Intuitionistic Pre*Irresolute Maps in Intuitionistic Topological Spaces

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Abstract

The major goal of this work is to introduce the concepts of Intuitionistic Pre * Irresolute maps and their contra version in ITS using the concepts of Intuitionistic Pre * Open and Intuitionistic Pre * Closed sets. Further we give characterization for these maps and discuss the relationshipwithotherknownintuitionisticmaps.Alsowefindtheequivalentconditionsforthese maps.

Keywords: IntuitionisticPre*Irresolutemap,IntuitionisticPre*Continuousmap,Contra Intuitionistic Pre * Irresolute map.

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1. Introduction

In 1996, D. Coker [1] introduced the concept of intuitionistic sets and also he has introduced theconceptofintuitionistictopologicalspaces.In2016G.SasikalaandM.Navaneethakrishnan [4]definedintuitionisticPreopensetsinintuitionistictopologicalspaces.We[5]givesthe definition of intuitionistic pre * open sets in intuitionistic topological spaces.

In this study, we define intuitionistic pre * Irresolute maps and their contra version. We also demonstratethattheintuitionisticpre*IrresolutemapisintermediatebetweenQuasiintuitionistic Pre * Continuous and intuitionistic pre * continuous maps.

2. Preliminaries

Definition-2.1[1]Let$X$beanon-emptyset,an*intuitionisticset*(ISinshort)Äisanobjecthaving theformÄ=<X,$Ä_1$,$Ä_2$>,where$Ä_1$and$Ä_2$aresubsetsof$X$ satisfying$Ä_1 \cap Ä_2 = \phi$.Theset$Ä_1$and $Ä_2$ are called the set of members of Äand set of non-members of Ärespectively.

Definition - 2.2[1] Let$\mathcal{X}$be a non-empty set, $\ddot{A}= <\mathcal{X},\ddot{A}_1,\ddot{A}_2>$ and $\mathcal{B}= <\mathcal{X},\mathcal{B}_1,\mathcal{B}_2>$be an IS's and let $\{\ddot{A}_i : i \in J\}$ be arbitrary family of IS's, where $\ddot{A}= <\mathcal{X},\ddot{A}_1,\ddot{A}_2>$. Then the followings are hold.

a) $\ddot{A}\subseteq\mathcal{B}$ iff $\ddot{A}_1\subseteq\mathcal{B}_1$ and $\ddot{A}_2\supseteq\mathcal{B}_2$.

b) $\ddot{A}=\mathcal{B}$ iff $\ddot{A}\subseteq\mathcal{B}$ and $\ddot{A}\supseteq\mathcal{B}$.

c) $\ddot{A}^c=<\mathcal{X},\ddot{A}_2,\ddot{A}_1>$ is calledthecomplementof$\ddot{A}$and$\ddot{A}^c$is alsodenotedby$\mathcal{X}-\ddot{A}$.

d) $\cup\ddot{A}_i=<\mathcal{X},\cup\ddot{A}_{i1},\cap\ddot{A}_{i2}>$.

e) $\cap\ddot{A}_i=<\mathcal{X},\cap \ddot{A}_{i1},\cup\ddot{A}_{i2}>$.

f) $\ddot{A}- \mathcal{B}= \ddot{A}\cap\mathcal{B}^c$.

g) $\ddot{\phi}_I=<\mathcal{X},\phi,\mathcal{X}>$ and $\ddot{\mathcal{X}}_I=<\mathcal{X},\mathcal{X},\phi>$.

Definition 2.3[1]Assumethat $\mathcal{X}_J$ is anon-emptyset and$m_{xj}\in\mathcal{X}_J$ beafixedelementthenthe $\mathcal{J}$S M_J is defined by $M_J = <\mathcal{X}_J,\{m_{xj}\},\{m_{xj}\}^c>$ is called an intuitionistic point.

Definition-2.4[1]Let$\mathcal{X}$ beanon-emptyset and τ_{IT}bethefamilyof intuitionisticsetsof$\mathcal{X}$ then τ_{IT} is called an *intuitionistic topology* (IT in short) on$\mathcal{X}$ if it is satisfying the following axioms:

1) $\ddot{\mathcal{X}}_I,\ddot{\phi}_I\in\tau_{IT}$.

2) $\ddot{A}\cap \mathcal{B}\in\tau_{IT}$ forevery$\ddot{A}$, $\mathcal{B}\in\tau_{IT}$.

3) $\cup\ddot{A}_i\in\tau_{IT}$for anyarbitrary family$\{\ddot{A}_i:i\in J\}\subseteq\tau_{IT}$.

The pair $(\mathcal{X},\tau_{IT})$ is called *intuitionistic topological space* (ITS in short) and IS in τ_{IT} is known as theintuitionisticopenset(IOSinshort)in$\mathcal{X}$,thecomplementoftheIOSiscalledtheintuitionistic closed set (ICS in short) in$\mathcal{X}$.

Definition-2.5[1]Let$(\mathcal{X},\tau_{IT})$beanITSand$\ddot{A}$beaISin$\mathcal{X}$ thentheintuitionisticinterioroperator of $\ddot{A}$(Iint($\ddot{A}$) in short) and intuitionistic closure operator of $\ddot{A}$(Icl($\ddot{A}$) in short) are defined by:

$$Iint(\ddot{A})= \cup\{\mathcal{G}:\mathcal{G}\text{ is an IOS in }\mathcal{X}\text{ and }\ddot{A}\supseteq\mathcal{G}\}.$$

$$Icl(\ddot{A})=\cap \{\mathcal{G}:\mathcal{G}\text{ is an ICS in }\mathcal{X}\text{ and }\ddot{A}\subseteq\mathcal{G}\}.$$

Definition - 2.6[2] $(\mathcal{X},\tau_{IT})$ be an ITS and $\ddot{A}$ be a IS in$\mathcal{X}$ then $\ddot{A}$ is said to be *intuitionistic generalizedclosed*(Ig-closedinshort)setifIcl($\ddot{A}$)$\subseteq\mathcal{U}$whenever$\ddot{A}\subseteq\mathcal{U}$ and$\mathcal{U}$ isIOSin$\mathcal{X}$.The complement of the Ig – closed set is called the *Ig- open set* in$\mathcal{X}$.

Definition - 2.8[4,5] Let$(\mathcal{X},\tau_{IT})$ bean ITSand $\ddot{A}$bean intuitionisticset then

a) $\ddot{A}$is intuitionisticpreopen(IPO)setin$\mathcal{X}$ifA$\subseteq$Iint(Icl(A)).

b) $\ddot{A}$is intuitionisticpre*open(IP*O) setin$\mathcal{X}$ifA$\subseteq$Iint(Icl*(A)).

ThecomplementoftheIPO andIP*O sets arecalledtheIPCandIP*Csetsin$\mathcal{X}$.

Theorem- 2.9[5] Let(X,τ_{IT}) bean ITSthen thefollowingsarehold.

a) EveryIO setis IP*O set.
b) EveryIP*O setisIPOset.
c) ArbitraryunionofIP*O setsisIP*O set.
d) Intersectionof IP*Csets is IP*Cset.

Theorem- 2.10[5] Let(X,τ_{IT}) bean ITSand Äand BbeaIS of Xthen thefollowingsare hold.

a) IP*int(ϕ_I)= $\phi_I$andIP*int(X_I)=X_I.
b) IfÄis IP*-open setthen Ä= IP*int(Ä).
c) Ä⊆Bthen IP*int(Ä) ⊆IP*int(B).
d) IP*cl(ϕ_I) =$\phi_I$andIP*cl(X_I)= X_I.
e) IfÄis IP*-closedsetthen Ä=IP*cl(Ä).
f) Ä⊆BthenIP*cl(Ä)⊆IP*cl(B).

Theorem- 2.11[1,5]Let(X,τ_{IT}) bean ITSand ÄbeaISof Xthen thefollowingsare hold.

a) Iint(X–Ä) =X– Icl(Ä)and Icl(X– Ä)=X– Iint(Ä).
b) IP*int(X– Ä)=X– IP*cl(Ä) andIP*cl(X– Ä) =X– IP*int(Ä).

Theorem -2.12[6]Letf :$X \to Y$is saidto be

a) I-continuous mapiff^{-1}(V)is IOsetin XforeveryIOsetV in Y.
b) IP*-continuous mapiff^{-1}(V)isIP*O setin$X$foreveryIO setV in Y.
c) IP-continuous iff^{-1}(V)is$\mathcal{I}$ƤOsetin Xforevery$\mathcal{I}$O setVin$Y$.
d) QuasiIP*- continuous iff^{-1}(V)isIO setin XforeveryIP*OsetV$_J$in$Y$.
e) PerfectlyIP*-continuous iff^{-1}(V)is I-clopensetin$X$foreveryIP*O setVin$Y$.
f) StronglyIP*-continuous iff^{-1}(V)is IP*−clopensetin$X$foreveryIS V in$Y$.

3. IntuitionisticPre*IrresoluteMaps

Definition–3.1. AmapffromX toY issaidtobeIntuitionisticPre*Irresoluteataintuitionistic pointm∈XifforeachIP*OsetBof$Y$containingf(m),thereisanIP*OsetAin$X$suchthatm ∈Aand f(A)⊆B.

Definition – 3.2. A map f from ITS (X,τ_{IT}) into another ITS (Y,σ_{IT}) is Called Intuitionistic Pre * Irresolute Map if f^{-1}(M)is IP*O in X for each IP*O set M in Y.

Example–3.3. Let$X$=Y={a,b,c}.ConsidertheIT'sτ_{IT}={X_I,ϕ_I,<X,{b},{a,c}>,<X,{a,b},{c}>}

andσ_{IT}={Y_I,ϕ_I,<Y,{a},{b,c}>,<Y,{a,b},{c}>}thenIP*O(X)=τ_{IT}andIP*O(Y)=σ_{IT}.Letf: (X,τ_{IT})→(Y,σ_{IT})beamapdefinedby,f(a)=b,f(b)=a,f(c)=c.Here,f^{-1}(Y_I)=X_I,f^{-1}(ϕ_I)=ϕ_I,f^{-1}(<Y,{a},{b,c}>)=<X,{b},{a,c}>,f^{-1}(<Y,{a,b},{c}>)=<X,{a,b},{c}>.Therefore,inverseimage of each IP*O set in Y under f is IP*O sets in X. Therefore f is IP*- Irresolute map.

Theorem–3.4. Amapf:(X,τ_{IT})→(Y,σ_{IT})isIP*-Irresolutemapiffinverseimageofevery IP*C set in Y is IP*C set in X.

Proof: Suppose f : (X,τ_{IT}) → (Y,σ_{IT}) is IP*- Irresolute map. Let M be any IP*C set in Y. (i.e) M^c is IP*O set in Y then by definition – 3.2, $f^{-1}(M^c)$ = $[f^{-1}(M)]^c$ is IP*O set in X. Therefore, f^{-1}(M) is IP*Csetin$X$.Conversely,supposeMbeanyIP*Osetin Y.(i.e)M^cisIP*Csetin$Y$.Therefore$f^{-1}(M^c)$ = $[f^{-1}(M)]^c$ is IP*C set in X. Therefore, f^{-1}(M) is IP*O set in X. Hence f is IP*- Irresolute.

Theorem–3.5. EveryconstantmapisIP*-Irresolutemap.

Proof: Supposeamapf:(X,τ_{IT})→(Y,σ_{IT}) isaconstantmapdefinedbyf(m)=$n_0$forallIPm∈ $X$and $n_0$isafixedpointin$Y$. LetU beanIP*Osetin$Y$then f^{-1}(U)=X_I (or)$\phi_I$accordingasx_0∉ U.Thusf^{-1}(U)isIP*O setin$X$. Hencef is IP*- Irresolutemap.

Theorem–3.6. EveryIP*-Irresolutemapis IP*-continuous.

Proof: Supposeamapf:X →Y isIP*-irresolute.LetMbeanyIOsetin$Y$.Therefore,MisIP*O in Y. Therefore f^{-1}(M) is IP*O set in X. Hence, f is IP*- Irresolute map.

Theconverseoftheabovetheoremneednotbetrueas shows inthefollowingexample.

Example–3.7. Let$X$={a,b}and$Y$={1,2}.ConsidertheIT'sτ_{IT}={X_I,ϕ_I,<X,{b},ϕ>,<X,ϕ, {b}>}andσ_{IT}={Y_I,ϕ_I,<Y,{1},ϕ>}thenIP*O(X)=τ_{IT}andIP*O(Y)={Y_I,ϕ_I,<Y,{1},ϕ>, <Y,{1},{2}>}. Letf : (X,τ_{IT}) → (Y,σ_{IT}) beamap defined by, f(a) = 2, f(b) = 1. Here, f^{-1}(Y_I) =X_I, f^{-1}(ϕ_I) =ϕ_I, f^{-1}(<Y,{1},ϕ>) =<X,{b},ϕ> are all IP*O sets in X. Therefore, f is IP*- continuous. But f^{-1}(<Y,{1},{2}>) = <X,{b},{a}> is not a IP*O set in X. Therefore, f is not a IP*- irresolute.

Theorem–3.8. Let(X,τ_{IT})and(Y,σ_{IT})beanITS inwhicheveryIP*O setis IOS.Thenf:(X,τ_{IT})

$\rightarrow(Y,\sigma_{IT})$ is an IP*-irresolutemap iffis IP*- continuousmap.

Proof: Let M be any IP*O set in Y then by hypothesis, M is an IO set in Y. Since, f is IP*- continuous then $f^{-1}(M)$ is IP*O set in X. Hence f is IP*- irresolute map.

Theorem– 3.9. Let$(X,\tau_{IT})$and (Y,σ_{IT})bean ITSthen thefollowingsarehold.

a) EveryQuasiIP*-continuous mapisIP*-Irresolutemap.
b) EveryPerfectlyIP*-continuousmapisIP*-Irresolutemap.
c) EveryStronglyIP*-continuousmapisIP*-Irresolute map.

Proof: (a)Let $f : (X,\tau_{IT}) \rightarrow (Y,\sigma_{IT})$ be a Quasi IP*- continuous map. Let M be any IP*O set in Y then $f^{-1}(M)$ is IO set in X. Since, every IO set is IP*O set. Therefore, $f^{-1}(M)$ is IP*O set in X for each IP*O set M in Y. Hence, f is IP*- Irresolute map.

(b) Letf: $(X,\tau_{IT})\rightarrow(Y,\sigma_{IT})$beaPerfectlyIP*-continuousmap.LetM be anyIP*Osetin Y then $f^{-1}(M)$isI-clopensetin$X$.(i.e),$f^{-1}(M)$isI-opensetin$X$.Since,everyIOsetisIP*Oset.Therefore, $f^{-1}(M)$ is IP*O set in X. Hence, f is IP*- Irresolute map.

(c) Letf :$(X,\tau_{IT}) \rightarrow (Y,\sigma_{IT})$beaStronglyIP*- continuous map. LetM beany IP*O setin Y. (i.e) MisI-setin$Y$.Thereforef^{-1}(M)isIP*-clopenset in X.(i.e),$f^{-1}(M)$isIP*-opensetin$X$.Hence, f is IP*- Irresolute map.

Theconverseoftheabovetheoremneednotbetrueas shows inthefollowingexample.

Example–3.10. Let$X$=Y={a,b,c}.ConsidertheIT'$s\tau_{IT}$={X_I,ϕ_I,<X,{a},{c}>,

<X,{c},{a,b}>,<X,{a,c},ϕ>}andσ_{IT}={Y_I,ϕ_I,<Y,{a},{b,c}>,<Y,{a,b},{c}>}thenIP*O(X)=

{X_I,ϕ_I,<X,{a},{c}>,<X,{c},{a,b}>,<X,{a,c},ϕ>,<X,{a},ϕ>,<X,{a},{b,c}>,<X,{a,c},{b}>} and

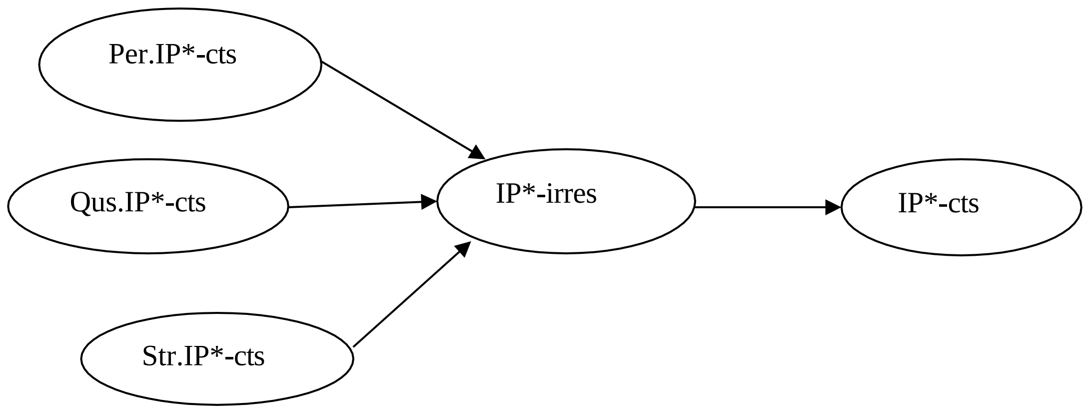
IP*O(Y)=σ_{IT}.Letf:$(X,\tau_{IT})\rightarrow(Y,\sigma_{IT})$beamapdefinedby,f(a)=a,f(b)=c,f(c)=b.Here,$f^{-1}(Y_I)$ = X_I, $f^{-1}(\phi_I)$ = ϕ_I, f^{-1}(<Y,{a},{b,c}>) =<X,{a},{b,c}>, and f^{-1}(<Y,{a,b},{c}>) = <X,{a,c},{b}>. Therefore, inverse image of each IP*O set in Y under f is IP*O sets in X. Therefore f is IP*- Irresolutemap.But, f^{-1}(<Y,{a},{b,c}>)andf^{-1}(<Y,{a,b},{c}>)arenotbelongsto τ_{IT}.Therefore,f is not a Quasi IP*- continuous map.

Example – 3.11. In example – 3.10, f is IP*- irresolute map. But, f^{-1}(<Y,{a},{b,c}>) and f^{-1}(<Y,{a,b},{c}>) are not belongs to τ_{IT} and τ_{IT}^{c}. Therefore, f is not a Perfectly IP*- continuous.

Example – 3.12. In example – 3.10, f is IP*- irresolute map. But, inverse image of every intuitionistic set of

Y is does not belongs to IP*C(X) and IP*O(X). Therefore, f is not a Strongly IP*- continuous map.

Remark–3.13. Thefollowingdiagramshowstherelationshipof IP*-irresolute mapwithother IP*- continuous maps.



Theorem–3.14. Letf:$X \to Y$beamap,thenthefollowings are equivalent.

a) fisanIP*-irresolutemap.

b) fisanIP*-irresoluteateveryintuitionisticpointof$X$.

c) f^{-1}(M)is IP*C setin $X$for everyIP*CsetM in$Y$.

Proof: (a) ⇒(b), Let f : $X \to Y$be IP*- irresolute map. Let m be any IP in $X$and Mbe an IP*O set in Y containing f(m). (i.e), f(m) $\in \omega$then m $\in f^{-1}(\omega)$. Since f is IP*- irresolute, N = f^{-1}(M) is an IP*O set in Xcontaining a point m such that f(N) ⊆M. Hence f is an IP*- irresolute at every IP of X.

(b) ⇒(c),LetMbeanIP*Csetin$Y$ thenM^cisanIP*Osetin Y.Letm$\in f^{-1}(M^c)$thenf(m) $\in M^c$. Since f is an IP*-continuous at every IP of$X$then there is an IP*Oset ωin$X$containing a point m such that f(m) $\in f(\omega) \subseteq M^c$. Therefore, $\omega \subseteq f^{-1}(M^c)$. Hence $f^{-1}(M^c) = \cup\{\omega: m \in f^{-1}(M^c)\}$. SincearbitraryunionofIP*OsetisIP*Osetthen$f^{-1}(M^c)$isIP*Osetin$X$.Thus$f^{-1}(M)=f^{-1}[(M^c)^c]$
$=[f^{-1}(M^c)]^c$isIP*C setin X. Hencef^{-1}(M) isIP*Csetin$X$for everyIP*C setMin$Y$.

(c) ⇒(a) Suppose,f^{-1}(M)isIP*Csetin$X$ foreveryICsetMin$Y$.Thenbytheorem–3.4,fisIP*- irresolute map.

Theorem–3.15. Letf:$(X,\tau_{IT}) \to (Y,\sigma_{IT})$ beamap,thenthefollowingsareequivalent.

a) fisanIP*-irresolutemap.

b) f^{-1}(IP*int(A)) ⊆IP*int(f^{-1}(A))foreachIS A of$Y$.

c) IP*int(f(B))⊆f(IP*int(B))foreveryIS Bof$X$.

Proof: (a) ⇒ (b), Let f : X →Y be IP*- irresolute map. Let A be any IS of Y then IP*int(A) is IP*O set in Y. Since f is IP*- irresolute then f^{-1}(IP*int(A)) is IP*O set in X. Therefore, f^{-1}(IP*int(A)) = IP*int(f^{-1}(IP*int(A))). Hence, f^{-1}(IP*int(A)) ⊆ IP*int(f^{-1}(A)).

(b) ⇒(c),LetBbeanyISof$X$thenf(B)isISof$Y$.Byourassumption,f^{-1}(IP*int(f(B)))⊆IP*int(f^{-1}(f(B))). Hence, IP*int(f(B)) ⊆ f(IP*int(B)).

(c) ⇒(a),Let Gbe anyIP*Osetof Y thenf^{-1}(G) isIS of X.By(c),IP*int(f(f^{-1}(G)))⊆f(IP*int(f^{-1}(G))).(i.e)f^{-1}(IP*int(G))⊆IP*int(f^{-1}(G)).Since GisIP*Osetthenf^{-1}(G)⊆IP*int(f^{-1}(G)).Also, IP*int(f^{-1}(G)) ⊆f^{-1}(G). Therefore, f^{-1}(G) is IP*O set in X. Hence f is IP*- irresolute map.

Theorem–3.16. Letf:(X,τ_{IT})⟶(Y,σ_{IT}) beamap,thenthefollowingsareequivalent.

a) fisanIP*-irresolutemap.

b) IP*cl(f^{-1}(A))⊆f^{-1}(IP*cl(A))foreachIS A of$Y$.

c) f(IP*cl(B))⊆IP*cl(f(B))foreveryIS Bof$X$.

Proof:(a)⇒(b),Letf:X →Y beIP*-irresolutemap.LetAbeanyISof$Y$ thenIP*cl(A)isIP*C set in Y. Since f is IP*-irresolute then f^{-1}(IP*cl(A)) is IP*C set in X. Therefore, f^{-1}(IP*cl(A)) = IP*cl(f^{-1}(IP*cl(A))). Hence, IP*cl(f^{-1}(A)) ⊆ f^{-1}(IP*cl(A)).

(b) ⇒ (c), Let B be any IS of X then f(B) is IS of Y. By our assumption, IP*cl(f^{-1}(f(B))) ⊆ f^{-1}(IP*cl(f(B))). Hence, f(IP*cl(B)) ⊆ IP*cl(f(B)).

(c) ⇒ (a), Let G be any IP*C set of Y then f^{-1}(G) = f^{-1}(IP*cl(G)). By our assumption, f(IP*cl(f^{-1}(G)))⊆IP*cl(f(f^{-1}(G))). Therefore,IP*cl(f^{-1}(G)) ⊆f^{-1}(IP*cl(G))= f^{-1}(G).Also, f^{-1}(G)⊆IP*cl(f^{-1}(G)). Therefore, f^{-1}(G) is IP*C set in X. Hence f is IP*- irresolute map.

Theorem – 3.17. Let f : (X,τ_{IT}) → (Y,σ_{IT}) and g : (Y,σ_{IT}) → (Z,μ_{IT}) be IP*- irresolute map then their composition g∘f : (X,τ_{IT}) → (Z,μ_{IT}) is also IP*- irresolute map.

Proof: Let ω be an IP*O set in $\mathcal{Z}$. Since g is IP*- irresolute map then $g^{-1}(\omega)$ is IP*O set in $\mathcal{Y}$. Since f be IP*- irresolute map. Therefore $f^{-1}(g^{-1}(\omega)) = (g\circ f)^{-1}(\omega)$ is IP*O set in $\mathcal{X}$. Hence, g∘f is IP*- irresolute map.

Corollary –3.18. Thecomposition oftwoIP*-irresolutemapsis

a) IP*-continuousmap.

b) IP-continuousmap.

Proof: SupposefandgareIP*-irresolutemapthenbytheorem–3.17,g∘fisIP*-irresolutemap.

(a) By theorem – 3.6, g∘f is IP*- continuous map. (b) Since, every IP*- continuous map is IP- continuous map. Therefore, g∘f is IP- continuous map.

Theorem–3.19. Let$(\mathcal{X},\tau_{IT})$,$(\mathcal{Y},\sigma_{IT})and(\mathcal{Z},\mu_{IT})$be threeITS,f:$(\mathcal{X},\tau_{IT})\rightarrow(\mathcal{Y},\sigma_{IT})$andg:$(\mathcal{Y},\sigma_{IT})$ $\rightarrow(\mathcal{Z},\mu_{IT})$aremapsthen thefollowingsarehold,

a) Iffis IP*-irresolutemapandgis IP*-continuous maptheng∘fis IP*-continuousmap.

b) Iffis IP*-irresolutemapandgis IP*-continuous maptheng∘fis IP-continuousmap.

Proof: (a)Let ωbe an IO set in $\mathcal{Z}$. Since g is IP*- continuous map then $g^{-1}(\omega)$ is IP*O set in $\mathcal{Y}$. Since f be IP*- irresolute map then $f^{-1}(g^{-1}(\omega)) = (g\circ f)^{-1}(\omega)$is IP*O set in $\mathcal{X}$. Hence, g∘f is IP*- continuous map.

(b) By(a), g∘fis IP*-continuousmap. Since, every IP*-continuous map isIP-continuousmap. Therefore, g∘f is IP- continuous map.

4. ContraIntuitionisticPre*IrresoluteMaps

Definition–4.1. AmapffromITS$(\mathcal{X},\tau_{IT})$intoanotherITS$(\mathcal{Y},\sigma_{IT})$iscalledContraIntuitionistic Pre * Irresolute Map if $f^{-1}(M)$is IP*C set in $\mathcal{X}$for each IP*O set M in $\mathcal{Y}$.

Example–4.2. Let$\mathcal{X}$={a,b,c}and$\mathcal{Y}$={1,2,3}.ConsidertheIT'$s\tau_{IT}=\{\tilde{\mathcal{X}}_I,\tilde{\phi}_I,$ <$\mathcal{X}$,{a},{b,c}>,<$\mathcal{X}$,{b},{a,c}>,<$\mathcal{X}$,{a,b},{c}>}andσ_{IT}={$\tilde{\mathcal{Y}}_I$, $\tilde{\phi}_I$,<$\mathcal{Y}$,{2},{1,3}>}thenIP*C($\mathcal{X}$)= {$\tilde{\mathcal{X}}_I$,$\tilde{\phi}_I$,<$\mathcal{X}$,{c},{a,b}>,<$\mathcal{X}$,{a,c},{b}>,<$\mathcal{X}$,{b,c},{a}>} andIP*O($\mathcal{Y}$)=σ_{IT}.Letf:$(\mathcal{X},\tau_{IT})\rightarrow(\mathcal{Y},\sigma_{IT})$ bea mapdefinedby,f(a)=3,f(b)=1,f(c)=2.Here,$f^{-1}(\tilde{\mathcal{Y}}_I)=\tilde{\mathcal{X}}_I$,$f^{-1}(\tilde{\phi}_I)=\tilde{\phi}_I$,$f^{-1}$(<$\mathcal{Y}$,{2},{1,3}>)

=<𝑋,{c},{a,b}>∈IP*C(𝑋). Therefore,f iscontraIP*-irresolutemap.

Theorem–4.3. Amapf :(𝑋,τ_{IT})→(𝑌,σ_{IT})is Contra IP*-Irresolute Mapiffinverse image of every IP*C set in 𝑌 is IP*O set in 𝑋.

Proof: Suppose f : (𝑋,τ_{IT}) → (𝑌,σ_{IT}) is Contra IP*- Irresolute Map. Let M be any IP*C set in𝑌. (i.e)M^cisIP*Osetin𝑌thenf^{-1}(M^c)=[f^{-1}(M)]^{c}isIP*Csetin𝑋.Therefore,f^{-1}(M)isIP*Osetin 𝑋. Conversely, suppose M be any IP*O set in 𝑌. (i.e) M^c is IP*C set in 𝑌. Therefore f^{-1}(M^c) = [f^{-1}(M)]^{c}isIP*Osetin𝑋. Therefore, f^{-1}(M)isIP*Csetin𝑋. HencefisContraIP*- IrresoluteMap.

Theorem–4.4. EveryContraIP*-IrresolutemapisContraIP*-continuous map.

Proof: Suppose a map f : (𝑋,τ_{IT}) → (𝑌,σ_{IT}) is Contra IP*- irresolute. Let M be any IO set in 𝑌. Since, every IO set is IP*Oset. Therefore, M is IP*O in 𝑌. Since f is Contra IP*- Irresolute map. Therefore f^{-1}(M) is IP*C set in 𝑋. Hence, f is Contra IP*- continuous map.

Theconverseoftheabovetheoremneednotbetrueas shows inthefollowingexample.

Example–4.5. Let𝑋={a,b}and𝑌={1,2}.ConsidertheIT’sτ_{IT}={$𝑋_I$,ϕ_I,<𝑋,{a},ɸ>, <𝑋,ɸ,{a}>}andσ_{IT}={$𝑌_I$,ϕ_I,<𝑌,{1},ɸ>}thenIP*C(𝑋)=τ_{IT}andIP*O(𝑌)={$𝑌_I$,ϕ_I, <𝑌,{1},ɸ>,<𝑌,{1},{2}>} . Let f : (𝑋,τ_{IT}) → (𝑌,σ_{IT}) bea map defined by, f(a) = 1, f(b) = 2. Here, f^{-1}($𝑌_I$) =$𝑋_I$, f^{-1}(ϕ_I) = ϕ_I, f^{-1}(<𝑌,{1},ɸ>) =<𝑋,{a},ɸ> are all IP*C sets in 𝑋. Therefore, f is IP*- continuous map. But f^{-1}(<𝑌,{1},{2}>) = <𝑋,{a},{b}> is not a IP*C set in 𝑋. Therefore, f is not a Contra IP*- irresolute map.

Theorem–4.6. Let(𝑋,τ_{IT})and(𝑌,σ_{IT})beanITS inwhicheveryIP*O setis IOS.Thenf:(𝑋,τ_{IT}) →(𝑌,σ_{IT}) isan ContraIP*- irresolutemap if fisContraIP*- continuousmap.

Proof: LetMbeanyIP*Osetin𝑌 thenbyhypothesis,MisanIOsetin𝑌.Since,fisContraIP*- continuous then f^{-1}(M) is IP*C set in 𝑋. Hence f is Contra IP*- irresolute map.

Theorem– 4.7. Let(𝑋,τ_{IT})and (𝑌,σ_{IT})bean ITSthen thefollowingsareholds

a) EveryPerfectlyIP*-continuous mapis ContraIP*-Irresolutemap.
b) EveryStronglyIP*-continuous mapis ContraIP*-Irresolutemap.

Proof:(a)Letf :(𝑋,τ_{IT}) → (𝑌,σ_{IT}) beaPerfectly IP*- continuousmap. LetMbeany IP*Osetin

Y then $f^{-1}(M)$ is I- clopen set in X. (i.e), $f^{-1}(M)$ is IC set in X.Since, every IC set is IP*C set. Therefore, $f^{-1}(M)$ is IP*C set in X. Hence, f is Contra IP*- Irresolute map.

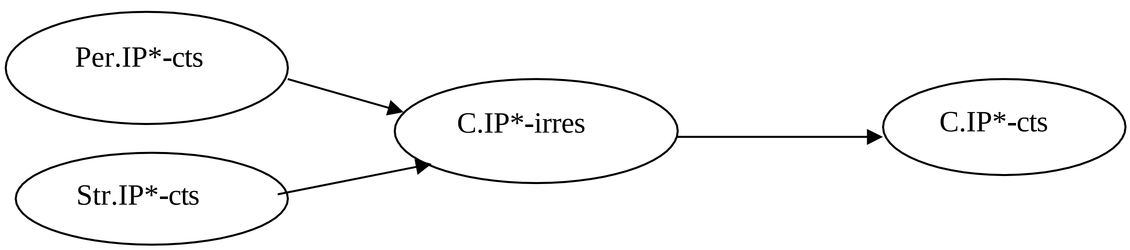
(b)Letf: $(X,\tau_{IT})\rightarrow(Y,\sigma_{IT})$beaStronglyIP*-continuous map.LetM be anyIP*O setin Y.(i.e) M is IS in Y. Therefore, $f^{-1}(M)$ is IP*- clopen set in X. (i.e), $f^{-1}(M)$ is IP*C set in X. Hence, f is Contra IP*- Irresolute map.

Theconverseoftheabovetheoremneednotbetrueas shows inthefollowingexample.

Example–4.8. Inexample–4.2,fisContraIP*-irresolutemap.But,$f^{-1}(<Y,\{2\},\{1,3\}>)= <X,\{c\},\{a,b\}>\in\tau_{IT}$. Therefore,f isnotaPerfectly IP*- continuousmap.

Example – 4.9. In example – 4.2, f is Contra IP*- irresolute map. But, inverse image of every intuitionistic set of Y is does not belongs to IP*C(X) and IP*O(X). Therefore, f is not a Strongly IP*- continuous map.

Remark–4.10. Thefollowingdiagramshowstherelationshipof ContraIP*-irresolutemap with other IP*-continuous maps.



Theorem–4.11. Iff:$(X,\tau_{IT})\rightarrow(Y,\sigma_{IT})$andg:$(Y,\sigma_{IT})\rightarrow(Z,\mu_{IT})$areContraIP*-irresolutemaps then $g\circ f : (X,\tau_{IT}) \rightarrow (Z,\mu_{IT})$ is also IP*- irresolute map.

Proof: LetωbeanIP*Osetin$Z$.Sinceg isContraIP*-irresolutemap then$g^{-1}(\omega)$ isIP*Csetin Y.SincefbeContraIP*-irresolutemap.Therefore$f^{-1}(g^{-1}(\omega))=(g\circ f)^{-1}(\omega)$isIP*Osetin$X$. Hence, $g\circ f$ is IP*- irresolute map.

Corollary –4.12. Thecomposition oftwoIP*-irresolutemapsis

c) IP*-continuousmap.

d) IP-continuousmap.

Proof: SupposefandgareContraIP*-irresolutemapthenbytheorem–4.11,$g\circ f$isIP*-irresolute map. (a) By theorem – 3.6, is IP*- continuous map. (b) Hence, $g\circ f$ is IP- continuous.

Theorem–4.13. Let(X,τ_{IT}),(Y,σ_{IT})and(Z,μ_{IT})be threeITS,f:(X,τ_{IT})→(Y,σ_{IT})andg:(Y,σ_{IT})

→(Z,μ_{IT})aremapsthenthefollowingsarehold,

a) If f is IP*- irresolute map and g is Contra IP*- irresolute map then g∘f is Contra IP*- irresolute map.

b) If f is IP*- irresolute map and g is Contra IP*- irresolute map then g∘f is Contra IP*- continuous map

Proof: (a)Let ωbe an IP*O set in Z. Since g is Contra IP*- irresolute map then $g^{-1}(\omega)$ is IP*C set in Y. Since f be IP*- irresolute map then $f^{-1}(g^{-1}(\omega)) = (g\circ f)^{-1}(\omega)$ is IP*C set in X. Hence, g∘f is IP*- irresolute map.

(b)By (a), g∘f is Contra IP*- irresolute map. By theorem – 4.4, g∘f is Contra IP*- continuous map.

5. Conclusions

WediscussedtheIP*-irresolutemapsandContraIP*-irresolutemapsinthispaper.Weintend to conduct research in the future on IP*- Open maps, IP*- Closed maps, Super IP*- Open maps and so on.

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