Investigation of Markovian Process in Server Break-down

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Abstract

In day to day life goods produced at a number of places are consumed at different locations and then the problem faced by producer or dealer is to transfer goods from numerous places of origin to various places of consumption such that cost of transportation, time required, damages to the product etc. are optimum. This is called classical transportation problem. In this study we have studied transportation problem solve by Markov Decision Process. Introduction to the operations research is made along with over view of optimization techniques of single objective transportation problem. For literature review in the subject of multi objective fractional as well as linear multi objective transportation problems. In basic concepts of Markov Decision Process are given for further usage in the thesis work. We have stated algorithms for solving different types of multi objective programming problems. In entire work we have used basically linear membership functions, hyperbolic membership function and exponential membership function. These membership functions are Markov Decision Process. In case of linear membership function weighted arithmetic mean, weighted quadratic mean, geometric mean is used to find compromise solution. Almost everywhere we have found that the compromise solutions are fairly close to optimal solution when the problem is solved as single objective function.

We have solution procedure for multi objective linear transportation is given using Markov Decision Process. Introduction to fractional programming is made and solutions of multi objective linear fractional programming problems are discussed. We have solutions of multi objective linear fractional transportation problem. In this topic fractional objective functions are approximated as linear functions using Taylor's theorem and then applied method to solve the multi objective linear transportation problem discussed earlier. We focus on solution to fractional transportation problem with Markov Process coefficients. To multi objective fractional transportation problem with goal programming approach. We deal with general integer programming problem along with fractional transportation problems. The main result of this thesis is a characterization of increasing Marko processes satisfying certain conditions. At the end bibliography and research papers published by authors are given.

Keywords: Markov Process, Transportation, Queen analyses Server system.

I. Introduction and literature review

The bulk or multi-server service Markov's extensive range of applications—which include high performance serial buses, data transmission via satellites, ATM switching elements, and model digital communication systems like packet switches and multiplexers—makes it indispensable. A time division multiple access (TDMA) approach can be used to represent some telecommunication systems utilizing a batch arrival model with a finite capacity of the buffer size. Numerous writers have done in-depth research on bulk arrival, bulk service, or both because of its applications in numerous important domains. The transportation system can be viewed as a stochastic process in which decisions are made at various points in order to maximize efficiency or minimize costs. Arkov.V.'s work [1] presents the application of Markov modeling for autonomous control of complex dynamic systems. It likely discusses how control strategy decision-making can be aided by the use of Markov models to evaluate and predict the behavior of these kinds of systems. Arkov, Breikin, and Kulikov [2] reported a work on the development of fuzzy Markov simulation algorithms for product testing equipment. This research most likely looks into the integration of fuzzy logic with Markov modeling to account for uncertainty and imprecision in the simulation of product testing procedures. Markov chains is discussed by Arthem Sapozhirikov [3]. With an emphasis on f-ergodicity—a generalization of the idea of ergodicity—this work likely explores the convergence qualities of Markov chains. The Boole Centre for Research in Information's research probably advances the study of stochastic processes and probability theory. Aumann, R.J. and Pearles, M. [4] address a deviational problem in economics in 1965. Most frequently, a specific problem with equilibrium states or deviations from expected behavior in economic theory is the focus of this paper. Decision theory and mathematical economics are advanced in this study that was published in the Journal of Mathematics and Applications.

An extensive abstract on the characteristics and specificities of Markov chains is provided by Avranchenkov, K.E. and Sanchez, E. [5] for the IPMU 2000 conference. The main traits and behaviors of Markov chains are probably summed up in this abstract, which advances knowledge of stochastic processes and the range of uses for them in other domains. Markov chains are covered by Avranchenkov, K.E. [6] in a paper from INRIA Sophia Antipolis. With a possible focus on theoretical features, algorithmic considerations, or applications in computer science and allied fields, this study most likely offers an overview or analysis of Markov chains. A article on optimal inventory policy is presented by K. J. Arrow, T. Harris, and J. Marschak [7] in the journal Econometrica. This groundbreaking work advances the fields of operations research and supply chain management by probably introducing or analyzing mathematical models for identifying the best inventory management tactics. Written by K. J. Arrow, S. Karlin, and H. Scraf, [8] "Studies in Applied Probability and Management Science" was published by Stanford University Press in 1958. This most likely consists of several research papers or monographs covering various topics in applied probability and management science, including queueing theory, optimization, and decision analysis. In the Journal of Applied Probability, J. R. Artale jo and A. Gomez-Corral [9] address the steady-state solution of a single-server queue with linear repeated requests. The performance characteristics of queuing systems with particular arrival patterns are probably examined in this research, which advances knowledge of queuing theory. Artalejo, J. R., and A. In a book released by Springer, Gomez-Corral [10] presents a computational method to retrial queuing systems. This paper provides an extensive examination of retrial queuing systems, covering computer approaches, mathematical modeling, and analysis methodologies, adding to the theoretical and practical elements of queuing theory. The Annals of Operations Research publish an article by J. R. Artalejo, A. Krishnamoorthy, and M. J. Lopez-Herrero [11] on the numerical analysis of (s, S) inventory systems with repeated tries. It is likely that the computational techniques for inventory systems with stochastic demand and periodic replenishment rules are the main emphasis of this paper. In the Aligarh Journal of Statistics, Arshad, M., Khan, S. U., and Ahsan, M. J. [12] present a method for resolving concave quadratic programs. Most likely, the approach or algorithm presented in this study can effectively solve optimization problems with concave quadratic objectives that are subject to linear constraints. Anstreicher, K. M., Den Hertog, D., Roos, C., and Terlaky, T. [12] in Algorithmica. Most likely, the optimization approach presented in this study is designed specifically to solve convex quadratic programming problems quickly. The Naval Research Logistics Quarterly published an enhanced dual algorithm with constraint relaxation for all-integer programming developed by Austin, L. M. and Ghandforoush, P. [13]. This work probably presents an integer programming approach for addressing problems when the decision variables can only have integer values. A rural transit vehicle management system and condition predictor model are presented by Anderson, MD, and AB. Sandlin [14] in the Journal of Public Transportation. This study probably discusses a model for condition prediction and fleet management for rural transportation vehicles.

A rural transit vehicle management system and condition predictor model are presented by Anderson, MD, and AB. Sandlin [16] in the Journal of Public Transportation. The efficiency and dependability of rural transit services may be enhanced by the model described in this study, which is probably intended to manage fleets of rural transit vehicles and forecast their condition. The Naval Research Logistics Quarterly published a discussion of "Programming with Linear Fractional Functional" by Charnes and W.W. Cooper in [17]. This work adds to the field of mathematical programming and optimization by maybe introducing techniques for solving optimization issues using linear fractional functional. The article "Optimal estimation of executive compensation by linear programming" was published in the Journal of the Institute of Management Science by Charnes, W.W. Cooper, and R. Ferguson in [18]. This work likely adds to the fields of management science and human resources management by presenting a linear programming method for optimizing executive salary. "The Generalized Algorithm for Solving the Fractional Multi-Objective Transportation Problem" is introduced by Alexandra I. Tkacenko [19] in ROMAI J. In order to solve multi-objective transportation issues with fractional objectives, this study probably gives an algorithmic method. This helps with multi-objective decision-making and transportation optimization. "The Multi-objective Transportation Fractional Programming Model" is discussed by Alexandra I. Tkacenko [20] in the Computer Science Journal of Moldova. This work probably adds to optimization theory and transportation management by providing a mathematical model for resolving transportation problems with fractional objectives.

The Multiple Criteria Transportation Model is presented by Alexandra Tkacenko [21] in the publication Recent Advances in Applied Mathematics and Computational and Information Sciences. This work likely adds to transportation planning and management by introducing a transportation model that takes into account several factors for decision-making. "A Simple Method for Obtaining Weekly Efficient Points in Multiobjective Linear Fractional Programming Problems" is introduced by B. Metev and D. Gueorguieva [22] in the European Journal of Operational Research. This technique probably adds to optimization research by offering a simple way to find weekly efficient points in multiobjective linear fractional programming problems. "Fractional Goal Programming for Fuzzy Solid Transportation Problem with Interval Cost" is presented by B. Radhakrishnan and P. Anukokila [23] in Fuzzy Information and Engineering. This work probably contributes to optimization methods in fuzzy environments by employing fractional goal programming to handle transportation issues with fuzzy parameters and interval costs. In Reliability: Theory & Applications, Bansal [24] performs "Behavioral Analysis and Maintenance Decisions of Wood Industrial Subsystem Using Stochastic Petri Nets Simulation Modeling". To investigate behavioral features and maintenance decisions in wood industrial subsystems, this probably uses stochastic Petri nets simulation modeling. This helps with reliability engineering and maintenance management. Yadav et al. [25] probably adds to inventory optimization techniques by introducing an interval number strategy to maximize inventory management of degrading items with preservation technology investment. Yadav et al. [26] probably optimizes an inventory model that takes into account a number of variables, including selling price, demand that must be met quickly, carbon emissions, and investments in green technology, all of which support sustainable supply chain management. Typically, these studies entail maximizing the parameters of certain policies and utilizing renewal theory to derive analytical expressions for long-run costs per time unit. Chaudhary and Bansal [27] use the Gumbel-Hougaard family copula to investigate the dependability of a spirulina production unit.

Markov Decision Process

The Markov Decision Process (MDP) is a structured approach for managing complex planning challenges. It provides a mathematical framework for modeling decision-making in uncertain situations. MDP is a model used in task scheduling that reflects the dynamic and probabilistic nature of industrial environments. The core concept is that the system transitions between states based on probabilistic outcomes and the rewards associated with choosing a specific action.

Let A denote the set of possible actions, P the probability function of the transition, R the reward function, and X the state space. The probability of transitioning from the current state x ∈ X to state x′ ∈ X with action a ∈ A is represented by the transition probability P(x′|x, a). The reward function R(x, a,x′) shows how much the system gets in return for choosing to go from x to x′ with action an. M = (X, A, R, P) thus yields an MDP.

The ideal policy π* serves as the basis for MDP decision-making.

$$\pi^* (x) = \text{argmax } \sum_{x'} P(x'|x,a)[R(x,a,x') + \gamma V^* (x')], \quad (1)$$

that optimizes V(x), the predicted cumulative payoff.

$$V^*(x) = \max \sum_{x'} P(x'|x,a)[R(x,a,x') + \gamma V^* (x')], \quad (2)$$

where γ is the discount factor, which in some algorithms ensures convergence by weighing present rewards against future ones.

Model Description

There are two servers in the model under analysis, but only one of them can take a break at any one moment. There is always at least one server ready to go. The service time for every batch is independent of the batch size, which has an exponential distribution with parameter μ, and the arrival process is assumed to follow a Poisson distribution with parameter λ. The servers themselves can only take one break at a time and have an exponential distribution with parameter θ. The normal bulk service policy states that service doesn't start until at least "a" clients show up. The server will service every client in the queue if the queue length is "a" but not more than "b"; if there are more than "b" clients, the server will serve "b" clients first.

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The following presumptions are made in this model.

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i. After finishing service, if the server still has (a-1) clients in the system and the other server is in use, it stays in the system and waits for the queue size to reach 'a' for an arbitrary amount of time, known as the switchover period, which is exponentially distributed with parameter.

ii. If the server discovers that there are less than (a-1) clients in the system and all other servers are either occupied or unavailable, it will take a break with an exponentially distributed time, denoted by parameter theta. As a result, with this system, a server is never removed and is limited to one break per two consecutive service periods.

iii. The server will start serving immediately if a new arrival occurs during this switchover period. If no arrivals occur, the server will take a postponed break at the end of the switchover period.

iv. The inter-arrival time, service time, breaks, and switchover period's non-negative random variables have identical distributions and are independent of one another. The steady-state solutions and system parameters are found, as well as the average number of clients in the queue. Additionally, numerical examples showing the analytical conclusions for different values of the parameter are given.

Mathematical Formulation

The queueing system can also be expressed as a continuous time parameter Pjn (n > 0, j = 0,1,2,3) and Qjn ((0 ≤ n ≤ a-2), j = 1,2), where 'n' is the number of customers in the queue and 'j' is the status of the server, are examples of Markov chains with the following steady state probabilities.

P_{0n} is the chance that one server is unavailable while the other is on vacation.
P_{1n} is the likelihood that one server is occupied while the other is away on vacation.

P_{2n} is the likelihood that both servers are occupied,
P_{3a-1}: the likelihood that one server is in use while the other is in the switchover phase
Q_{1n} is the likelihood that one server is in use while the other is not,
Q_{2n} is the likelihood that one server is taking a vacation while the other is going through a switchover.

The limiting probabilities that follow are defined in relation to various states.

$P_{0n} = \lim_{n\to\infty} P_{0n}(t)$, $P_{1n}(t) = \lim_{n\to\infty} P_{1n}(t)$, $P_{2n}(t) = \lim_{n\to\infty} P_{2n}(t)$, $Q_{1n}(t) = \lim_{n\to\infty} Q_{1n}(t)$, $Q_{2n}(t) = lim_{n\to\infty} Q_{2n}(t)$ exists.

Equilibrium equations

The balance equations in the steady state satisfied by P_{jn} $(j=1,2,3)$and Q_{jn} $(j=1,2)$ for and $n \geq 0$ are given by

$$(\lambda + \mu)P_{00} = \mu P_{10} + \mu Q_{10} \tag{1}$$

$$(\lambda + \theta)P_{0n} = \lambda P_{0n-1} + \mu P_{1n} + \mu Q_{1n}(1 \leq n \leq a-2) \tag{2}$$

$$(\lambda + \theta)P_{0a-1} = \lambda P_{0a-2} + \mu P_{1a-1} + \mu P_{3a-1} + \mu Q_{1a-1} \tag{3}$$

$$(\lambda + \mu + \theta)P_{10} = \lambda P_{0a-1} + 2\mu P_{20} + \mu \sum_{n=a}^{b} P_{1n} \tag{4}$$

$$(\lambda + \mu + \theta)P_{1n} = \lambda P_{1n-1} + 2\mu P_{2n} + \mu P_{1n+b}(1 \leq n \leq a-2) \tag{5}$$

$$(\lambda + \mu + 2\theta)P_{1a-1} = \lambda P_{1a-2} + \alpha P_{3a-1} + \mu P_{1a-1+b} \tag{6}$$

$$(\lambda + \mu + \theta)P_{1n} = \lambda P_{1n-1} + \mu P_{1n+b}(n \geq a) \tag{7}$$

$$(\lambda + 2\mu)P_{2n} = \lambda P_{2n-1} + \theta P_{1n+b} + 2\mu P_{2n+b}(n \geq 1) \tag{8}$$

$$(\lambda + \mu + \alpha)P_{3a-1} = 2\mu P_{2a-1} + \theta P_{1a-1}(n = a - 1) \quad (9)$$

$$\lambda Q_{20} = \theta P_{00} \quad (11)$$

$$\lambda Q_{2n} = \theta P_{0n} + \lambda Q_{2n-1}(1 \leq n \leq a - 1) \quad (12)$$

$$(\lambda + \mu)Q_{1n} = \lambda Q_{1n-1} + \theta P_{1n}(1 \leq n \leq a - 1) \quad (13)$$

$$(\lambda + \mu)Q_{10} = \theta P_{10} + \lambda Q_{2a-1} \quad (14)$$

Calculation of Equilibrium Solutions

It is defined by the forward shifting operator $E(P_{1n}) = P_{1n+1}$. E will represent this operator. From equation (7), we get: (μ Eb+1 $-(\lambda + \mu + \theta)E + \lambda) = P1n = 0$ $(n \geq a)$ $P_{1n} = 0$ The characteristic equation of the previously described equation has a single real root within the circle |Z| = 1. Rouche's theorem can be used to determine if is smaller than 1 if $\rho = \frac{\lambda + \theta}{b\mu}$.

If r_0 is the root of the aforementioned characteristic equation and $|r_0| < 1$, then

$$P_{1n} = r^{n-a+1}P_{1a-1}(n \geq a) \quad (15)$$

Using equation (9), we get,

$$(2\mu\ E^{b+1} - (\lambda + 2\mu)E + \lambda)\ P_{2n} = -\ \theta P_{1n+b+1}$$

The characteristic equation of this equation, following simplification using equation (15), has just one real root, r_1, which is situated in the interval (0,1) when ρ = and, in accordance with Rouche's theorem.

$$P_{1n} = r_0^{n-a+1}P_{1n-1}(n \geq a) \quad (16)$$

Where A_1 is a constant and $k = \frac{-\theta r_0^{b-a+2}}{(\lambda+2\theta)r_0-\lambda}$

Substituting n = a-2, a-3, a-4,...1 in equation (5) and solving recursively with (15) and (16)

$$P_{1n} = (A_1 B_n(r_1) + k\, B_n(r_0) + r^{n-a})\, P_{1a-1}(1 \leq n \leq a-2)$$

(17)

Where $B_n\,(x) = \frac{2\mu R}{\lambda(x-R)}\,(x^n - \left(\frac{x}{R}\right)^{a-1} R^n)$, $R=\frac{\lambda}{\lambda+\mu+\theta}$ and $k=\frac{-\theta r_0^{b-a+2}}{(\lambda+2\theta)r_0-\lambda}$

Likewise, utilizing (17) to solve equation (13) recursively

$$Q_{1n} = (A_2 r_2^n + A_1 g_n(r_1) + k g_n(r_0) + k_0 r_2^{n-a}) P_{1a-1}, \quad (1 \leq n \leq a-1) \qquad (18)$$

Where A_2 is a Constant $r_2=\frac{\lambda}{\lambda+\mu}$

$g_n\,(x) = \frac{2\mu\theta R}{\lambda(x-R)}(\frac{x^{n+1}}{(\lambda+\mu)\mu-\lambda} + \left(\frac{x}{R}\right)^{a-1}\frac{R^n}{\theta})$ and $k_0=\frac{\theta r_0}{(\lambda+\mu)r_0-\lambda}$

By adding (2), (12) and using the equations (1), (11)

$$P_{0n} + Q_{2n} = {}^{\mu}\sum_{k=0}^{n}(P_{1n} + Q_{1n}),\ (0 \leq n \leq a-2)$$

Substituting the values of P_{1n} and Q_{1n} in equations (17) and (18)

$$P_{0n}+Q_{2n}= {}^{\mu}(A_2 r^n + A_1, B_n(r_1) + g_n(r_1)] + k, B_n(r_1) + g_n(r_1)] + (1+k_0) r^{n-a}) P_{1a-1}$$

Let $B_n(x) + g_n(x)\ C_n(x) = \frac{2\mu x^n}{(\lambda+\mu)x-\lambda}$

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$$P_{0n} + Q_{2n} = {}^{\mu}\sum_{k=0}^{n} (A_2 r^n + A_1 C_n(r_1) + k C_n(r_0) + (1+ k_0) r^{n-a}) P_{1a-1} \quad 2 \quad 0$$

Simplifying and expanding the aforementioned equation further yields

$$P_{0n} + Q_{2n} = \frac{\mu}{\lambda}\sum_{k=0}^{n}(A_2 r_2^n + A_1 C_n(r_1) + k C_n(r_0) + (1+k_0) r_a^{n-a})\, P_{1a-1}$$

(10) yields the probability for the switchover period.

$P_{3\,a-1} = [A_1T(r_1) + k\,A_1T(r_0) + R_1\,]\,P_{1a-1}$

(19)

Where $T(x)=\frac{2\mu x^{a-1}}{(\lambda+\mu+a)}$ *and* $R_1=\frac{\theta}{\lambda+\mu+a}$

Additionally, by summing (17) and (18), we get, utilizing the outcomes of equation (19),

$$P_{1n} + Q_{1n} = (A_2r_2^n + A_1D(r_1)r_1^n + \mathrm{F}(r_0)r_0^n))P_{1a-1}(0 \le n \le a - 1)$$

Utilizing the outcomes of P_{3a-1}, P_{2n}, and Q_{1n} in equation (8), the values of the constants were determined.

$$(\lambda + 2\mu)(A_1 + k \quad +\mathrm{k})= \lambda \quad [A_1T(r_1) + kT(r_0) + \quad R_1]+ \theta r_0^{-a+1}\left(\frac{r_0^a-r_0^{b+1}}{1-r_0}\right)+ \quad 2\mu[A_1\left(\frac{r_1^a-r_1^{b+1}}{1-r_1}\right)+ \quad \mathrm{k}\left(\frac{r_0^a-r_0^{b+1}}{1-r_0}\right)] + \lambda[(A_2r_2^n + A_1g_1(r_1) + kg_n(r_0) + k_0r_0^{n-1})]$$

Using $g_{a-1}(x)=\frac{2\mu x^{a-1}}{(\lambda+\mu)\mathrm{x}-\lambda)}$ and simplifying

$$A_2= \frac{1}{r_2^{a-1}}[A_1S(r_1) + kS(r_0) - \frac{\theta r_0}{\lambda}\left(\frac{1-r_0^{b-a+1}}{1-r_0}\right) - (R_1 + k_0r_0^{n-1})]$$

Chance that one server is occupied while the other is away on vacation (P_{1B})

Any overworked server who finds that there are less than 'a-1' customers in the system will be allowed to take one vacation. In the event that P_{1B} indicates the probability of one server being up while the other is away on vacation, then

$$P_{1B} = \left(A_2 + \frac{1-r_2^a}{1-r_2} + A_2D(r_1)\frac{1-r_1^a}{1-r_1} + F(r_0)\frac{1-r_0^a}{1-r_0} + \frac{r_0}{1-r_0}\right)P_{1\,a-1}$$

The likelihood that one server is unavailable while the other is on vacation (P_{0B}) Following the completion of the service, if the server finds that another server is unavailable and that the number of clients is less than "a-1," he will not be able to access the system again until the queue size reaches "a." All of the system's servers will always be operational. If the probability that one server is unavailable while the other is on vacation is represented by P0B, then

$$P_{0B} = \frac{\mu}{\lambda}\left(A_2\left\{\frac{\mathrm{a}}{1-\mathrm{r}_2} - \frac{(1-r_2^a)}{(1-r_2)^2}\right\} + A_1D(r_1\{\frac{\mathrm{a}}{1-\mathrm{r}_1} - \frac{r_1(1-r_1^a)}{(1-r_1)^2}\} + F(r_0)\{\frac{\mathrm{a}}{1-\mathrm{r}_0} - \frac{r_0(1-r_0^a)}{(1-r_0)^2}\}\right)P_{1\,a-1}$$

Possibility that one server is in use during the switchover period and the other is not (P_{3a-1})

If one of the servers is busy and another server finds "a-1" clients there, the server will remain in the system until the queue size changes to "a". The term "server switchover period" refers to this. In the event that P_{3a-1} indicates the probability of one server being operational during a switchover, then

$$P_{3a-1} = [A_1 T(r_1) + \text{k}\, A_1 T(r_0) + R_1]\, P_{1a-1}$$

Numerical Analysis

For some chosen values of the operational parameters of a, b, λ, μ, and θ, the system performance measures are calculated and the numerical results are given in tables. It has been observed that when the arrival rate, λ, rises, so does the wait length in the system. Furthermore, a variety of values of a, b, and λ are shown to satisfy the normalized requirement, which is the total probability $P_{0B}+P_{1B} + P_{2B} + P_{3a-1}\approx 1$. The comparisons between this model and previous models are shown in Table 1. This proposed model is seen to have fewer batches sitting in the queue than other existing models.

Table 1: The Performance measures for $\boldsymbol{\theta}$= 0.2 and $\boldsymbol{\mu}$= 1

λ		L_q	P_{0B}	P_{1B}	P_{2B}	P_{3a-1}
5	*a = 10* *b = 25*	5.16056	0.5117	0.3915	0.00966	0.000041
10		8.81293	0.3423	0.5743	0.0108	0.001700
15		12.1141	0.1748	0.6096	0.1165	0.012760
20		23.5442	0.0490	0.8205	0.1202	0.010300
6	*a = 20* *b = 30*	9.1025	0.7304	0.2301	0.0020	0.000080
12		11.2353	0.4347	0.5455	0.0117	0.003421
18		16.1302	0.2939	0.6405	0.0578	0.004500
24		26.5239	0.1488	0.7287	0.1224	0.098789
10	*a = 30* *b = 50*	14.4438	0.7024	0.3231	0.0002	0.000049
20		19.6429	0.5106	0.4056	0.0121	0.009878
30		28.6061	0.3238	0.6076	0.0224	0.023140
40		45.5809	0.1034	0.7227	0.1100	0.044531

Table 2 L_q **for various values of , *a* when *b* = *50*, θ= *0.5* and μ = *1***

λ	*a=10*	*a=20*	*a=30*	*a=40*	*a=45*
5	5.2399	9.0993	15.2845	19.2809	21.0005
10	7.5620	11.6587	15.6054	19.9154	22.0534
15	12.0918	12.7643	15.9769	20.1236	22.4333
20	16.4365	15.8790	17.9896	21.4732	23.2970
25	18.9994	19.4367	20.9076	22.8553	24.9982

Cost Model

This section looks at this model's costs while accounting for different server and client waiting time charges. Let's say that every server has a fixed price per hour.

C_0= a waiting cost per unit of service
W_0= and a cost per unit of service
C_1 =for each server).

Should M represent the anticipated overall expense per unit of time required to run the system, then $M = 2C_0 + W_0L_q + C_1\,\mu\,(2P_{2B} + P_{1B} + P_{3a-1})$. The duration of the line and the operating system's anticipated total cost per unit of time For different values, *M* is compared with both a single and repeated vacation of M/M(a,b)/(2,1).

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Visual Representation

The estimated number of customers in line (KYL) for the proposed model is plotted against that of existing vacation models that are currently in use. It is observed that the expected customer base for vacation spots is smaller than that of the existing Markov models. This illustrates how client wait times are shortened by the proposed model.

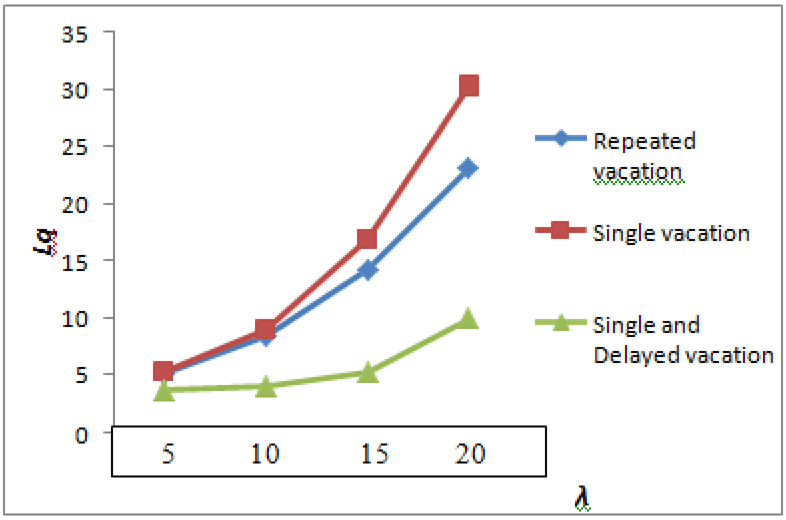

Figure Comparison of L_q for various values of a = 10, b = 25 when θ **= 0.1 and** μ **=1**

Conclusion

A M/M(a,b)/(2,1) Markov model with servers on vacation that is dependent on batch sizes and switchover period state is examined in this work. Figure shows that the length of the queue increases together with the arrival rate λ. Table demonstrates that the value of L_q for the suggested model is lower than that of the earlier vacation Markov models for the same set of values for λ, θ, a, and b. This demonstrates how the suggested Markov system shortens client wait times.

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