

Intelligent Attention Networks for Quantum Tomography

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Abstract:-The rapid evolution of quantum computing technologies has made the reconstruction of noisy many-body quantum states a critical research problem. As quantum systems continue to grow in complexity, generative neural network models have emerged as a promising approach for accurately estimating quantum states. In this study, we present the **Intelligent Attention Network (IAN)**, an attention-driven generative framework designed to reconstruct quantum states by learning the density matrix of noisy mixed quantum systems.

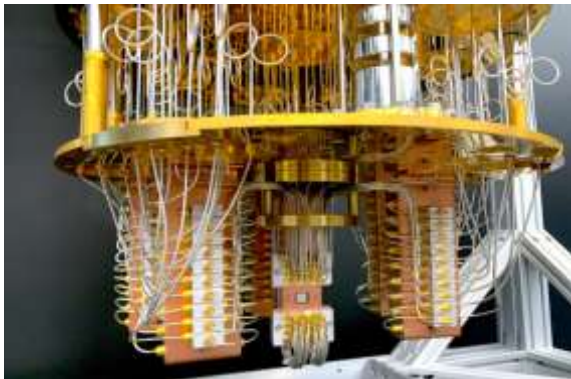


Fig:- IBM's Quantum computer

Introduction:-The IAN architecture draws inspiration from the transformer-based attention mechanism introduced by **Vaswani et al.** in the influential paper *"Attention Is All You Need"* (NIPS 2017). Originally developed for natural language processing, this attention mechanism is highly effective at capturing long-range dependencies and contextual relationships within sequential data, enabling it to outperform conventional NLP models.

Experimental evaluations demonstrate that IAN achieves higher reconstruction accuracy than previously proposed neural-network-based quantum tomography methods under comparable conditions. Furthermore, the framework successfully reconstructs the density matrices of noisy quantum states generated on **IBM Quantum (IBMQ)** hardware. The superior performance of IAN is attributed to its ability to effectively capture and represent quantum entanglement across the entire quantum system.

The rapid advancement of contemporary quantum hardware [1] has made the accurate characterization and verification of large-scale quantum systems an increasingly significant challenge. **Quantum state tomography (QST)** provides a complete framework for describing quantum states [2]. However, because the **Hilbert space** of an N_q -qubit system grows exponentially with the number of qubits, conventional tomography methods, including **Gaussian maximum likelihood estimation (MLE)** [3], demand an exponential amount of measurement data as well as exponential computational resources. As a result, exact reconstruction of the density matrix is practically limited to relatively small quantum systems with $N_q \lesssim 10$. For example, the quantum tomography implementation available in IBM's Qiskit framework currently supports only $N_q = 3$ qubits.

Due to these computational limitations, many experimental studies employ indirect approaches for estimating quantum errors, such as different forms of **randomized benchmarking** [4]. Although recent research has introduced methods that estimate quantum state properties directly from measurement outcomes with encouraging scalability [5], these

techniques generally depend on global entangling operations acting simultaneously on all qubits. Such operations remain beyond the capabilities of most existing experimental quantum platforms. Consequently, there is a growing demand for scalable and experimentally feasible techniques capable of characterizing noisy and highly entangled many-body quantum states.

In recent years, machine learning—particularly deep neural network-based generative models—has emerged as a promising approach for quantum state reconstruction [6–8]. This direction originated from the work presented in [9], where a **restricted Boltzmann machine (RBM)** was trained to represent complex quantum many-body states without requiring exponentially large memory or parameter counts. Despite these advances, the representational capacity and training scalability of **RBM**s are generally confined to pure and positive quantum states [6, 9–12], individual quantum oscillators [13], and relatively small mixed-state systems [14]. These limitations reduce their effectiveness for reconstructing quantum states generated by today's large-scale noisy quantum computing devices.

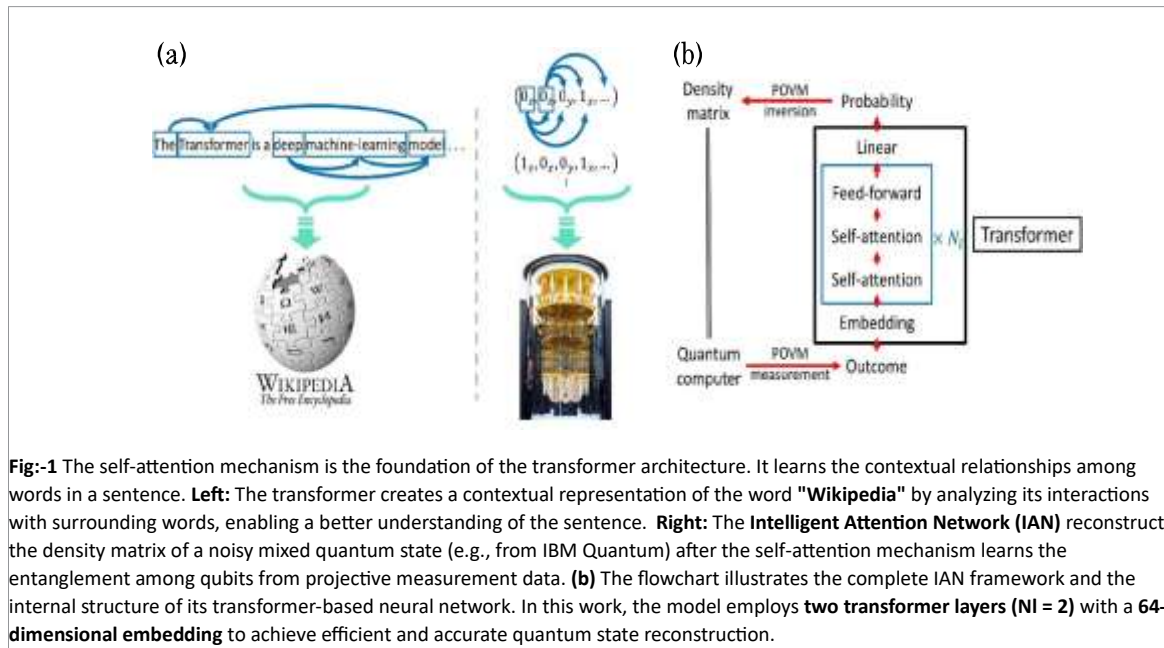


Fig:- 1

The Transformer architecture [15] is an **autoregressive neural network** model that utilizes the attention mechanism [17–19] to effectively capture dependencies within sequential data. Compared with earlier state-of-the-art **Natural Language Processing (NLP)** models, such as **Recurrent Neural Networks (RNNs)** [20, 21], the Transformer has demonstrated significant improvements in both computational efficiency and predictive accuracy. Prior to the introduction of the Transformer, most NLP applications relied on the RNN architecture [22, 23], where contextual information was preserved by passing an encoded memory of previously processed words through the sequence as each new word was analyzed.

Despite its success, the RNN-based approach is inherently limited in modeling long-distance dependencies because the encoded memory gradually loses information as the sequence length increases, leading to an **exponential** reduction in correlation strength [24]. To overcome this limitation, the **Transformer architecture** employs a **self-attention mechanism** that simultaneously evaluates the relationships among all words in a sentence. This enables the model to capture both short-range and long-range semantic dependencies more effectively, resulting in improved contextual understanding and superior performance across a wide range of NLP tasks.

As illustrated schematically in **Figure 2(b)**, the proposed quantum state reconstruction framework receives a collection of **positive operator-valued measurement (POVM)** outcomes, obtained either from numerical simulations or real quantum hardware, as its input and produces the corresponding reconstructed **density matrix** as the output. The underlying **Transformer** model is trained using a dataset consisting of **Ns** one-shot local **POVM** measurement outcomes $\{a_i\}^{Ns}$ of a quantum state ρ , where each measurement is a vector

$a_i \in \{1, \dots, N_a\} \otimes N$ sampled from the distribution:

$$p_a(\vec{a}) = \text{Tr}[\mathbf{M}^a \mathbf{p}], \quad (1)$$

The POVM T -matrix is defined as $T^{a,a'} = \text{Tr}[\mathbf{M}^a \mathbf{M}^{a'}]$. If $T^{a,a'}$ is invertible, the reconstructed density matrix ρ_T can be computed from the learned POVM distribution $p_T(\vec{a})$ in a post-processing step: $\rho = \sum_a p(\vec{a}) T^{-1} \mathbf{M}^a$. (2)

In both the experimental implementation and classical simulation studies, the N_q -qubit Greenberger–Horne–Zeilinger (GHZ) state is selected as the target quantum state.

$$\text{GHZ}_N = \frac{1}{\sqrt{2}} (|0\rangle^{\otimes N_q} + |1\rangle^{\otimes N_q}), \quad (3)$$

Quantum systems with sizes varying from $N_q = 3$ to 90 qubits are considered in this study. The choice of the GHZ state is motivated by its importance as a pure quantum state in quantum communication and by the availability of comparable results in the literature, including techniques that reconstruct only quantum state properties rather than the complete density matrix [5, 7].

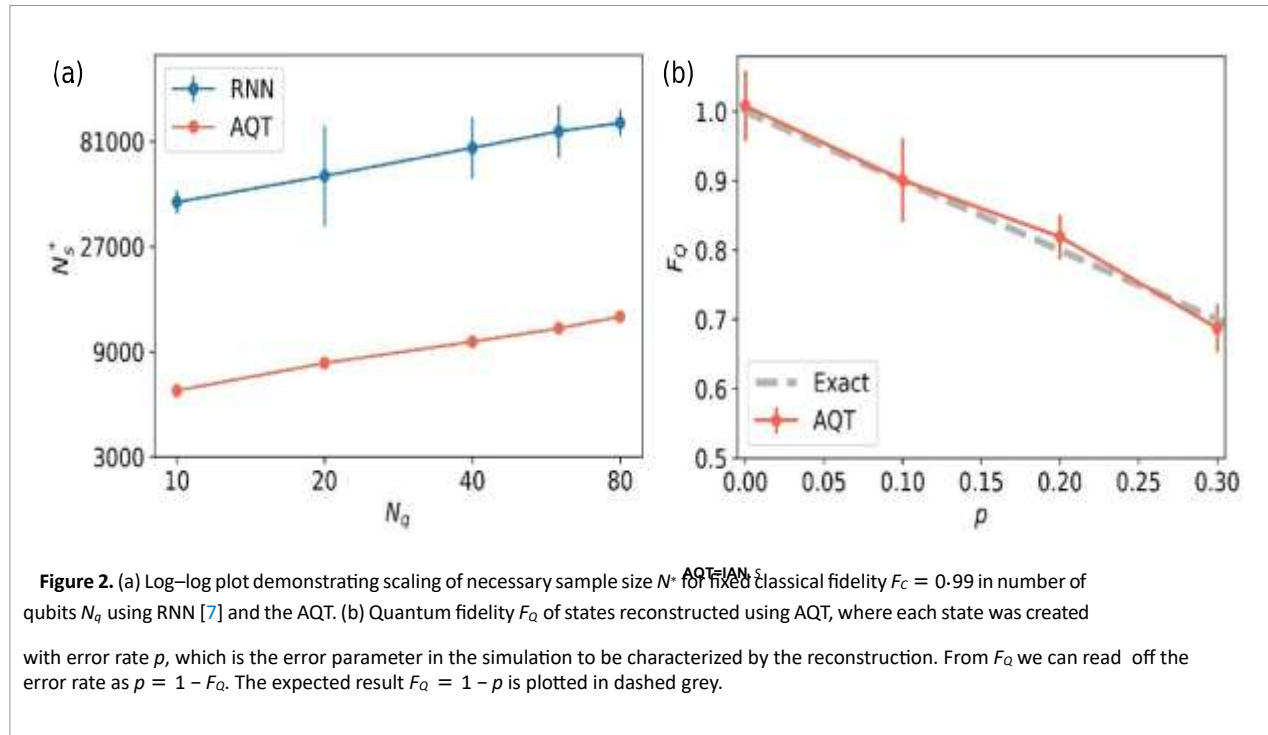


Fig-2 AQT=IAN

We now investigate the IAN’s performance on a mixed state with a built-in simulated error. We consider a 3-qubit GHZ system and assume there is one faulty qubit, which we pick to be qubit-0. We assume that the faulty qubit flips ($0 \leftrightarrow 1$) with probability p . More precisely, this represents the mixed state:

$$\rho_{\text{err}}(p) = (1 - p)|\text{GHZ}\rangle_3 \langle \text{GHZ}|_3 + p|\psi\rangle_3 \langle \psi|_3, \quad (4)$$

To further assess the quantum state characterization capability of the proposed framework, the density matrix of a 6-qubit GHZ state is reconstructed, exceeding the quantum tomography limit currently supported by Qiskit (see Figure 4). Instead of using measurement data collected from IBMQ hardware, classically simulated measurement outcomes generated from an ideal noise-free GHZ state are employed. This choice is primarily motivated by the limited availability of publicly accessible POVM measurement data on the IBMQ platform.

The reconstructed density matrix shown in Figure 4 is obtained using a total of 200000 measurement samples and achieves a quantum fidelity of 0.977. Furthermore, the reconstructed IC-POVM probability distribution (p_1) (see

Equation (4)) exhibits excellent agreement with the theoretical GHZ distribution, which is consistent with the observations presented in Figure 2(a).

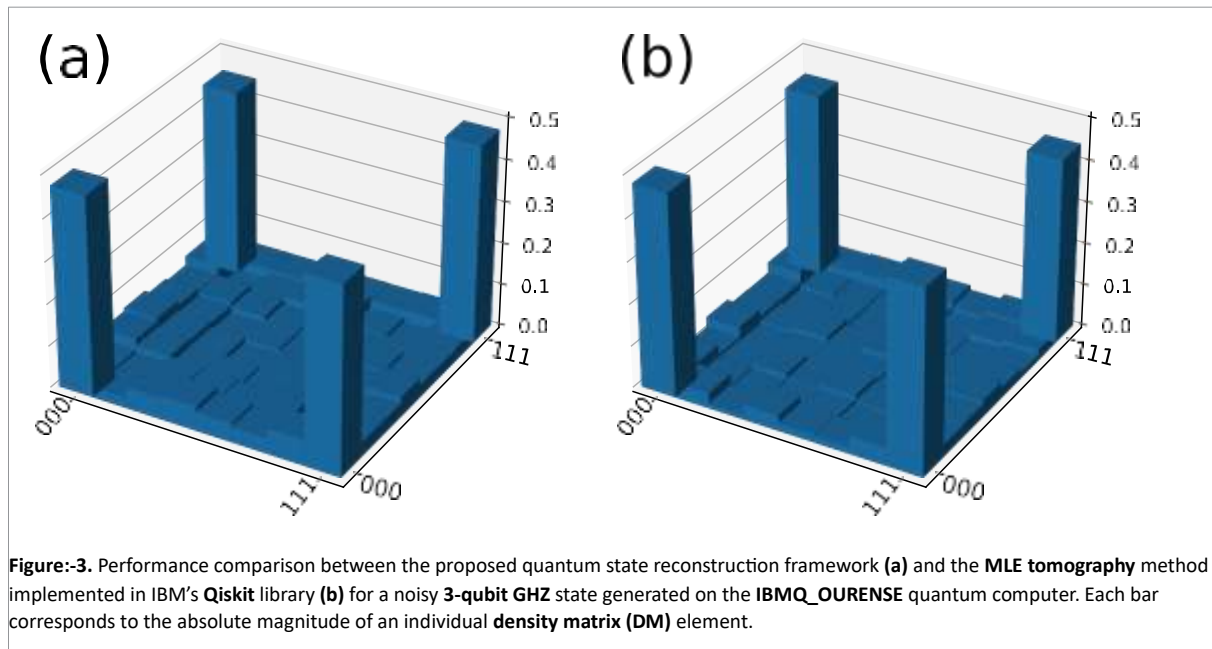


Fig:-3

To the best of our knowledge, IAN is among the first machine learning-based frameworks capable of accurately reconstructing density matrices corresponding to quantum states generated by experimentally realized quantum computing platforms. Compared with existing neural network-based quantum tomography methods, IAN achieves a significant reduction in sample complexity while effectively reconstructing noisy, full-rank quantum states directly from experimental measurement data.

For small-scale quantum systems, IAN provides density matrix reconstruction performance comparable to **Maximum Likelihood Estimation** (MLE) and, at the same time, extends its capability to POVM-based quantum state characterization for larger system sizes. Moreover, IAN relies only on experimentally feasible local **POVM** measurements and does not require globally acting entangling gates, which remain difficult to implement on current quantum hardware.

Another key advantage of IAN is that it does not depend on prior assumptions regarding the purity or entanglement structure of the quantum state being reconstructed. Its expressive learning architecture enables the characterization of a broad range of arbitrary quantum states. Owing to these capabilities, IAN represents an important advancement in integrating machine learning with near-term quantum computing and provides an efficient quantum tomography framework for narrowing the gap between classical simulations and experimental implementations in the era of **noisy intermediate-scale quantum (NISQ) computing**.

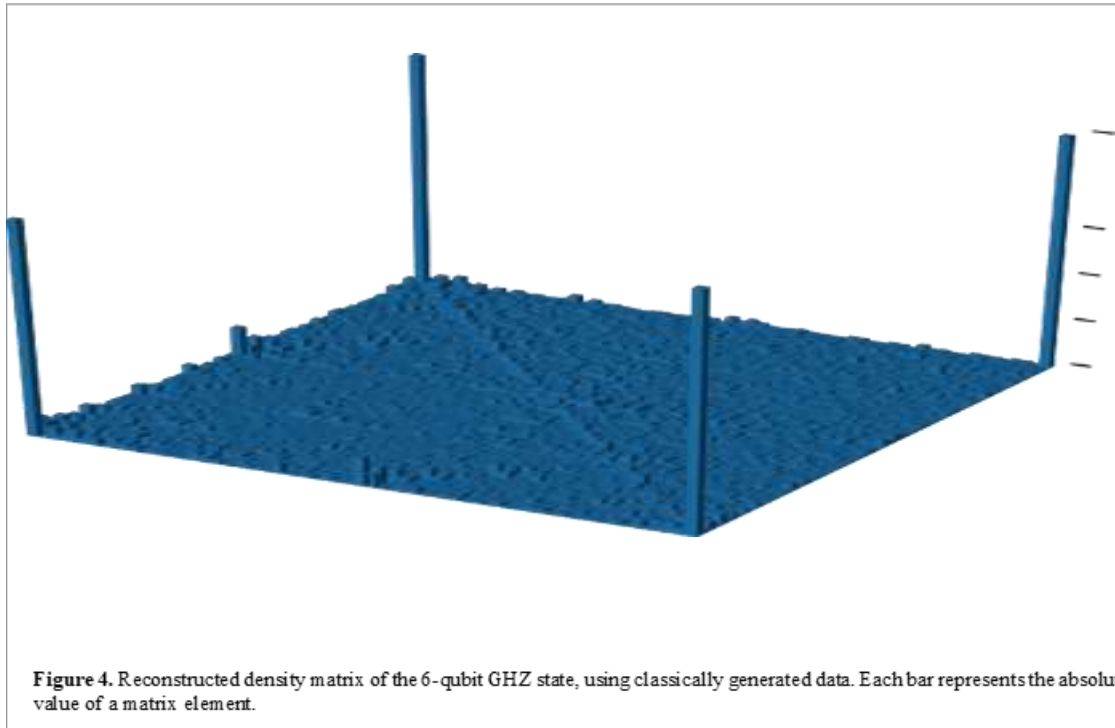


Fig:-4

IAN provides a promising foundation for future research in quantum state tomography. Since IAN learns a **POVM-based representation** of a quantum state that can be utilized to estimate operator expectation values, its application to **shadow tomography** represents an important direction for future investigation. The present study primarily focuses on the **GHZ** state, allowing direct comparison with previously reported methods that do not perform complete density matrix reconstruction. However, IAN is not restricted to this specific pure quantum state. Future studies investigating how the required number of measurements, N_s^* , scales with the number of qubits, N_q , for quantum states exhibiting more complex entanglement structures could provide deeper insights into machine learning-based quantum tomography. In addition, evaluating IAN on larger experimental quantum systems and across different quantum computing architectures will help determine its scalability and practical applicability. Another promising research direction is to extend the current framework beyond the basic simulated error model toward more advanced quantum error characterization and assessment techniques. Such an extension could complement existing **gate-set tomography** approaches [29, 30] and further improve the reliability of quantum state reconstruction on noisy quantum hardware.

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