includes both the prestress moment and the uniform direct compression. The modulus of rupture is taken as 0.62 times the square root of concrete strength in MPa for the screening study. The ratio of service moment to cracking moment becomes one component of the utilization index. A value close to unity signals that modest force loss, live load variation, restraint, or thermal gradient may invalidate the uncracked stiffness assumption.

$$M_{cr} = P_e e + \left(\frac{P_e}{A} + f_r \right) Z \quad (8)$$

A parabolic tendon with zero end eccentricity and midspan eccentricity e is converted to the equivalent uniform uplift in (9). The immediate midspan deflection

of the simply supported member is then calculated with (10). Positive deflection denotes downward movement. For long term response, sustained dead load and 30 percent of live load are evaluated with an effective modulus based on 60 percent of the creep coefficient, while the remaining live load is treated as transient. The final prestressing force is used for uplift. This approach captures the direction and relative magnitude of major effects, but a rigorous construction stage analysis would integrate changing prestress and creep curvature through time.

$$W_p = \frac{8P_e e}{L^2} \quad (9)$$

$$\delta_{mid} = \frac{5(w_s - w_p)L^4}{384E_c I} \quad (10)$$

The deflection screen is the absolute value of predicted long term midspan movement divided by L over 300. Absolute movement is used because excessive upward camber can be as disruptive as downward sag for finishes, drainage, bearings, and connected members. The model does not include cracking, creep recovery, nonstructural restraint, composite slabs, thermal curvature, support settlement, or erection tolerances. Accordingly, cases near the limit require a refined analysis even when all other checks have reserve.

D Flexural resistance and shear reinforcement

Ultimate flexural resistance is evaluated by strain compatibility for a rectangular compression zone. The effective tendon strain is combined with the strain increment caused by section curvature, and the bonded tendon stress is capped at 0.90 times specified tensile strength. Passive tension steel is taken as 0.22 percent of gross section area and is capped at 500 MPa. Force equilibrium gives the neutral axis and equivalent compression block in (11). Nominal moment is then obtained from the internal force couples in (12), with a flexural reduction factor of 0.90 in the design resistance.

$$0.85f'_c b a = A_p f_{ps} + A_s f_s \quad (11)$$

$$M_n = A_p f_{ps} (d_p - \frac{a}{2}) + A_s f_s (d_s - \frac{a}{2}) \quad (12)$$

The flexural feasibility condition requires the reduced nominal resistance to exceed 1.10 times the factored moment demand. The additional 10 percent is a study reserve that separates bare code feasibility from the optimization boundary. It also prevents a carbon minimum from resting exactly on an ultimate limit that is later examined probabilistically. Ductility is not established solely by the strength equation. Final detailing must check tendon ratio, passive reinforcement, strain classification, development, confinement, and rotation capacity under the applicable code.

Shear is included so that an apparently efficient flexural member does not omit a major quantity of passive steel. Concrete contribution is represented by a square root strength expression, the vertical component of the parabolic prestress is credited near the support, and the required two leg stirrup ratio follows (13). A minimum ratio is enforced, and 10 mm two leg stirrups are limited to 300 mm spacing in the inventory. The simplified check does not address disturbed end regions, anchorage bursting, web crushing, tendon curvature forces, or fatigue. These must be designed independently.

$$\frac{A_v}{s} \geq \frac{V_u / \phi_v - V_c - V_p}{f_{yv} d_p} \quad (13)$$

E Embodied carbon objective

The environmental objective is a cradle to gate indicator, not a complete life cycle assessment. Equation (14) combines concrete volume, prestressing steel mass, passive steel mass, and an allowance for ducts and anchorages with scenario factors. Reference concrete factors are 320, 360, and 410 kg CO₂e per cubic metre for 40, 50, and 60 MPa classes. The C25 and C40 scenarios multiply those concrete factors by 0.78 and 0.62. Prestressing steel and passive steel are assigned 2.50 and 1.70 kg CO₂e per kilogram, respectively. These values are controlled assumptions for comparison; project decisions should use verified environmental product declarations with matched system boundaries.

$$G = V_c \gamma_c + m_p \gamma_p + m_s \gamma_s + G_a \quad (14)$$

Carbonation uptake, transport to site, stressing equipment, formwork, curing heat, construction waste, maintenance, service life extension, demolition, and recycling credits are excluded. The omission is intentional because those terms require location and project data not supplied to the study. The result is therefore reported as a member level indicator in kg CO₂e per beam. It is suitable for comparing alternatives generated under the same boundary, but it must not be interpreted as a product declaration or the total footprint of a completed structure.

F Constraint aggregation and Pareto selection

Each limit state is expressed as demand divided by its screening limit. The governing utilization U in (15) is the maximum of transfer compression, transfer tension, service compression, service tension, cracking, long term deflection, and ultimate flexure ratios. A design is feasible when U is not greater than one, the effective tendon stress remains within the defined range, the geometric eccentricity preserves 90 mm tendon cover to the bottom, and the span to depth ratio lies between 10 and 20. The maximum operator preserves the physical identity of the controlling condition.

$$U = \max(u_{tr,c}, u_{tr,t}, u_{ser,c}, u_{ser,t}, u_{cr}, u_{\delta}, u_M) \quad (15)$$

The optimization minimizes both the carbon indicator and utilization as stated in (16). One design dominates another when it has no larger value of either objective and is strictly better in at least one. The nondominated set forms the Pareto front. A normalized distance to the ideal point selects a knee solution for detailed discussion. This selection is transparent and reproducible, but it is not a universal preference. A client prioritizing minimum initial carbon may choose a point nearer U equals one, whereas a critical facility may accept more material for lower utilization and higher reliability.

$$\min \mathbf{F}(\mathbf{x}) = [G(\mathbf{x}), U(\mathbf{x})] \quad \text{subject to} \quad U \leq 1 \quad (16)$$

G Exploratory reliability formulation

Deterministic utilization does not show the probability that uncertain resistance falls below uncertain demand. The ultimate limit state in (17) uses the same 10 percent reserve adopted in deterministic screening. Concrete strength is lognormal with a coefficient of variation of 0.10; effective prestress is normal with a coefficient of variation of 0.07; dead and live loads are normal with coefficients of variation of 0.08 and 0.18; and the resistance model is lognormal with a coefficient of variation of 0.06. Negative load samples are truncated. Service tension and deflection limit states are evaluated in parallel.

$$g_u(\mathbf{X}) = \phi M_n(\mathbf{X}) - 1.10 M_u(\mathbf{X}) \quad (17)$$

$$P_f = \Pr[g(\mathbf{X}) < 0], \quad \beta = -\Phi^{-1}(P_f) \quad (18)$$

A seeded Monte Carlo simulation with 100000 samples estimates the probability of a negative margin. The continuity estimator adds one half event to avoid reporting a mathematically zero probability from a finite run. Reliability index beta is calculated from the inverse standard normal distribution in (18). The assumed distributions are illustrative and are not calibrated to a particular region, contractor, tendon supplier, or load survey. Their purpose is to reveal sensitivity and compare margins, not certify a target reliability under IS, ACI, AASHTO, or Eurocode provisions.

TABLE IV
PRESTRESS LOSS COMPONENTS USED IN THE COMPARATIVE MODEL

Component	Model basis	Time scale	Project replacement data
Friction and anchorage	2.5% of jacking stress	Immediate	Duct and stressing plan
Elastic shortening	0.50 modular ratio term	Immediate	Stressing sequence
Shrinkage	Humidity, size, strength, mix	Months to years	Curing and exposure
Creep	Stress, humidity, size, strength, mix	Years to decades	Concrete compliance
Steel relaxation	2.5% final fraction	Logarithmic	Strand class and stress

TABLE V
NORMALIZED STRUCTURAL SCREENING CONSTRAINTS

Limit state	Utilization definition	Feasible
Transfer compression	max stress / 0.60 fci	≤ 1.00
Transfer tension	max tension / 0.50 sqrt fci	≤ 1.00
Service compression	top stress / 0.45 fck	≤ 1.00
Service tension	bottom tension / 0.45 sqrt fck	≤ 1.00
Cracking	service moment / cracking moment	≤ 1.00
Deflection	absolute long term delta / L over 300	≤ 1.00
Ultimate flexure	1.10 demand / reduced resistance	≤ 1.00

H Stage consistency and model boundary

The transfer, service, and ultimate calculations share geometry but do not share an identical structural state. At transfer, concrete has the specified early strength, the section carries its self weight, the tendon has experienced only immediate loss, and nonstructural restraint is assumed absent. At service, 28 day material properties are used, superimposed dead and live actions are present, and the final effective tendon force includes time-dependent losses. At ultimate, bonded strain compatibility determines tendon stress rather than holding it at the service value. Preserving these stage definitions prevents the common error of combining a force from one age with material properties or loads from another.

The stress screens are evaluated at midspan because the simply supported uniform-load moment and tendon eccentricity reach their maxima there. That location is sufficient for the idealized global flexural comparison, but it is not sufficient for detailed design. Near the anchorage, direct compression, local bearing, bursting, spalling, duct curvature, and reduced eccentricity create a different stress field. Intermediate tendon high points, openings, diaphragms, and concentrated loads may also govern. A project model should evaluate enough stations to capture changes in section, tendon profile, load, and restraint.

Equivalent uplift represents the vertical component of the curved tendon force for the selected parabolic profile. It is used only in the deflection and simplified shear inventory; the fiber-stress equations use the prestress force and eccentricity directly. This avoids counting the same prestress action twice. In a continuous member, equivalent loading generates primary and secondary effects that depend on support restraint and stiffness. The simply supported formulation contains no secondary prestress moment and therefore cannot be transferred unchanged to continuous frames or segmental bridges.

The carbon objective is calculated only after a case has been structurally evaluated. An infeasible design is not made acceptable because its material indicator is low. Conversely, feasibility does not grant environmental preference; all retained alternatives remain available for comparison. This separation mirrors constrained

multiobjective optimization, in which objectives rank solutions inside an admissible domain. It also allows the designer to introduce additional constraints such as fire resistance, vibration, fatigue, durability, congestion, maximum depth, or minimum reliability without redefining the carbon equation.

Numerical safeguards are intentionally simple. Square-root material expressions receive positive strengths, tendon eccentricity is capped by cover, negative loads are removed in probabilistic sampling, and the effective tendon stress has a physical lower bound. The neutral-axis solution is bracketed within the section. Each result stores the seven component utilization ratios as well as their maximum so that a feasible flag can be audited. Calculations are made with unrounded values; rounding occurs only in captions and tables.

These safeguards do not replace independent engineering review. A programming error can remain internally consistent, and a mathematically correct implementation can represent the wrong physical system. Verification therefore combines equation-level recomputation, limiting cases, monotonic trend checks, and visual examination of response surfaces. Validation remains a separate comparison with measurements. The distinction is especially important when optimization searches thousands of cases, because the algorithm will exploit any omitted constraint or unintended sign more efficiently than a manual design process.

V COMPUTATIONAL STUDY DESIGN

A Discrete variables and search domain

The design space contains 48,600 unique combinations. Full enumeration was selected instead of a stochastic search because the number of discrete cases remains manageable and every feasible point can be inspected. This avoids dependence on population size, mutation rate, convergence tolerance, or a particular optimizer seed. It also makes the Pareto front exact within the stated grid. The enumeration should not be mistaken for a proof of a continuous global optimum; a design between two grid levels may perform better. The grid is a transparent screening resolution from which local refinement can begin.

Three spans are combined with five web widths, ten overall depths, three concrete strengths, nine strand counts, four eccentricity ratios, and three concrete scenarios. Width and depth are constant along the span. The tendon is represented by one equivalent bonded group with a parabolic centroidal profile. Strand count changes in increments of two to reflect symmetric tendon layouts, although actual anchorage hardware may impose a different grouping. The eccentricity ratio is measured from the gross section centroid and is truncated when necessary to preserve 90 mm between the tendon centroid and the bottom surface.

The span to depth constraint is intentionally broad. A lower ratio of ten avoids an excessively deep member, while an upper ratio of twenty removes slender sections for which deflection and vibration would usually require more refined consideration. These bounds do not prescribe architectural proportions. They merely prevent the numerical search from exploiting geometries outside the intended beam class. Likewise, the width range does not establish adequate anchorage-zone dimensions; local bursting and spalling reinforcement must be developed from the selected proprietary system.

TABLE VI
DISCRETE PARAMETER SPACE

Variable	Levels	Unit	Count
Clear span L	12 15 18	m	3
Width b	300 350 400 450 500	mm	5
Overall depth h	750 to 1650 at 100 increments	mm	10
Concrete strength f _{ck}	40 50 60	MPa	3
Strand count	8 to 24 at 2 increments	number	9
Eccentricity ratio e over h	0.25 0.30 0.35 0.40	none	4
Concrete scenario	C0 C25 C40	category	3
Relative humidity	70 for search 50 and 85 for sensitivity	percent	1 plus 2

B Materials loads and construction stage assumptions

Concrete elastic modulus is 5000 times the square root of cylinder strength in MPa. Strength at transfer is 75 percent of 28 day strength. Prestressing reinforcement is a 15.2 mm low relaxation seven wire strand with 140 mm² area, 1860 MPa specified tensile strength, and 195 GPa modulus. Jacking stress is 75 percent of specified tensile strength. Passive longitudinal reinforcement equals 0.22 percent of gross section area and has 500 MPa yield strength. The passive ratio is held constant to isolate the effects of geometry and prestress; detailed design would optimize it separately and satisfy minimum, maximum, spacing, and ductility rules.

Concrete unit weight is 25 kN per cubic metre. Every beam carries 8 kN per metre superimposed dead load and 20 kN per metre nominal live load in addition to self weight. The service combination is unfactored dead plus live loading. The ultimate combination is 1.2 times dead plus 1.6 times live, and a 0.90 flexural reduction factor is applied. The selected factors establish a consistent research comparison and are not asserted as the governing combination for any specific occupancy, bridge class, jurisdiction, or consequence category.

The transfer stage includes self weight and the force remaining after friction, anchorage, and elastic shortening. The final service stage includes all five loss components and the complete service load. This staged distinction is important. Applying final losses at transfer would understate early prestress effects, while applying transfer force at service would overstate decompression

and cracking resistance. No composite deck, continuity moment, erection bracing, or temporary construction load is included.

TABLE VII
COMMON ANALYTICAL ASSUMPTIONS

Item	Value	Interpretation
Strand fpu and area	1860 MPa and 140 mm ²	IS 14268 compatible class
Jacking stress	0.75 fpu	Common study value
Concrete modulus	5000 square root fck MPa	Screening relation
Transfer strength	0.75 fck	Fixed age proxy
Concrete unit weight	25 kN per m ³	Normal weight concrete
Superimposed dead load	8 kN per m	Uniform
Live load	20 kN per m	Uniform
Service combination	Dead plus live	Unfactored comparison
Ultimate combination	1.2 dead plus 1.6 live	Research basis
Nominal analysis life	50 years	Loss development plot

C Enumeration feasibility filter and data control

Each candidate is processed in the same order: gross properties and loads; immediate loss; time dependent loss; transfer stresses; service stresses; cracking; immediate and long term deflection; bonded tendon flexural resistance; shear reinforcement inventory; carbon inventory; and normalized constraints. A case is retained only when all structural utilization ratios do not exceed one, effective stress lies between 0.55 and 0.80 times strand tensile strength, the cover condition is met, and the span to depth ratio remains within the stated bounds. This produced 13,859 feasible cases, or 28.5 percent of the enumerated grid.

Numerical data are kept in base N and mm units inside the analysis. Moments are converted to kN m only for reporting, and volumes are converted to cubic metres before carbon factors are applied. The flexural neutral axis is found by bisection over 90 iterations. Bisection is slower than a Newton update but remains stable when tendon stress reaches its cap or passive steel changes from elastic to yielded response. Repeated execution gives identical deterministic results. The Monte Carlo generator is independently seeded and its seed is reported so that the probabilistic sample is also reproducible.

The feasibility filter is deliberately conservative about the uncracked model. A candidate that exceeds the cracking moment is removed instead of being analyzed with an artificially unchanged gross stiffness. This means the database represents fully prestressed or conservatively partially prestressed behavior within the adopted service tension screen. A future extension can add cracked transformed-section iteration, tension stiffening, crack width, redistribution, and fatigue to admit a larger partially prestressed domain.

VI VERIFICATION AND NUMERICAL QUALITY ASSURANCE

A Independent equilibrium and unit checks

Verification was performed at equation, member, and database levels. The selected 15 m compromise design is 15 m 300 x 1450 mm fck 60 MPa 16 strands e/h 0.25 C40. Its effective direct compression, eccentric bending term, and service gravity term were recomputed independently from the stored geometry and force. Their signed sums reproduce the reported top and bottom fiber stresses. The compression-block resultant was also compared with the sum of tendon and passive-steel tension. The final discrepancy is governed by numerical display precision rather than equilibrium error.

Dimensional checks provide a second line of control. A stress term uses force divided by area or moment divided by section modulus and therefore returns N per mm². The parabolic equivalent load uses N mm divided by mm² and returns N per mm, numerically identical to kN per metre. Deflection contains load per length times length to the fourth divided by modulus times second moment and returns mm. Carbon terms are converted to cubic metres or kilograms before intensity factors are applied. These checks prevent the common thousandfold errors caused by mixing kN m with N mm.

TABLE VIII
INDEPENDENT CALCULATION AUDIT FOR THE SELECTED DESIGN

Quantity	Model	Independent check	Difference
Direct stress P_e over A	6.0146 MPa	6.0146 MPa	0.000
Eccentric stress $P_e e$ over Z	9.0219 MPa	9.0219 MPa	0.000
Gravity stress M over Z	10.4006 MPa	10.4006 MPa	0.000
Top service stress	7.3933 MPa	7.3933 MPa	0.00e+00
Bottom service stress	4.6360 MPa	4.6360 MPa	0.00e+00
Section force equilibrium	4228.26 kN	4228.26 kN	2.203e-14%
Carbon component sum	2692.707 kg	2692.707 kg	0.00e+00

B Limiting cases and monotonic behavior

Limiting-case tests were used to verify signs and trends. With eccentricity set to zero, the prestress bending term and equivalent uplift vanish while direct compression remains. With tendon area set conceptually to zero, the equations reduce to an ordinary uncracked concrete beam under gravity loading. Increasing gravity moment increases top compression and decreases bottom compression under the adopted sign convention. Increasing eccentric prestress has the opposite effect. These trends appear in the plotted stress block and exclude a silent sign reversal.

Geometry trends were also inspected. At constant width, depth, loading, force, and material, section modulus grows with depth squared and second moment grows with depth cubed. Stress and deflection therefore fall

nonlinearly as depth increases. At fixed geometry, increasing strand count raises cracking resistance and ultimate resistance but also moves the transfer stress closer to its limit. At fixed design, reducing relative humidity increases shrinkage and creep loss in the adopted model. These monotonic relationships were checked across the enumerated arrays before Pareto filtering.

The computed feasible count was reconciled by span and by concrete scenario, and every retained candidate was rechecked against the maximum of its component ratios. No candidate marked feasible had an individual utilization greater than one. This database-level assertion is more informative than spot checking only an attractive design because optimization can expose combinations that are never examined in a traditional hand iteration.

C Prestress loss benchmarking

The component model predicts total 50 year losses of 19.9, 16.3, and 13.7 percent at relative humidities of 50, 70, and 85 percent, respectively. These values lie in a credible preliminary-design range for low-relaxation bonded systems, but agreement in magnitude is benchmarking rather than validation. The NCHRP, PCI, and AASHTO studies cited earlier show substantial scatter around loss predictions because actual modulus, release strength, curing, aggregate, environment, and construction sequence vary [8]-[13].

Figure 4 shows that immediate effects create the initial reduction, shrinkage develops over months, creep continues more slowly, and relaxation accumulates logarithmically. Figure 5 exposes how the humidity assumption changes only the concrete time-dependent terms in the current model. This separation is useful during calibration: friction should be replaced from a tendon profile and jacking plan, anchorage seating from system data, and shrinkage and creep from mixture-specific testing or a recognized model. A single total percentage cannot be updated with the same traceability.

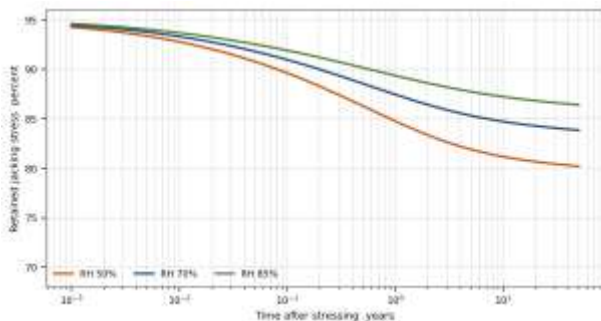


Fig. 4. Predicted retention of jacking stress over 50 years for the selected geometry under three constant relative humidity scenarios.

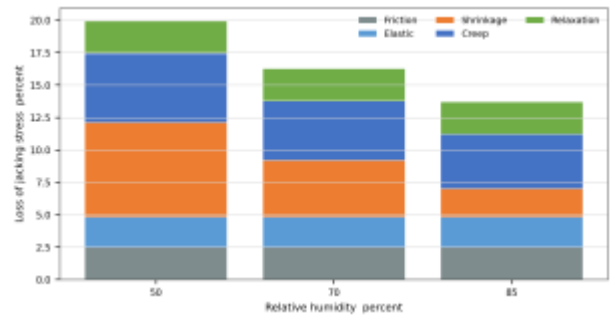


Fig. 5. Decomposition of final prestress loss for the selected design. Values are comparative model predictions, not measurements.

TABLE IX
FINAL LOSS SENSITIVITY TO RELATIVE HUMIDITY

RH	Friction	Elastic	Shrinkage	Creep	Relaxation	Total
50	2.50	2.32	7.31	5.32	2.50	19.94
70	2.50	2.32	4.38	4.57	2.50	16.27
85	2.50	2.32	2.19	4.19	2.50	13.70

VII PARAMETRIC RESULTS AND INTERPRETATION

A Feasible domain and governing conditions

Of 48,600 candidates, 13,859 passed all geometric, effective-force, serviceability, and strength filters. Feasibility increased from 2,843 cases at 12 m to 5,316 at 15 m and 5,700 at 18 m, but the counts should not be read as evidence that the longer span is easier. The fixed depth grid removes many shallow short-span options through the lower span-to-depth bound, whereas the 18 m span uses more of the upper-depth range. Feasible fraction is therefore meaningful only within each discrete domain.

Among feasible designs, the controlling ratio changes with geometry and prestress. Transfer compression becomes important when tendon area and eccentricity are large relative to section modulus. Transfer tension becomes important when high eccentric force produces an upward prestress moment before superimposed loading is present. Service tension and cracking control shallow or lightly prestressed sections. Ultimate flexure appears near low steel areas, while deflection is less frequently critical within the adopted depth bounds. This distribution confirms that prestressed design is a staged problem; no one ratio identifies the feasible domain by itself.

Figure 9 illustrates the boundary for a 15 m, 350 mm wide, 50 MPa beam under the C25 scenario at e over h equal to 0.35. Moving to greater depth generally lowers utilization, but adding strands is beneficial only until transfer stresses begin to rise. The resulting boundary is curved rather than a straight minimum-steel line. That curvature is the numerical expression of competing transfer and service actions.

TABLE X
GOVERNING CHECK AMONG FEASIBLE DESIGNS

Governing check	Cases	Share percent
Cracking	8,080	58.3
Transfer compression	2,983	21.5
Transfer tension	2,306	16.6
Service tension	482	3.5
Service compression	8	0.1
Long term deflection	0	0.0
Ultimate flexure	0	0.0

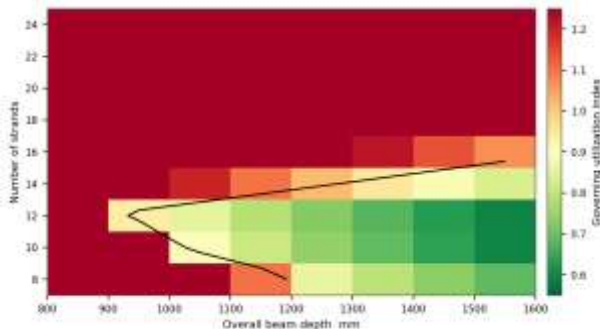


Fig. 9. Feasibility map for a representative 15 m beam. The black contour is the utilization boundary U equal to one.

B Service stress sensitivity to strand area and eccentricity

Figure 6 holds the selected section and material constant while varying strand count continuously for visualization and eccentricity ratio between 0.22 and 0.42. The zero-stress contour divides sections retaining bottom compression from sections developing bottom tension under full service load. Both additional strand and larger eccentricity move the response toward compression, but the contours flatten at high values because loss grows with concrete stress at the tendon level. The selected compromise has a bottom compression of 4.64 MPa and therefore does not rely on the tensile allowance to remain uncracked.

Bottom compression alone is not a complete performance objective. A point far above the zero-stress contour uses more high-strength steel, raises anchorage demand, and may generate undesirable transfer tension at the top fiber. A practical tendon layout should therefore be chosen by reading the service contour together with transfer utilization. The selected e over h of 0.25 is lower than the largest available value because the deeper section already provides sufficient section modulus and cracking reserve.

Figure 8 separates service cracking moment from ultimate design resistance. Both increase with strand count, although ultimate resistance also depends on the strain-compatible tendon stress and passive reinforcement. For low strand counts, strength can be close to the screening demand; at high counts, the marginal capacity gain becomes less valuable because the section is already serviceable and transfer stress or

carbon may govern. The graph therefore supports an intermediate tendon quantity rather than a maximum-force rule.

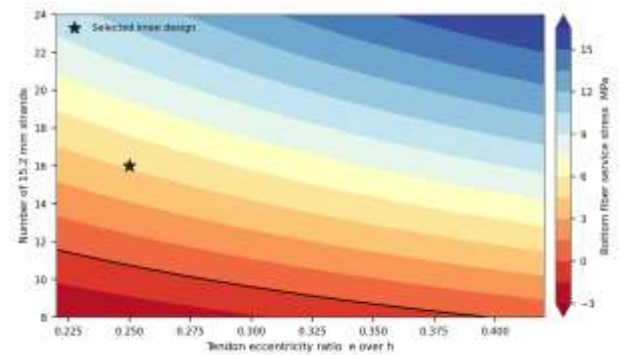


Fig. 6. Bottom fiber service stress for the selected 15 m section as strand count and tendon eccentricity vary. Compression is positive.

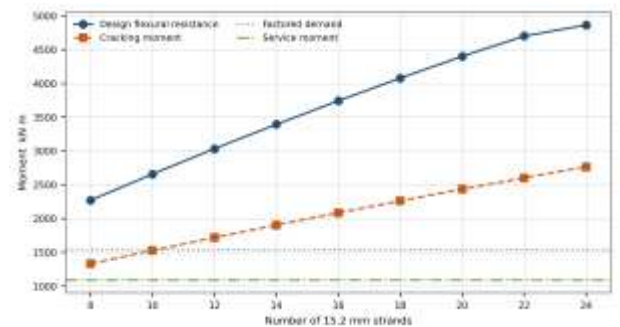


Fig. 8. Effect of strand count on cracking moment and flexural resistance for the selected section.

C Long term deformation response

Figure 7 shows the predicted long-term midspan movement as eccentricity changes for three spans. A positive value denotes downward movement and a negative value denotes residual upward camber. Increased eccentricity strengthens the prestress uplift and moves each curve downward. Because gravity and prestress terms both scale with span but interact with a depth selected in proportion to span, the spacing between curves is not a simple fourth-power relation. The plot demonstrates why deflection should be calculated with the chosen tendon profile rather than inferred from span-to-depth ratio alone.

The selected 15 m compromise predicts a small residual upward movement. That result should not be interpreted to the nearest millimetre. Actual camber is sensitive to modulus at stressing, construction age, storage duration, temperature, creep under the prestress moment, sequential loading, and restraint. The valuable result is the reserve relative to the L over 300 screening magnitude and the location of the sign reversal. Designs near zero calculated movement are often attractive for floor level control, but uncertainty can change the sign and should be incorporated into erection tolerances.

None of the three plotted curves establishes vibration performance. Prestressing can reduce cracking and retain stiffness, yet a slender floor may still be governed by frequency, acceleration, walking excitation, or perceptible vibration. For floor systems, a dynamic

check should follow the static serviceability screen. For bridges, repeated-load deflection and fatigue of tendons, anchorages, ducts, and passive reinforcement require separate combinations.

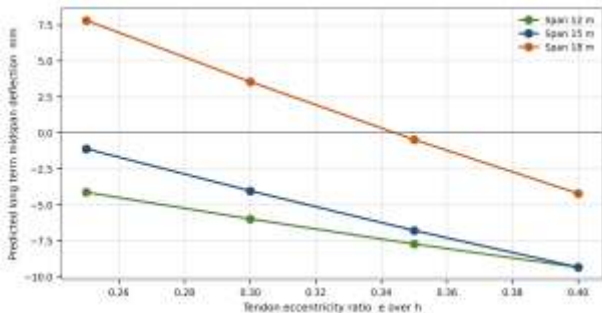


Fig. 7. Predicted long term midspan movement versus tendon eccentricity. Positive values indicate downward deflection.

D Spanwise compromise designs

Table XI reports one normalized-distance Pareto compromise for each span. All three select a 300 mm width and the C40 environmental scenario, but depth, strand count, and eccentricity change with span. This does not establish that the narrowest beam and deepest binder substitution are universally preferred. Width is not penalized for reinforcement congestion, formwork repetition, fire resistance, vibration, anchorage-zone confinement, or lateral stability in the current objective. The C40 factor is also a hypothetical inventory scenario. A supplier-specific mixture may move to another scenario once measured time-dependent properties and verified carbon data are used.

The spanwise carbon indicator rises faster than span alone because both member volume and reinforcement inventory increase. Nevertheless, utilization remains within a relatively narrow band for the compromise points, showing that Pareto selection can maintain a similar reserve at different scales. A family of solutions is more useful than one equation-based preliminary size because it gives the designer alternatives when depth, width, strength, or tendon hardware is constrained.

TABLE XI
SELECTED PARETO COMPROMISE DESIGNS BY SPAN

L m	b x h mm	fck	n	e/h	Mix	Loss percent	Carbon kg	Governing U
12	300 x 1150	60	12	0.25	C40	16.1	1688	0.523 Cracking
15	300 x 1450	60	16	0.25	C40	16.3	2693	0.525 Transfer compression
18	300 x 1650	60	18	0.30	C40	17.1	3663	0.565 Cracking

E Pareto tradeoff and carbon decomposition

The 15 m feasible population and its nondominated edge are shown in Fig. 10. The minimum-carbon design emits 1708 kg CO₂e under the controlled boundary and has utilization 0.895. The selected knee emits 2693 kg CO₂e and has utilization 0.525. The lowest-utilization point on the front emits 5894 kg CO₂e at utilization 0.421. These are different answers to different preferences. Reporting only the minimum would hide the material cost of

reserve; reporting only the knee would hide the additional saving available near the constraint boundary.

Relative to the defined feasible conventional baseline, the compromise reduces the cradle-to-gate indicator by 24.9 percent. Figure 11 shows that the reduction is dominated by the lower concrete scenario factor and changed section inventory. Prestressing steel remains a visible contribution even though its mass is much smaller than concrete mass because its per-kilogram factor is higher. Passive reinforcement includes the longitudinal steel and the simplified shear inventory. The decomposition prevents a low concrete factor from concealing a shift into additional high-intensity steel.

Carbon reduction is conditional on equivalent function. The structural filter provides the same set of analytical limits, but it does not prove equal durability, fire resistance, fatigue life, robustness, inspectability, or construction waste. A project comparison should add those functional requirements and use consistent service life and system boundaries. If a low-clinker mixture delays stressing or requires heat curing, the associated energy and programme effects must also enter the inventory.

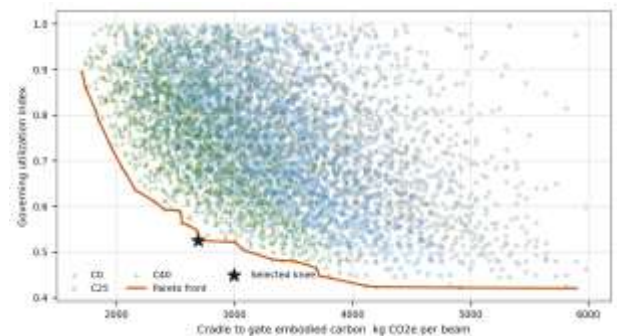


Fig. 10. Feasible 15 m designs, nondominated front, and the selected compromise between embodied carbon and utilization.

TABLE XII
REPRESENTATIVE 15 M DESIGN ALTERNATIVES

Alternative	b x h	fck	n	e/h	Mix	Carbon	U	Loss
Minimum carbon	300 x 1050	40	12	0.30	C40	1708	0.895	20.7
Selected compromise	300 x 1450	60	16	0.25	C40	2693	0.525	16.3
Minimum utilization	500 x 1450	60	22	0.25	C0	5894	0.421	13.6
Conventional baseline	400 x 1250	50	12	0.30	C0	3584	0.697	13.8

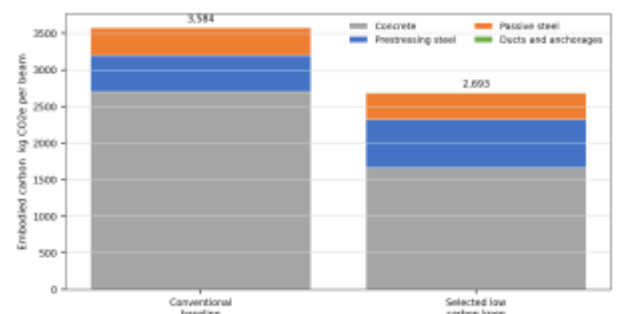


Fig. 11. Cradle-to-gate carbon decomposition for the conventional baseline and selected low-carbon compromise.

TABLE XIII
MEMBER CARBON INVENTORY UNDER THE CONTROLLED
BOUNDARY

Design	Concrete	Prestress	Passive steel	Accessories	Total kg CO ₂ e
Baseline	2700.0	494.5	379.4	9.9	3583.8
Compromise	1658.7	659.4	361.5	13.2	2692.7

VIII EXTENDED SENSITIVITY AND IMPLEMENTATION ANALYSIS

A Influence of concrete scenario

The three concrete scenarios influence two parts of the calculation. Their carbon multipliers change the inventory directly, while their shrinkage and creep multipliers change effective prestress and every downstream service response. This coupling prevents the low-carbon scenario from receiving an environmental credit while retaining reference-concrete deformation. In the enumerated domain, the direct carbon reduction is generally larger than the structural penalty introduced by the conservative time-dependent multipliers, so C40 appears frequently on the low-carbon edge. That pattern is a consequence of the assumed scenario values, not evidence that the deepest substitution is always optimal.

Table XIV separates the number of feasible 15 m designs from the least-carbon point available within each scenario. Feasible count alone should not be used to rank mixtures because one scenario may contain many redundant high-material sections. The minimum indicator shows the best grid point under that scenario, while median utilization describes the central reserve among its retained population. Together they reveal whether a carbon reduction is obtained by moving the whole feasible population or by relying on a narrow boundary region.

If measured creep or shrinkage for a candidate mixture is lower than the conservative C40 multipliers, its effective force and service reserve would improve while the inventory credit remains. If early strength is delayed or measured deformation is higher, some C40 cases would move outside the feasible domain. This is why the mixture acceptance plan must connect environmental documentation with structural tests at stressing age and over time. A standalone environmental product declaration does not supply the modulus or deformation functions needed for prestress design.

TABLE XIV
EFFECT OF CONCRETE SCENARIO WITHIN THE 15 M FEASIBLE
POPULATION

Code	Scenario	Feasible	Minimum carbon	Median U	Loss at minimum
C0	Reference concrete	1,785	2198	0.753	18.7
C25	Moderate low carbon scenario	1,770	1866	0.753	19.4
C40	Deep low carbon scenario	1,761	1708	0.755	20.7

B One at a time sensitivity around the selected design

One-at-a-time perturbation provides a local explanation of the multivariable Pareto result. It does not capture interactions and should not replace enumeration, but it makes the direction and approximate scale of each effect easy to audit. Table XV holds all selected design inputs constant while changing one scenario, humidity, eccentricity ratio, or strand count. Repeated rows corresponding to the selected value serve as internal anchors.

Changing only the concrete scenario alters carbon strongly and loss modestly. Changing humidity alters loss and service response but leaves the cradle-to-gate inventory unchanged because operational exposure is outside that boundary. Eccentricity changes stress and deflection without changing material quantity in the simplified inventory. Strand count changes effective force, cracking reserve, transfer compression, resistance, and high-strength steel carbon simultaneously. These different pathways show why a composite objective based only on total loss or only on total weight would miss important tradeoffs.

One-at-a-time tables must be interpreted within geometric validity. An eccentricity ratio outside the enumerated grid is included here only to expose response direction, and the effective eccentricity is still capped by bottom cover. An actual tendon profile must satisfy minimum radius, duct spacing, bursting reinforcement, anchor geometry, and construction tolerance along its entire length. Similarly, a strand-count change may require a different anchorage or multiple ducts, changing friction and accessory quantities that are fixed in the comparative model.

TABLE XV
LOCAL SENSITIVITY AROUND THE SELECTED 15 M DESIGN

Variation	Loss percent	Bottom stress	Movement mm	Cracking U	Transfer U	Carbon kg
Mix C0	15.2	4.83	-1.00	0.519	0.525	3709
Mix C25	15.7	4.74	-1.05	0.521	0.525	3121
Mix C40	16.3	4.64	-1.10	0.524	0.525	2693
RH 50 percent	19.9	3.98	-0.70	0.542	0.525	2693
RH 70 percent	16.3	4.64	-1.10	0.524	0.525	2693
RH 85 percent	13.7	5.10	-1.39	0.512	0.525	2693
e over h 0.20	15.2	2.99	1.96	0.572	0.451	2693
e over h 0.25	16.3	4.64	-1.10	0.524	0.525	2693
e over h 0.30	17.6	6.18	-4.02	0.486	0.598	2693
e over h 0.35	19.1	7.61	-6.78	0.456	0.669	2693
Strands 12	14.3	1.14	2.65	0.636	0.370	2525
Strands 14	15.3	2.91	0.75	0.574	0.448	2609
Strands 16	16.3	4.64	-1.10	0.524	0.525	2693
Strands 18	17.2	6.32	-2.90	0.483	0.602	2777
Strands 20	18.2	7.96	-4.66	0.449	0.678	2861

C Concrete strength as a coupled variable

Higher concrete strength changes more than ultimate compression capacity. It increases the elastic modulus used for service deflection, raises the transfer and service stress screens, changes modulus of rupture and cracking moment, reduces the reference creep coefficient, and changes the baseline carbon factor. These effects can favor higher strength in a structurally constrained beam even when each cubic metre carries a larger environmental factor. The selected compromise uses 60 MPa concrete because the additional stiffness and stress reserve allow a narrow section with relatively low utilization.

A strength increase does not automatically improve every transfer condition. If higher strength is achieved through a higher paste content or earlier stressing, actual creep and autogenous shrinkage may not follow the simple strength modifiers. Strength also does not solve an anchorage-zone geometry problem or tendon congestion. The design should therefore distinguish specified 28 day strength, required strength at stressing, measured modulus, tensile behavior, and time-dependent deformation instead of allowing one *f_{ck}* value to represent all material performance.

The embodied-carbon consequence of strength depends on structural efficiency. Comparing equal volumes generally favors a lower carbon intensity, but comparing equal function may favor a smaller high-strength section. The discrete results capture this member-level tradeoff only for the rectangular geometry and fixed loads. Whole-building optimization may also credit reduced floor-to-floor height, smaller foundations, fewer columns, or longer spans. Conversely, it may penalize specialized curing, material transport, or low production volume. These system effects are outside the member boundary but can be larger than the difference between adjacent grid points.

D Grid resolution and local refinement

Full enumeration is exact only at the declared levels. Width changes by 50 mm, depth by 100 mm, strand count by two, and eccentricity ratio by 0.05. A continuous optimum may lie between them, and discrete tendon hardware may introduce finer or coarser practical steps. A recommended refinement begins with the nondominated designs, then searches one half of the original spacing around each variable while preserving constructible strand layouts and cover. Refinement should stop when the improvement is smaller than uncertainty in quantity factors or when neighboring alternatives are indistinguishable for decision purposes.

Excessive numerical resolution can create false precision. A difference of a few kilograms CO₂e is not meaningful when concrete factors are scenario values and member quantities exclude site waste. Likewise, a utilization difference in the third decimal place is not

decisive when creep, shrinkage, modulus, load, and resistance models are uncertain. The purpose of local refinement is to identify a stable neighborhood and practical rounding, not to advertise an exact mathematical optimum.

Discrete robustness can be checked by perturbing each design one fabrication step. A solution is preferable when a small reduction in eccentricity, one missing strand, a modest strength shortfall, or a higher loss estimate does not produce abrupt infeasibility. In this sense the shape of the feasible neighborhood matters alongside the point utilization. Future work can quantify this with a robustness radius, a worst-case objective over bounded uncertainty, or chance constraints embedded directly in the search.

TABLE XVI
RECOMMENDED REFINEMENT OF THE DISCRETE SEARCH

Variable	Initial spacing	Local refinement	Practical constraint
Width	50 mm	25 mm around Pareto points	Anchorage width and formwork module
Depth	100 mm	25 or 50 mm	Architectural zone and self weight
Strand count	2 strands	System-compatible grouping	Anchor and duct capacity
Eccentricity ratio	0.05	Profile coordinates	Cover radius and tolerances
Concrete strength	10 MPa	Supplier classes or measured data	Transfer age modulus and carbon
Concrete scenario	3 categories	Product-specific EPD and tests	Consistent structural and LCA data

E Construction tolerances and force verification

Calculated eccentricity is a centroidal design value, whereas installed ducts deviate within positional tolerances. A vertical profile error changes prestress moment and equivalent uplift without changing measured jacking force. The structural consequence is greatest where bending moment and intended eccentricity are high. Shop drawings should therefore define profile ordinates, support chairs, permissible deviation, minimum cover, and hold points before concreting. Congested end zones require special inspection because displaced ducts can conflict with bursting reinforcement or reduce cover.

Jack pressure and theoretical elongation provide complementary checks. Pressure indicates applied force through calibration, while elongation integrates tendon strain and can expose friction, obstruction, seating, or length errors. Neither should be treated as exact. Jack calibration, strand modulus, actual free length, temperature, duct friction, wedge draw-in, and reading technique all affect comparison. Acceptance criteria should come from the governing specification and tendon supplier, and discrepancies should be investigated before grouting or cutting tails.

Effective force verification after anchoring is valuable for highly sensitive or repeated members. Lift-off testing, load cells, vibrating-wire instruments, elasto-

magnetic systems, or other accepted methods may be appropriate depending on access and required accuracy. Instrumentation changes the project only when it is linked to an action plan. The plan should define baseline readings, environmental corrections, trigger thresholds, responsible personnel, and the analysis update made when a reading departs from expectation.

Grouting is a structural durability operation for bonded tendons. Material qualification, mixing, fluidity, bleed control, venting, pressure, volume balance, outlet confirmation, records, and inspection all affect whether the tendon develops reliable bond and corrosion protection. The present resistance model assumes complete bond. A voided or poorly protected duct is not represented by a small adjustment to prestress loss and requires investigation under project-specific procedures.

F Monitoring and model updating

A staged monitoring plan can convert uncertain inputs into project evidence. At stressing, record concrete strength, modulus if tested, concrete and air temperature, jack identification and calibration, pressure, elongation, seating, and tendon sequence. At transfer or anchor lock-off, record beam camber and support condition. During storage and erection, record age, humidity exposure, applied loads, and camber. After superimposed dead load and occupancy, repeat elevation or deflection measurements at meaningful intervals.

Measurements should be compared with prediction bands rather than one deterministic curve. Early deviation can indicate an incorrect initial force, geometry, or modulus; a later change in slope can indicate time-dependent behavior, load history, restraint, or deterioration. Updating should preserve physical meaning. For example, fitting one arbitrary total-loss factor may reproduce a force reading but cannot distinguish friction from creep or predict a changed construction stage reliably.

Bayesian updating or weighted least squares can estimate uncertain parameters when measurement error and prior ranges are defined. Identifiability must be checked because camber alone may not separate effective force from modulus and creep. Combining tendon force, concrete strain, temperature, and deflection improves inference. The updated model should then predict an observation not used in calibration. This prospective check is essential before updated parameters are used for safety decisions.

Monitoring is most valuable when the information affects design or maintenance. For a repeated precast or post-tensioned production line, early members can update later stressing targets and camber predictions. For a unique bridge, long-term data can inform bearing adjustment, overlay placement, inspection intervals, or load management. Data ownership, sensor durability,

calibration, and archival format should be planned with the structural questions, not added after construction.

IX RELIABILITY AND ROBUSTNESS SCREENING

A Random variable model

Reliability screening was performed for the selected 15 m compromise and the minimum-carbon boundary design. The same 100000 standardized sample positions and seed were used for both designs so that differences arise from the designs rather than sampling noise. Concrete strength and the resistance-model factor are lognormal. Effective prestress, dead load, live load, and the service stress multiplier are normal, with negative physical load values truncated. The deflection model factor is lognormal. Variables are treated as independent because no project data were available to establish correlation.

Independence is not necessarily conservative. Concrete strength, modulus, shrinkage, creep, and transfer age are correlated through mixture and curing. Prestress measurement error may be correlated among tendons, and occupancy load may be spatially correlated. The analysis therefore quantifies robustness to a disclosed hypothetical uncertainty model, not code-calibrated structural reliability. ISO 2394 principles require consequence class, model uncertainty, inspection, and target reliability to be selected for the actual application [36].

TABLE XVII
ILLUSTRATIVE UNCERTAINTY MODEL

Variable	Distribution	Mean basis	COV	Applied response
Concrete strength	Lognormal	Nominal f_{ck}	0.10	Ultimate service deflection
Effective prestress	Normal truncated	Calculated f_{pe}	0.07	All three limits
Dead load	Normal truncated	Calculated dead	0.08	Demand and movement
Live load	Normal truncated	20 kN per m	0.18	Demand and movement
Resistance model	Lognormal	1.00	0.06	Ultimate
Service stress model	Normal	1.00	0.05	Service tension
Deflection model	Lognormal	1.00	0.15	Deflection

B Probability estimates and interpretation

The minimum-carbon design produces 3,849 ultimate and 31,974 service-tension failures in the illustrative sample, corresponding to reliability indices of 1.77 and 0.47. Its deterministic utilization of 0.895 therefore does not translate into a large probabilistic reserve when live load, prestress, and model factors vary. The result is not surprising: minimum-carbon selection intentionally approaches the deterministic boundary.

No failure was observed for the compromise design in any of the three sampled limits. The continuity estimator

reports a probability of approximately 5.0×10^{-6} and beta 4.42; this is a finite-sample reporting bound, not proof that the actual failure probability equals that value. Estimating rarer events would require importance sampling, subset simulation, first-order reliability analysis with verified gradients, or a much larger sample. The defensible conclusion is only that the compromise is substantially more robust than the boundary design under the stated assumptions.

Deflection shows no sampled failures for either design because the deterministic movement has considerable reserve relative to L over 300. Service tension is the most fragile limit for the minimum-carbon design even though cracking gives the largest deterministic ratio. This difference occurs because uncertainty in live load and effective prestress acts directly on the bottom-fiber stress margin. It demonstrates the value of evaluating multiple probabilistic limit states instead of applying one uncertainty multiplier to the maximum deterministic utilization.

TABLE XVIII
MONTE CARLO RELIABILITY SCREEN WITH 100000 SAMPLES

Design	Limit state	Failures	Probability	Beta
Minimum carbon	Ultimate flexure	3,849	3.849×10^{-2}	1.77
Minimum carbon	Service tension	31,974	3.197×10^{-1}	0.47
Minimum carbon	Deflection	0	5.000×10^{-6}	4.42
Compromise	Ultimate flexure	0	5.000×10^{-6}	4.42
Compromise	Service tension	0	5.000×10^{-6}	4.42
Compromise	Deflection	0	5.000×10^{-6}	4.42

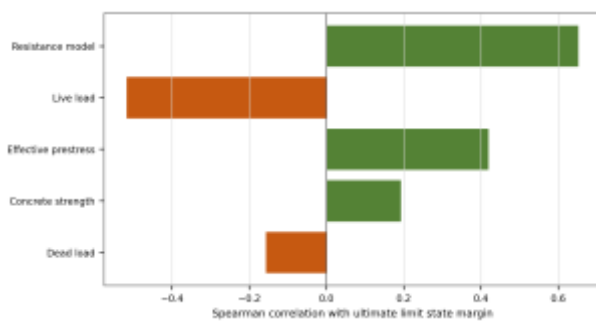


Fig. 12. Rank sensitivity of the selected compromise ultimate margin to the five principal stochastic inputs.

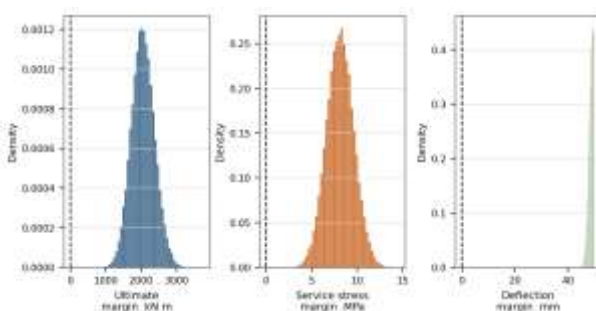


Fig. 13. Monte Carlo margin distributions for the selected compromise. The dashed line denotes the failure boundary.

C Sensitivity and robust selection

The largest absolute rank correlation with ultimate margin is resistance model at 0.652, followed by live

load at -0.518. Positive correlation means the variable increases calculated margin; negative correlation means it raises demand or otherwise reduces margin. Rank correlation is a global monotonic indicator and does not apportion variance exactly, but it identifies where better information can most efficiently improve confidence.

The resistance model and live load dominate the current ultimate screen, followed by effective prestress. This ordering suggests three practical actions: retain reserve beyond a bare deterministic ratio, use a project-appropriate loading model, and verify effective force through calibrated elongation and lift-off or other accepted measurements where applicable. Increasing concrete test frequency alone would not remove the principal uncertainty if model bias and live load remain broad.

Robust selection can be introduced formally by adding reliability index or a percentile response as a third objective. The present paper keeps reliability separate so that the deterministic Pareto geometry remains visible. A three-objective study could reveal solutions that are slightly heavier than the minimum-carbon point but much less sensitive to a particular variable. Such a study should use code-calibrated distributions, parameter correlations, and consequence-specific targets rather than the illustrative inputs in Table XVII.

X ENGINEERING DISCUSSION

A Why depth and eccentricity outperform force alone

The parameter study supports a mechanics-based interpretation. Tendon force contributes direct compression proportional to P over A and counteracting bending proportional to P_e over Z . Increasing force raises both terms linearly and raises high-intensity steel inventory. Increasing eccentricity raises the moment term without raising direct compression or tendon mass, but it can worsen top tension at transfer and is limited by cover and curvature. Increasing depth raises Z quadratically and I cubically while adding concrete and self weight. Efficient design balances these different exponents rather than maximizing any one variable.

The selected 15 m compromise uses a deeper, narrower section than the conventional comparison. Its larger section modulus allows a moderate eccentricity ratio and controlled strand count to keep both transfer and service stresses away from their limits. The depth adds concrete volume, but the lower scenario factor and reduced width limit its carbon consequence. If architectural depth were fixed, the Pareto set would move toward greater width, strand area, or strength, and the reported saving would change.

A narrow deep section can introduce lateral stability and construction challenges not represented in two-dimensional flexure. Pretensioned girders may roll during handling, and post-tensioned members may need bracing before slabs or diaphragms are engaged.

Anchorage zones may also set a minimum web width. Thus, optimization output should be treated as a candidate geometry subject to constructability filters, not an automatically buildable section.

B Time dependent material behavior and low carbon concrete

Carbon reduction through binder substitution is attractive because concrete dominates member volume. Yet prestressed concrete places unusual emphasis on early strength, elastic modulus, creep, and shrinkage. Stressing too early can increase elastic deformation and creep loss. Delayed strength can change programme and formwork reuse. Increased autogenous shrinkage in a low water-binder high-strength mixture can affect restraint cracking even when drying shrinkage is controlled. These effects require mixture-specific qualification rather than strength-only acceptance.

The C25 and C40 scenarios include conservative increases in creep and shrinkage multipliers, but those multipliers are not material laws. Some limestone calcined clay, slag, fly ash, and hybrid cement systems may show comparable or lower long-term deformation depending on replacement level, curing, aggregate stiffness, water content, and admixture system [14]-[18]. The correct workflow is to obtain verified carbon intensity and measured mechanical time functions from the same production mixture. Combining the carbon value of one product with the deformation behavior of another can generate a false optimum.

Durability may outweigh cradle-to-gate saving. Chloride exposure, carbonation, sulfate, freeze-thaw, alkali-silica reaction, duct grouting, anchor protection, and drainage affect service life. A modest initial carbon increase that prevents tendon corrosion or premature replacement can reduce whole-life impact. Bonded post-tensioning also depends on complete grout filling and durable end anchorage protection. These quality controls should appear as functional constraints in a project life-cycle assessment.

C Code application in Indian and international practice

IS 1343 provides the principal Indian framework for prestressed concrete, supplemented by IS 456 for general concrete requirements and IS 14268 for uncoated stress-relieved low-relaxation strand [1], [37], [38]. A project designer must apply the exact permissible stresses, loss provisions, load combinations, cover, durability, shear, anchorage-zone, and detailing clauses relevant to the member and construction method. The study ratios were chosen to compare alternatives consistently; they are not a clause-by-clause design certificate.

International codes organize several checks differently. ACI 318, ACI 423 guidance, AASHTO LRFD, Eurocode 2, fib Model Code, and the PCI Design Handbook contain distinct classifications, resistance factors, load combinations, time-dependent approaches, and detailing

rules [2]-[7]. A design that passes one screening set is not automatically interchangeable across those documents. The integrated architecture remains useful because the code-specific equations can be substituted without changing the overall sequence of stage analysis, inventory, Pareto filtering, and reliability evaluation.

For a publishable experimental extension in India, measured data should include cube or cylinder strength at stressing and 28 days, modulus, shrinkage prisms, creep specimens, strand relaxation certificate, jack calibration, actual elongation, seating loss, concrete temperature, relative humidity, and beam camber at multiple ages. Load testing should record curvature, crack initiation, strain distribution, deflection, tendon strain where feasible, and failure mode. Such a dataset would permit calibration of the analytical modifiers and genuine validation of the selected design.

D Decision significance and practical tradeoffs

The minimum-carbon design saves 52.3 percent relative to the baseline but sits much closer to the analytical boundary. The compromise saves 24.9 percent and retains considerably more reserve. This is a classic diminishing-return decision: the final increment of carbon saving carries a disproportionate increase in sensitivity. A rational choice depends on consequence of failure, inspection access, uncertainty quality, repetition count, and whether one design will be produced many times.

Standardization can reverse a member-by-member optimum. A slightly heavier repeated section may reduce formwork changes, tendon hardware variation, fabrication error, and site delay. Conversely, a one-off transfer girder with strict depth may justify a more tailored design. The Pareto front should therefore be reviewed alongside cost, schedule, repetition, procurement, and quality assurance rather than reduced to an automatic point selection.

The framework also clarifies where research effort has value. If service tension governs, better live-load definition and force monitoring may be more beneficial than a small increase in concrete strength. If transfer tension governs, adjusting stressing age, sequence, tendon profile, or temporary loading may be preferable to adding reinforcement alone. If carbon is dominated by concrete, verified binder data and geometry refinement deserve attention. If steel dominates, strand utilization, passive ratio, and recycled-content procurement become stronger levers.

XI PROPOSED PRELIMINARY DESIGN PROCEDURE

The following procedure translates the research framework into an auditable preliminary design workflow. It is intended to organize engineering judgment, not replace calculations required by the governing standard.

- 1) Define the structural system, clear span, support condition, construction stages, exposure, fire requirement, vibration criterion, design life, and governing codes before selecting a section.
- 2) Establish permanent and variable actions for transfer, handling, erection, service, fatigue where relevant, accidental conditions, and ultimate resistance. Keep load origin and units traceable.
- 3) Select initial geometry and a tendon profile compatible with cover, anchorage hardware, duct curvature, deviators, openings, reinforcement congestion, and construction access.
- 4) Specify concrete and strand properties at each relevant age. Obtain project values for modulus, shrinkage, creep, relaxation, friction, wobble, anchorage seating, and stressing sequence when sensitivity warrants them.
- 5) Calculate immediate losses and transfer stresses using the force and loads present at transfer. Check top and bottom fibers, end zones, bursting, spalling, local bearing, and temporary stability.
- 6) Calculate time-dependent effective force, service stresses, decompression or cracking, crack control where permitted, camber, short- and long-term deflection, vibration, and durability-dependent service conditions.
- 7) Check bonded tendon flexural resistance, ductility, shear, torsion, fatigue, passive reinforcement, development, anchorage, robustness, and all detailing provisions under the selected standard.
- 8) Create a quantity inventory and apply geographically and temporally representative environmental product data with matched boundaries. Retain concrete, prestress, passive steel, ducts, anchors, energy, transport, waste, maintenance, and end-of-life terms appropriate to the study goal.
- 9) Generate feasible alternatives and present a Pareto set for structural reserve, carbon, cost, or other objectives. Do not report a single optimum without showing sensitivity to assumptions and practical constraints.
- 10) Evaluate uncertainty for boundary designs and important repeated members. Verify critical variables during construction, compare measured response with predictions, and update the calculation record when deviations occur.

TABLE XIX

DIAGNOSTIC ACTIONS FOR COMMON GOVERNING OUTCOMES

Observed control	Likely mechanism	Candidate response
Transfer top tension	High eccentric force	Reduce eccentricity or force revise sequence increase transfer strength
Transfer compression	High P over A or Pe over Z	Increase section modulus reduce force alter profile
Service tension	Insufficient effective Pe	Improve profile reduce loss add depth or strand refine loading
Cracking	Mservice close to Mcr	Increase section modulus or prestress perform cracked analysis
Long term movement	Creep and load imbalance	Refine stages stiffness profile creep data and tolerances
Ultimate flexure	Insufficient force couple	Adjust tendon and passive steel depth strength and ductility
Embodied carbon	Concrete or steel inventory	Optimize geometry obtain verified EPDs avoid burden shifting

Observed control	Likely mechanism	Candidate response
Low reliability	High uncertainty or low reserve	Collect data add reserve use reliability calibrated design

The sequence should be iterative. A serviceability change alters force, geometry, and inventory; an environmental change can alter modulus and time-dependent behavior; and a construction change can alter immediate loss. Version control is therefore part of technical quality. Each iteration should record inputs, model source, code edition, mixture data, tendon system, software version, and the exact design selected from the alternatives.

For peer review, the most useful deliverable is not a spreadsheet containing unexplained final cells. It is a calculation map linking every reported stress, loss, resistance, quantity, and objective to an equation and a source input. The tables and figures in this paper demonstrate that principle. They allow a reviewer to see the direction of each effect and to challenge individual assumptions without reconstructing the entire analysis.

XII LIMITATIONS AND REQUIRED VALIDATION

The study is intentionally broad and analytical. Its primary limitation is the use of a rectangular simply supported member with gross-section elastic service response. Continuous beams develop secondary prestress moments, redistribution, and support-zone cracking. Flanged or voided sections change section efficiency and shear behavior. External or unbonded tendons require member-level compatibility for tendon stress increase. The numerical conclusions should not be transferred directly to those systems.

Loss functions are simplified and use constant humidity. They do not model temperature, staged casting, composite action, differential shrinkage, creep recovery, steam curing, heat of hydration, or construction interruptions. Friction and anchorage loss are fixed fractions rather than profile-integrated calculations. The model also omits grout stiffness and defects, tendon corrosion, fatigue, fire, impact, seismic cyclic response, torsion, bearing behavior, end-zone stress, local anchorage design, and progressive collapse.

Environmental factors are controlled scenarios rather than verified declarations. The boundary excludes transport, site energy, formwork, curing, waste, maintenance, replacement, carbonation, demolition, and credits beyond the system. Consequently, the carbon indicator should rank only alternatives evaluated under the same assumptions. The reliability variables are illustrative and independent, and their distributions are not calibrated to a specific code or population.

TABLE XX
STUDY LIMITATIONS AND NEXT RESEARCH ACTIONS

Topic	Current treatment	Required extension
Structural form	Rectangular simply supported beam	Test flanged continuous and segmental members
Bond condition	Internal bonded tendon	Add unbonded and external compatibility
Service stiffness	Gross uncracked section	Iterate cracked stiffness and crack width
Prestress losses	Bounded component functions	Calibrate with force and deformation measurements
Environment	Constant relative humidity	Use temperature humidity histories and sealing
Shear and end zones	Inventory-level check	Nonlinear or strut-and-tie local design
Carbon	Cradle to gate scenarios	Project EPDs and whole-life assessment
Reliability	Independent illustrative variables	Code-calibrated models and correlations
Validation	Literature benchmarking only	Full-scale staged monitoring and load test

A validation programme should use at least two nominally identical beams per mixture so that repeatability can be separated from mixture effect. Tendon force should be inferred from calibrated jack pressure and elongation at stressing, then checked by an accepted force-monitoring technique at selected ages. Concrete companion specimens should remain in environmental conditions representative of the beams. Measurements should begin before transfer and continue through the rapid early period and the slower long-term period.

Model adequacy should be assessed with bias, coefficient of variation, residual plots, and prediction intervals rather than a single coefficient of determination. Calibration and validation datasets should be separated. If parameters are estimated from the same beam used to claim accuracy, the result measures fit rather than predictive performance. Publishing unsuccessful or anomalous results is important because anchorage seating, grout behavior, and environmental variability can dominate field response.

XIII CONCLUSIONS

A transparent analytical framework was developed to couple staged prestress loss, transfer and service behavior, flexural and shear resistance, material inventory, multiobjective selection, and exploratory reliability for bonded post-tensioned rectangular beams. Within the assumptions and discrete domain of this study, the following conclusions are drawn.

- Full enumeration evaluated 48,600 designs and retained 13,859, demonstrating that a large portion of plausible geometric combinations is rejected once transfer, service, cracking, deflection, effective-force, and ultimate checks are imposed together.
- Prestress loss should be maintained as visible components. Friction and anchorage, elastic shortening, shrinkage, creep, and relaxation have different causes, time

scales, and project data; one total percentage cannot be calibrated or audited with the same clarity.

- The selected 15 m compromise is 15 m 300 x 1450 mm fck 60 MPa 16 strands e/h 0.25 C40. It has predicted final loss 16.3 percent, carbon indicator 2693 kg CO₂e, and governing utilization 0.525.

- The compromise reduces the controlled cradle-to-gate indicator by 24.9 percent relative to the defined conventional baseline. This saving is conditional on scenario carbon factors and equal analytical function, not a universal product claim.

- The 15 m minimum-carbon point reaches 1708 kg CO₂e and utilization 0.895. Its illustrative Monte Carlo service-tension reliability index is only 0.47, showing that deterministic feasibility near a boundary can have limited robustness.

- The selected compromise produced no sampled failures in 100,000 trials under the illustrative model. The reported beta 4.42 is a finite-sample bound and should not be interpreted as code-calibrated reliability.

- Depth, eccentricity, and force are complementary variables. Greater tendon force alone can worsen transfer stresses and steel-related carbon; greater eccentricity can increase transfer tension; and greater depth improves stress and deflection efficiently but raises concrete quantity and may conflict with architecture.

- Low-carbon concrete must be assessed with the same mixture-specific early strength, modulus, shrinkage, creep, durability, curing, and environmental data used in the structural model. Strength and a generic carbon reduction factor are insufficient for final selection.

- The framework is suitable for preliminary research comparison and teaching. Project use requires clause-specific code checks, verified material and tendon-system data, anchorage-zone and durability design, construction-stage analysis, and experimental or field validation where response is sensitive.

The central design implication is that sustainability and serviceability should be optimized in the same calculation loop. When they are separated, a low-carbon material choice may be credited without its deformation behavior, or a structurally conservative tendon force may be selected without accounting for steel inventory and transfer risk. An integrated, auditable Pareto approach keeps those consequences visible and gives the engineer a defensible basis for choosing reserve.

APPENDIX A WORKED DESIGN AUDIT

This appendix provides a compact numerical audit of the selected 15 m design. The rectangular section is 300 mm wide and 1450 mm deep. Gross area is 0.4350 m², second moment is 0.0762 m⁴, and section modulus is 0.1051 m³. The tendon contains 16 strands with total area 2240 mm² and centroidal eccentricity 362.5 mm. Jacking force is 3124.8 kN.

The calculated loss components are 34.9 MPa friction and anchorage, 32.3 MPa elastic shortening, 61.1 MPa

shrinkage, 63.8 MPa creep, and 34.9 MPa relaxation. Their sum is 227.0 MPa, leaving effective tendon stress 1168.0 MPa and effective force 2616.4 kN.

Self weight is 10.88 kN per metre. Service moment is 1093.4 kN m and factored moment is 1537.0 kN m. Transfer stresses are -0.51 MPa at the top and 14.18 MPa at the bottom. Final service stresses are 7.39 MPa at the top and 4.64 MPa at the bottom, compression positive. Cracking moment is 2085.6 kN m.

Strain compatibility gives neutral axis depth 425.2 mm, tendon stress 1674.0 MPa at nominal resistance, and reduced resistance 3742.9 kN m. Predicted long-term midspan movement is -1.10 mm. The simplified shear inventory uses 10 mm two-leg stirrups at 300 mm spacing. The material quantities are 6.525 m³ concrete, 263.8 kg strand, 112.7 kg longitudinal passive steel, and 99.9 kg stirrups.

TABLE XXI
SELECTED DESIGN NORMALIZED CHECKS

Check	Utilization	Status	Reserve percent
Transfer compression	0.5253	Pass	47.5
Transfer tension	0.1518	Pass	84.8
Service compression	0.2738	Pass	72.6
Service tension	0.0000	Pass	100.0
Cracking	0.5242	Pass	47.6
Long term deflection	0.0220	Pass	97.8
Ultimate flexure	0.4517	Pass	54.8

APPENDIX B REPRODUCIBILITY RECORD

Deterministic enumeration uses the exact levels in Table VI and contains 48,600 rows before filtering. No random numbers enter the deterministic model or Pareto front. The Pareto set is formed after sorting by carbon and retaining each point with utilization lower than all points having no greater carbon. The selected compromise minimizes Euclidean distance to the ideal point after independently scaling carbon and utilization to zero-to-one ranges over the front.

The Monte Carlo record uses pseudorandom seed 41721 and 100000 samples. For a limit state with zero observed failures, one-half event is added to the numerator and one event to the sample denominator solely to avoid an infinite reported reliability index. This convention does not create evidence of a failure and does not replace rare-event analysis. All equations, factors, grids, carbon intensities, and distribution assumptions required to reproduce the reported values are stated in the paper.

Values shown in tables are rounded only for presentation. Feasibility and Pareto dominance use unrounded floating-point results. The computational model uses millimetres, newtons, and megapascals internally. A future open-data release should include one machine-readable row per candidate, a metadata dictionary with units, input version hashes, and separate

scripts for deterministic analysis, plotting, and probabilistic evaluation.

TABLE XXII
COMPUTATIONAL REPRODUCIBILITY METADATA

Record	Value	Note
Deterministic cases	48,600	Cartesian product in Table VI
Feasible cases	13,859	All filters evaluated on unrounded values
Pareto objectives	Carbon and utilization	Both minimized
Knee rule	Normalized ideal distance	Equal numerical weighting
Monte Carlo samples	100000	Shared sample definition
Random seed	41721	Reproducible pseudorandom sequence
Internal units	N mm MPa	Conversions only at reporting
Analysis status	Analytical benchmark	No experimental validation claimed

DATA AVAILABILITY STATEMENT

All reported numerical results are generated from the equations, parameter levels, factors, and random seed disclosed in this paper. No confidential project dataset or human-subject data were used. The calculated database can be regenerated from the stated Cartesian parameter grid.

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