INTERVAL PROBE GRAPH BY USING TREE

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Abstract

In this paper, we found some relations among interval probe graphs, proper probe graphs and interval graphs. By studying interval graph by using tree Moreover, we also characterize the differences between interval graphs and proper probe graphs.

Key Words

Interval graph, Interval bigraph, Probe interval graph, Ferrers bigraph, Ferrers dimension, Adjacency matrix of a graph.

1. INTRODUCTION

An undirected graph G=(V,E) is an *interval graph* if it is the intersection graph of a family of intervals on the real line in which each vertex is assigned an interval and two vertices are adjacent if and only if their corresponding intervals intersect. The study of interval graphs was spearheaded by Benzer in the course of his studies of the topology of the fine structure of genes. Since then interval graphs and their various generalizations were studied thoroughly. Also advances in the field of molecular biology, and genetics in particular, solicited the need for a new model. In , Zhang introduced another generalization of interval graphs called probe interval graphs, in an attempt to aid a problem called cosmid contig mapping, a particular component of the physical mapping of DNA. A *probe interval graph* is an undirected graph G=(V,E) in which the set of vertices V can be partitioned into two subsets P and N (called probes and nonprobes respectively) and there is an interval (on the real line) corresponding to each vertex such that vertices are adjacent if and only if their corresponding intervals intersect and at least one of the vertices belongs to P. Now several research works are continuing on this topic and some special classes of In fact, Golumbic and Trenk have devoted an entire chapter on probe interval graphs in their recent book on tolerance graphs. In fact, a probe interval graph is a tolerance graph with two distinct tolerances. McConnell and Spinrad obtained a nice algorithm to recognize a probe interval graph with a complexity at most n2. Moreover, motivated by the definition of probe interval graphs, generally, the concept of probe graph classes has been introduced. Given a class of graphs G, a graph G is a *probe graph of*G if its vertices can be partitioned into a set P of probes and an independent set N of nonprobes such that G can be extended to a graph of G by adding edges between certain nonprobes. In this way, many more probe graph classes have been defined and widely investigated, eg., probe split graphs, probe chordal graphs, probe tolerance graphs, probe threshold graphs and others . Another class of graphs that should be mentioned in this context is the class of tolerance graphs . In fact, a probe interval graph is a tolerance graph with two distinct tolerances.

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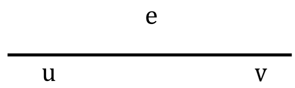
Definition 1.1

A graph G is a discrete structure consisting of nods called vertices and lines joining the nodes called edges.Two vertices are adjacent to each other if they are joint by an edge.The edge joining the two vertices is said to be an edge incident with them.We use V(G) and E(G) to denote the set of vertices and edges of G respectively.

Example:



G

Figure1.1

Here u and v are vertices and e is an edge incident with u and v.e can be denoted by uv or vu.

Definition 1.2:

A subgraph of a graph G is a graph whose vertex and edge sets are subsets of those of

G H Graph

Subgraph

Figure1.2

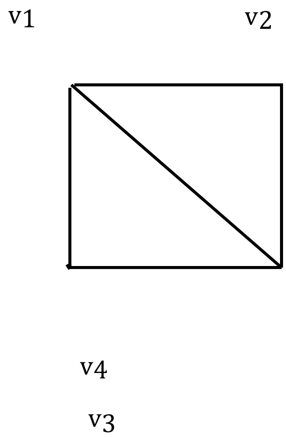
Definition 1.3:

A simple graph is agraph containing no loops and multiple edges

Definition 1.4:

The degree of a vertex is the number of edges incident with it,expect that a loop at a vertex contributes twice to the degree of that vertex.The degree of the vertex v is denoted by deg(v) or d(v).

Example:



Degree of v_1 and v_3 is 3

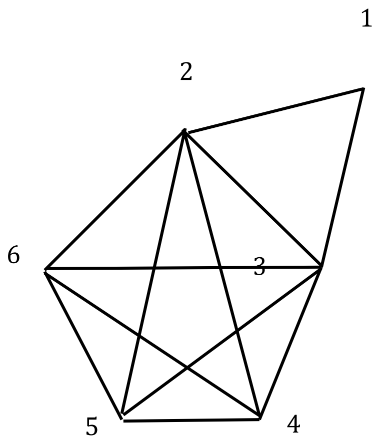
Degree of v_2 and v_4 is 2

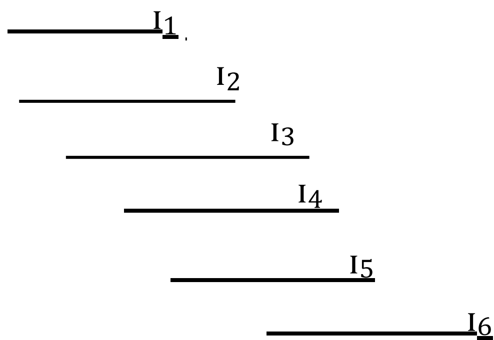
G

Definition 1.5:

Let V is a finite set of intervals of a real line.Then $G(V,E)$ denotes the graph whose vertex set is V and whose edge set is E.$(v_1, v_2) \in E$,if and only if $v_1 \cap v_2 = \varphi$.The graph $G(V,E)$ is an interval graph.

Example





Definition 1.6:

Let V be a vertex set of undirected graph,and P be a subset of V.Then $G(V,P,E)$ denotes a probe graph if its edge set $E \subset (P \times V)$.The elements of P are called probes. Definition 1.7:

Let V be a finite set of intervals of real line,and P be a subset of V.Then a probe graph $G(V,P,E)$ whose vertex set is V,whose probe set is P,and whose edge set is E,will be called a probe interval graph,if its vertex set is $E = \{(v_i, v_j)/v_i \cap v_j = \varphi \text{and} v_i \in P \text{or} v_j \in P\}$.

Definition 1.8:

An interval graph G that has a representation in which each intervals has the same unit length is called a unit interval graph.

Definition 1.9:

An interval graph G that has a representation in which no interval properly cotains another interval G is called a proper interval graph.

Definition 1.10:

Two elements $x, y \in X$ are comparable in P,if $x \prec y$ or $y \prec x$; Otherwise x and y are incomparable, which we denote $x \| y$.

Definition 1.11:

The linear order or chain is one with no incomparabilities and an anti-chain is an order with no comparabilities.

The dual of the ordered set $P = (X, \prec)$ is the order $P^d = (X, \prec^d)$ with $x \prec y \Leftrightarrow$ $y \prec^d x$.

Definition 1.12:

Two graphs are naturally associated with the order $P = (V, \prec)$. The comparability graph $G = (V,E)$ of P has edge set,

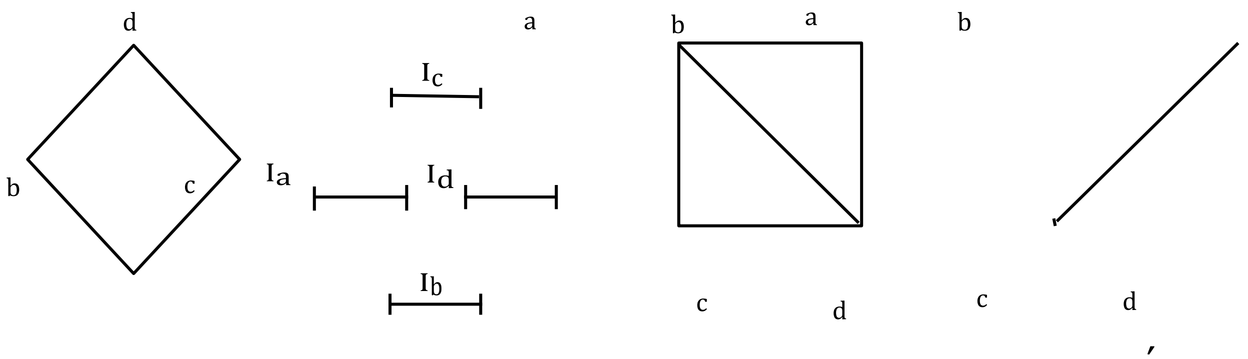
$E = \{xy/x \prec y \text{or} y \prec x\}$

'The incomparability graph G = (V,E) has edge set,

$E = \{xy/x\|y\}$

shows an order P and its comparability graph of G and its incoparability graph G .

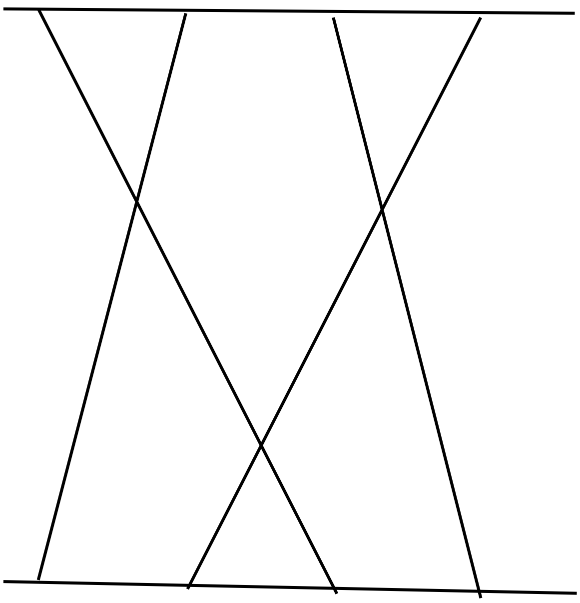
Example:



Definition 1.13:

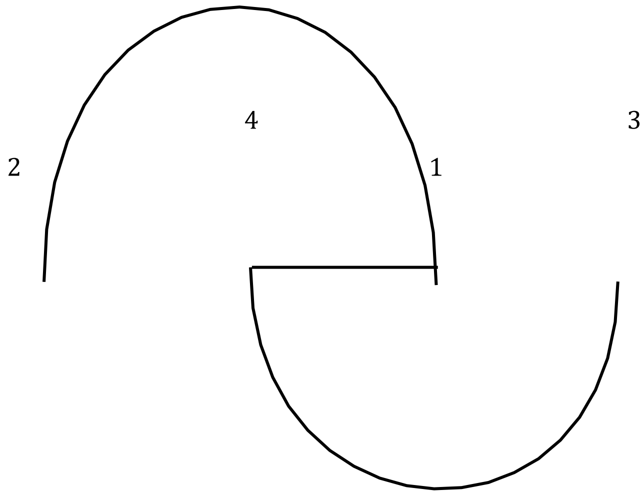
A graph $G = (V, E)$ is a permutation graph if there is a permutation Π of $V = \{1, 2, 3, ..., n\}$ so that for vertices i, j we have $ij \in E$ if and only if the order of i, j are reversed in Π.

Example:



1 2 3 4

2 4 1 3

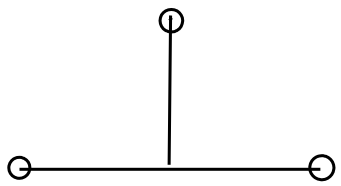


The path P_4 as a permutation graph

Definition 1.14:

An asteroidal triple in a graph G is a set of three non-adjacent vertices such that for any two of them,there exists a path between them in G that does not intersect the neigh- bourhood of the third.

Example:



Graph containing an asteroid triple

Figure 1.10

2. The trees that are Interval Probe Graph

A characterization of the trees that are interval probe graphs.We begin with a general lemma that shows the impossibility of an asteroidal triple of probe vertices.

Lemma 2.1.1.

The tree T_2 is an interval probe graph,and its central vertex must be a non-probe.

proof:

Let T_2 is an interval probe graph.

Suppose that T_2 has an interval completion in which the central vertex a is a probe.

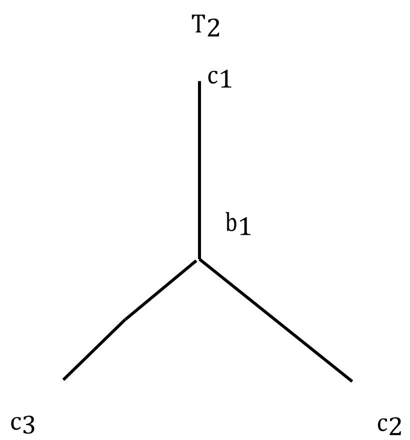


Figure 1.17

Using, Let G be an interval probe graph and Let G be an interval completion with respect to the probe partition P ∪N.If G has an asteroidal triple, then atleast one of these 3 vertices must be a non-probe which has a new neighbour in G .Since c_1, c_2, c_3 is an asteroidal triple of T_2,one of them,say c_1,is a non-probe and gets a new neighbour in G .Now, since c_1 is a non-probe ,vertex b_1 is a probe. If either $c_1b_2 \in E(G)$ or $c_1b_3 \in E(G)$ was a completed edge,Then G would have a chordless 4-

cycle,which is not allowed in an interval completion.

Either $c_1c_2 \in E(G)$ or $c_1c_3 \in E(G)$ must be a comleted edge,but either of these would give a forbidden chrodless 5-cycle in G Thus,c_1 has no new neighbour in G ,a contradiction.probe graph.

CONLUSION

In this paper, we figured out some results in Probe Graphs. A probe proper interval graph is the intersection graph of a set of intervals on the line such that every vertex is mapped to an interval, no interval contains another, and two vertices are adjacent if and only if the corresponding intervals intersect and at least one of them is a probe. We present the first linear-time algorithm that determines whether an input partitioned probe graph is a probe proper interval graph, and if the answer is positive then the algorithm constructs a corresponding set of intervals.

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