Investigation on Effect of Non-Linearity in Buckling and Post-Buckling of Fiber Laminated Shells

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**Abstract** - This research paper investigates the impact of non-linearity on the buckling and post-buckling behaviour of fibre-reinforced laminated shells, which are extensively utilized in aerospace, marine, and automotive applications due to their superior strength-to-weight ratio and tailored anisotropic properties. The study addresses the intricate effects of geometric non-linearities and material anisotropy, crucial factors influencing the stability and load-bearing capacity of these advanced composite structures. A comprehensive finite element analysis (FEA) framework is developed, incorporating both geometric and material non-linearities to accurately simulate the buckling and post-buckling response of laminated shells. Various parameters, including fibre orientation, stacking sequence, and boundary conditions, are systematically varied to assess their influence on the structural performance. The findings reveal that non-linearity significantly affects the buckling loads and post-buckling behaviour of laminated shells. Non-linear analyses predict lower buckling loads compared to linear models, underscoring the necessity of incorporating non-linear effects in design methodologies. Additionally, post-buckling analysis uncovers complex deformation mechanisms and stress redistribution phenomena, demonstrating the enhanced load-carrying capacity and resilience of the structures beyond initial buckling.

This investigation provides critical insights into the design and optimization of fibre-reinforced laminated shells, offering practical guidelines for improving their structural efficiency and reliability. The results contribute to the advancement of predictive modelling techniques and the development of high-performance composite materials, fostering innovation in engineering applications where lightweight and robust structures are paramount.

**Key Words:** Non-linearity, Buckling, Post-buckling, Geometric non-linearities, Material anisotropy, Finite element analysis (FEA), Structural stability, Composite materials, Aerospace structures, Marine structures, Automotive applications, Load-bearing capacity, Fiber orientation, Stacking sequence, Deformation mechanisms, Stress redistribution, Predictive modelling, Structural efficiency

1. **INTRODUCTION**

1.1. **GENERAL**

Thin-walled structure finds wide applications in many branches of engineering. Examples include aircraft, spacecraft, cooling towers, nuclear reactors, steel silos and tanks for bulk solid and liquid storage, pressure vessels, pipelines and offshore platforms. Because of the thinness of these structures, buckling is often the controlling failure mode. It is therefore essential that their buckling behavior be properly understood so that suitable design methods can be established. Buckling phenomena occurs when most of the strain energy which is stored as membrane energy can be converted to bending energy requiring large deformation resulting in catastrophic failure. Buckling analysis is a technique used to determine buckling loads, critical loads at which a structure becomes unstable and exhibits buckled mode shapes, buckling may be demonstrated by pressing the opposite edges of a flat sheet of cardboard towards one another. When a structure undergoes visibly large displacements transverse to the load then it is said to buckle. Buckling is a critical phenomenon in structural failure under compression load. The buckling strength of structures depends on many different factors such as initial imperfection, shell geometry, material plasticity, dynamics of loading, type of joint, boundary conditions at ends, large displacement and large strain. Local buckling of cylindrical shells is indicated by the growth of bulges, waves or ripples, and is commonly encountered in the component plates of thin structural members. If buckling deflections become too large then the structure fails, this is a geometric consideration, completely separated from any material strength consideration.

Buckling is a mathematical instability, leading to a failure mode. Theoretically, buckling is caused by a bifurcation in the solution to the equations of static equilibrium. At certain stage under an increasing load, further load is able to be sustained in one of two states of equilibrium, an un-deformed state or a laterally-deformed state. In practice, buckling is characterized by sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. Further load will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of the member's load-carrying capacity. If the deformations that follow buckling are not catastrophic the member will continue to carry the load that caused it to buckle. Figure 1. shows the buckle shape of pressure vessel.

This investigation is basically the study of buckling analysis of thin-walled cylinders. Thin-walled cylinders of
various constructions find wide uses as primary structural elements in simple and complex structural configurations. Using the membrane pre-buckling stress assumption, the effect of several boundary conditions on the buckling of thin-walled cylinders has been studied. The linear buckling assumes a membrane state of stress before buckling. It is well known that the linear buckling prediction of thin-walled cylinders is purely theoretical and it should be reduced in order to account for the influence of geometric imperfection and other inadequacy. As a consequence of discretization error, linear buckling analysis overestimates the buckling load and provides un-conservative results. Considering the combined effect of discretization error (a minor effect) and modeling error (a major effect), we should interpret the results of linear buckling analysis with caution. Figure 1.2 shows the behavior of linear buckling and nonlinear buckling. Nonlinear-buckling analysis requires that a load be applied gradually in multiple steps rather than in one step as in a linear analysis. Each load increment changes the structure shape and changes the structure stiffness. Although the load-control method is used in most types of nonlinear analyses, it would be difficult to implement in a buckling analysis. Limit point buckling is a flux in which the load-displacement curve reaches a ceiling and then exhibits negative stiffness and releases strain energy.

Figure 2: Example of Pressure Vessels Collapse

During limit point buckling there are no abrupt changes in the equilibrium path; however, if load is incessantly increased then the structure may jump or snap to another point on the load deflection curve. A limit point is characterized by the load-frequency curve passing through a frequency of zero with a zero slope. The load-deflection curve also has a zero slope at the point of maximum load.

1.2. NEED

power In many design projects, engineers must calculate the factor of safety (FOS) to ensure the design will withstand the expected loadings. Calculations require correctly recognizing the mechanisms of failure structure, due to the problem of buckling. Buckling is a dangerous mode of design failure. Buckling happens suddenly, without little if any prior warning, so there is almost no chance for corrective action. This buckling may be due to compressive loads created due to self-weight or outer members or internal pressure or may imperfections in the system. So proper check needs to be done for buckling of the components. If a component or part therefore is prone to buckling then its design must satisfy both strength and buckling safety constraints that is why buckling is important. In the present work Thin-walled cylinders geometries is study for buckling analysis. Analyzing all these conditions is a complicated task. The use of finite-elements analysis for investigation of buckling problem of thin-walled cylinders is becoming popular due to the improvement in computational hardware and materialization of highly specialized software. Depending on the degree of accuracy desired and limit of computational cost, two types of buckling analysis is being study that is linear buckling analysis and nonlinear buckling analysis. The need of the present work is to study the buckling phenomena of thin-walled cylinders using Shell43 & Shell181 elements from ANSYS software in plastic range.

1.3. OBJECTIVE

Taking into consideration the need of buckling analysis, the following objectives are carried out in the proposed work.

1. Study of the buckling phenomenon of thin-walled cylinders.
3. Study the ANSYS software based on finite element method in details.
4. Understanding and study the linear buckling and nonlinear buckling analysis in plastic range.
5. The main aim of this dissertation work is to study the buckling behavior of thin-walled cylinders along with parametric study of thin-walled cylinders in plastic range, considering the shell43 element and shell 181 elements.
1.4. THEME

Several of the civil engineering structures fail due to the problems of buckling. Buckling is a major problem in storage tanks, pressure vessels, pipeline, offshore structure, etc. The buckling strength of structures depends on many different factors such as initial imperfection, shell geometry, material plasticity, dynamics of loading, large displacement and large strain. Effect of these parameter on buckling behavior is large. But a few predominant factors affecting buckling need to be studies in details. The buckling in elastic range can be overcome easily but when buckling reaches plastic range, the failure becomes un-repairable. Thus, buckling analysis in plastic range is an important area to be research. In this work thin-walled cylinders are considered for buckling analysis in elastic and plastic range. And also study the effect of thickness on buckling strength, effect of non-linearity in the buckling strength of thin-walled cylinders.

2. LITERATURE SURVEY

2.1. INTRODUCTION

Buckling is an issue of concern for structural, mechanical, production, automobile, marine engineers. A vast range of research is available depending on the need of research. The literature concerned to the objective of present study is only being presented here. The literature review can be classified into three groups,

I. Types of Buckling of Structures
II. Types of Buckling Analysis
III. Work Related to Present Topic

2.2. LITERATURE REVIEW

1. Types of Buckling of Structures

A. Buckling of Thin-Walled Structures

A thin-walled structure has thicknessed much lesser than the other structural dimensions. Into this category fall plate assemblies, common hot- and cold- formed structural sections, tubes and cylinders, and many bridge and aero plane and structures. Thin-walled composite structure is widely used in civil, automotive and aerospace engineering applications due to its light weight with high stiffness and strength as well as property of corrosion resistance. Buckling of a plate occurs when the in-plane compressive load gets large enough to cause a sudden lateral deflection of the plate. Initially a plate under compressive load undergoes only in-plane deformations, but as this compressive load gets large, the plate reaches its critical buckling load, the load at which a sudden lateral deflection of the plate takes place.

B. Thin Shell Buckling

Thin-shell widely used in application of engineering and research such as aircraft, spacecraft, cooling tower, nuclear reactors, steel silos and tanks for bulk solid and liquid storage, pressure vessels, pipelines and offshore platforms. Local buckling of an edge-supported thin plate does not necessarily lead to total collapse as in the case of columns; generally, plates can withstand loads greater than critical. The p-q curve illustrates plates greatly reduced stiffness after buckling, so plates cannot be used in the post-buckling region unless the behavior in that region is known with confidence. It should be emphasized that the knee in the p-q curve is unrelated to any elastic- plastic yield transition; the systems being discussed are totally elastic. The knee is an effect of overall geometric rather than material instability.

C. Torsional Buckling

Torsional buckling of columns can arise when a section under compression is very weak in torsion, and leads to the column rotating about the force axis.

D. Flexural Torsional Buckling

More commonly, where the section does not possess two axes of symmetry as in the case of an angle section, this rotation is accompanied by bending and is known as flexural torsional buckling. Figure 4 shows the cross-sectional view of L-section before and after buckling.

E. Lateral Buckling

Lateral buckling of beams is possible when a beam is stiff in the bending plane but weak in the transverse plane and in torsion, as is the I-beam of the sketch. It often happens that a system is prone to buckling in various modes. These usually interact to reduce the load capacity of the system compared to that under the buckling modes individually. An example of mode interaction is the thin box section which develops local buckles at an early stage of loading, as shown greatly exaggerated here. The behavior of the column is influenced by these local buckles, and gross column buckle will occur at a load much less than the ideal Euler load. Figure 5 shows the lateral buckling of an I-section.

II. Types of Buckling Analysis

Two hypotheses regarding the type of material modeling are used: linear buckling analysis and nonlinear buckling analysis.
A. Linear Buckling Analysis

Linear buckling analysis depends on material linearity. Linear, or eigen value, buckling accounts for stress stiffness effects where compressive stresses tend to lessen a structure’s ability to resist lateral loads. As the compressive stresses increase the resistance to lateral forces decreases. At some load level, this negative stress stiffening overcomes the linear structural stiffness, causing the structure to buckle. This method corresponds to the textbook approach to elastic buckling analysis for instance; an eigen value buckling analysis of a column will match the classical Euler solution. However, imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength. Eigen value buckling analysis predicts the theoretical buckling strength and it often yields un-conservative results, and should generally not be used in actual day-to-day engineering analyses.

B. Non-Linear Buckling

Nonlinear buckling analysis is essentially an application of large deflection and plastic behavior. To determine buckling loads more accurately, nonlinear buckling analysis should be used. And therefore, it is recommended for design or evaluation of actual structures. This technique employs a nonlinear static analysis with gradually increasing loads to seek the load level at which the structure becomes unstable. Nonlinear buckling analysis is usually the more accurate approach and is recommended for design or evaluation of actual structures. Using the nonlinear technique, model can include features such as initial imperfections, plastic behavior, gaps, and large-deflection response. In addition, using deflection-controlled loading, we can even track the post-buckled performance of the structure (which can be useful in cases where the structure buckles into a stable configuration, such as "snap-through" buckling of a shallow dome). Nonlinear buckling analysis of a structure is an eigen value problem whose equation can be written as:

\[(K_n + \lambda \Delta K) \phi = [0]\]  \hspace{1cm} (2.1)

\(K_n = \) Structural stiffness matrix

\(\phi = \) Eigen Vector

\(\lambda = \) Eigen Value

\(\Delta K = \) Stress stiffness matrix

Consists of running a nonlinear, large deflection solution until the analysis stops converging, indicating an instability. Detailed review of the nonlinear behavior must be used to determine if analysis has reached a true structural instability. Factors such as the presence of imperfections, element formulation, step size, element mesh size, and nonlinear convergence settings will play a role in the prediction of the instability.

III. Work Related to Present Topic

C. Depaor & et al [1] have study presented on prediction of vacuum-induced buckling pressures of thin-walled cylinders. They investigate the effect of geometric imperfections on the buckling capacity of thin cylindrical shells subjected to uniform external pressure. A geometric survey was conducted on small- scale thin cylinders in order to measure geometric imperfections of the shell surface. The cylinders were tested to collapse in the laboratory and the results are compared to the results of the FE analysis. Both collapse pressure and post-buckling mode shape was accurately predicted by the FE analysis.

Elena-Felicia Beznia and I.Chirica [2] have study presented thin walled stiffened composite panels are among with the most utilized structural elements in ship structures. The composite layered panels with fibers are usually the most used in shipbuilding, aerospace industry and in engineering constructions as well. These structures possess the unfortunate property of being highly sensitive to geometrical and mechanical imperfections. According to these studies, it is possible to predict on how far into the post-buckling region it is possible to increase loading without losing structural safety. They studied the plates with delamination.

S.S. Wang & et al [3] have studied the effect of material nonlinearity on buckling and post buckling of fiber composite laminated plates and cylindrical shells and they have studied the methods of analysis developed in effective and efficient to address the coupled problem involving both composite material non-linearity and geometrical structural nonlinearity and their interaction. The material nonlinearity of the composite is modeled by power-law type, nonlinear shear constitutive equations for each lamina. The nonlinear effective composite equations in an incremental form are incorporated into a geometrically nonlinear analysis for studying buckling and post buckling deformation of composite laminate structures. A modified Riks solution scheme with an updated lagrangian formation was used to construct the equilibrium path during composite post buckling. Numerical example was given to illustrate the effect of material nonlinearity on buckling load, post buckling stiffness, and associated mode shape change of a composite structure under axial and pressure loading. Influence of laminated parameters, geometric imperfection, and loading mode on the post buckling equilibrium path and load-bearing strength of the composite structure with the nonlinear material properties were also studied.

Jin G. Teng [4] have studied the buckling analysis of thin shells in recent advances and trends. Thin shell structures find wide applications in many branches of engineering. Examples include aircraft, spacecraft, cooling tower, nuclear reactors, steel silos and tanks for bulk solid and liquid storage, pressure vessels, pipelines and offshore platforms. This investigation provides a review of recent research advances and trends in the area of thin shell buckling. In this investigation following topics are given emphasis, (a) imperfections in real structures and their influence (b) buckling of shells under local/non-uniform loads and localized compressive stresses (c) the use of computer buckling analysis in the stability design of complex thin shell structures.

Dinar Camotim and Cilmar Basaglia [5] have studied the buckling analysis of thin-walled steel structures using generalized beam theory (GBT) (state-of-the-art report). They investigate the generalized beam theory (GBT), originally intended for the analysis of prismatic thin-walled members, may be described as a beam theory that involves equilibrium
equations and boundary conditions expressed in terms of one-dimensional quantities, but, nevertheless, is able to incorporate folded-plate concepts as well, making it possible to account for local (wall bending) deformation and cross-section distortion. The main innovative feature of GBT is the fact that the member cross-section deformation is viewed as a linear combination of special shape functions, which are termed deformation modes and satisfy a number of orthogonality conditions. This GBT modal nature has significant advantages in terms of (i) the structural clarity of the solution obtained and (ii) its computational efficiency.

J. M. Sugirtha Singh and Dr. R. K. Thangaratnam [6] have studied presented buckling of functionally graded materials (FGM) as heterogeneous composite materials usually made from a mixture of metals and ceramics. This investigation presents the nonlinear formulation for functionally graded material (FGM) plates and shells using semi-loof shell element. Results for buckling and vibration analysis of functionally graded plates and shells were reported. Functionally graded materials have the advantage of their ability to withstand high temperature gradients unlike fiber matrix composites, which show mismatch of mechanical properties across an interface of two discrete materials bonded together and resulting in debonding at high temperatures in some cases. J. Woo, S. A. Meguid, L.S. Ong [6] have presented the analytical solution for the nonlinear free vibration behavior of plates made of functionally graded materials. The material properties of FGM are graded but continuous and are controlled by the variation of the volume fraction of the constituent materials. The effect of material properties, boundary conditions and thermal loading on the dynamic behavior of the plates was investigated. They studied the nonlinear analysis of FGM plates and shells using semi-loof shell element.

George J. Simitses [7] has presented the studied on buckling and post-buckling of imperfect cylindrical shells. G. J. Simitses study a state-of-the-art survey of the general area of buckling and post-buckling of thin-walled, geometrically imperfect, cylinders of various constructions, when subjected to destabilizing loads. The survey includes discussion of imperfection sensitivity and of the effect of various defects on the critical conditions.

Dr. Abdulkareem Al Humdany and Dr. Emad Q. Hussein [8] have presented buckling behavior of anti-symmetrically angle ply laminated composite plate under uniaxial compression using eigen value buckling analysis. They observe the buckling of a plate occurs when the in plane compressive load gets large enough to cause a sudden lateral deflection of the plate. Initially a plate under compressive load undergoes only in-plane deformations, but as this compressive load gets large, the plate reaches its critical buckling load, the load at which a sudden lateral deflection of the plate takes place. Analysis of plates buckling under in-plane loading involves solution of an eigen-value problem as opposed to the boundary value problem of equilibrium analysis. The effect of skew angle and compression loading on the buckling load and mode were also investigated. They also investigate the analytical determination of the critical buckling load of various types of plates. The buckling load will be determined for plates with various laminations (2,4,6,8 plies) with different ply directions. The results for the different lamination were compared to ascertain the influences of twist coupling and bending extension coupling.

S. J. Lee & et al [9] have studied the nonlinear finite-element analysis for evaluating the compressive-bending plastic buckling capacity of pipeline steel tube. The finite-element material model was used to simulate the behavior of the uniaxial tension test took from the surface of pipeline steel tube. They analyzed the compressive-bending buckling, by analytically and experimentally. They evaluate the methods were largely influenced by the material simulating techniques. The simulated material parameters being used to analyzed the compressive-bending experiments.

V. Piscopo [10] have studied the refined buckling analysis of rectangular plates under uniaxial and biaxial compression. And they compared relevant results with the classical ones and, form rectangular plates under uniaxial compression, a new direct expression, similar to the classical Bryan’s formula, was proposed for the Euler buckling stress.

Concluding Remark

The review of the literature shows considerable work on analysis of thin-walled cylinders in elastic range is available. On the other hand, very few works have been reported on nonlinear analysis of thin-walled cylinders. After searching 50+ research papers, the author could find only a single paper on plastic analysis. The aim of the present work is to study the nonlinear buckling analysis of thin-walled cylinders in plastic range. The shell43 element and shell181 element from ANSYS library is being proposed here for buckling analysis of thin-walled cylinder.

3. SYSTEM DEVELOPMENT

3.1. INTRODUCTION

In numerous studies have been made on buckling analysis of shell structure in elastic range. On the other hand, very few works have been reported on nonlinear buckling analysis of thin-walled cylinder under plasticity conditions. Most of the industrial structure fails due to the problems of buckling. Shell buckling is usually a major failure mode of thin-walled cylinders. Finite element method provided the solution for analyzing the problem of buckling. Finite element method is discretizing the structure into a number of elements which creates a finer mesh leading to greater accuracy. The material properties and geometry of a typical element are considered over these elements and expressed in terms of nodal displacements at nodes. In contemporary design practice, with the advent of large and fast software and advancement in numerical techniques, the problem of buckling can be solved. The aim of the present work is to study the linear and nonlinear buckling analyses of thin-walled cylinders using shell43 and shell181 elements.

3.2. ANALYTICAL MODEL

A thin-walled cylinder simply supported at the end is uniformly compressed in the axial direction as shown in Figure 6.
\[ \sigma = \frac{N_x}{t} = \frac{R E}{(1-\nu^2)} \]  

where

\[ R = (1 - \nu^2)\lambda^4 + a\left[(\pi^2 + \lambda^2)^2 - (2 + \nu)(3 - \nu)\pi^2\lambda^2\right] + \lambda^2n^2(3 + \nu) + n^2 - 2m^2 \]

\[ S = \lambda^2 \left[ \pi^2 (\pi^2 + \lambda^2)^2 + \frac{\lambda^2}{\pi^2}(\pi^2 + \lambda^2)^2 \right] + \frac{\pi^2}{\lambda^2} \left[ 1 + a(\pi^2 + \lambda^2)^2 \right] - \frac{\pi^2}{\lambda^2} \left[ \frac{\pi^2}{\lambda^2} - \frac{\lambda^2}{\pi^2}(\pi^2 + \lambda^2)^2 \right] \]

\[ \lambda = \frac{nE t}{R} \]  

\[ n = \frac{\pi^2}{\lambda^2} \]  

\[ N_x = \text{Axial Force} \]  

\[ E = \text{Young’s Modulus} \]  

\[ \nu = \text{Poisson’s Ratio} \]  

\[ t = \text{Shell thickness} \]  

\[ R = \text{Radius} \]

The simplified form is

\[ \sigma_v = \frac{N_x}{t} = 1 - \nu^2 \left[ \frac{(\pi^2 + \lambda^2)^2}{(\pi^2 + \lambda^2)^2} + (\pi^2 + \lambda^2)^2 \right] \]

When the value of \( n \) in eq. 3.9 is equal to zero, axisymmetric buckling occurs as

\[ \sigma_v = \frac{N_x}{t} = 1 - \nu^2 \left[ \frac{\pi^2}{\lambda^2} + \frac{\pi^2}{\lambda^2} \right] \]

\[ D = \frac{E t}{12(1-\nu^2)} \]  

Flexural Rigidity

Since \( \sigma_v \) is a continuous function of \( n \), the minimum value of eq. 3.10 can be written in the following form

\[ \sigma = \frac{E t}{k^2 \lambda (1-\nu^2)} \]

3.3. Computational Model- ANSYS Software

ANSYS is general-purpose finite element analysis (FEA) software package. Finite element analysis is a numerical method of deconstructing an intricate system into very small pieces called elements. The software implements equation that governed the behavior of these elements and solves them all creating a comprehensive explanation of how the system acts as a whole. These results then can be presented in tabulated or graphical forms. This type of analysis is typically used for the design and optimization of a system far too intricate to analysis by hand. Systems that may fit into this category are too intricate due to their geometry, scale, or governing equations. ANSYS provides a cost-effective way to explore the performance of products or processes in a virtual environment. The multifaceted nature of ANSYS provides a means to ensure that users are able to see the effect of a design on the whole behavior of the product, be it electromagnetic, thermal, mechanical etc. ANSYS software desires to define solution domain, the physical model, boundary conditions and the physical properties. Meshing divides the intricate model into small elements that become solvable in a too complex condition. Following are the general steps for solving any problem in ANSYS.

Building a Model

The ANSYS program has many finite-element analysis capabilities, ranging from a simple linear static analysis to a complex nonlinear transient dynamic analysis. Building a finite-element model requires more time than any other part of the analysis. First, job name and analysis titles have to be specified. Next, the preprocessor is used to define the element types, element real constants, material properties, and the model geometry. It is important to remember that ANSYS does not assume a system of units for intended analysis. Except in magnetic field analyses, any system of units can be used so long as it is ensured that units are consistent for all input data.

Element Types and Real Constants

The ANSYS element library contains more than 100 different element types. Each element type has a unique number and a prefix that identifies the element category. Select an appropriate element type for the analysis performed. Element real constants are properties that depend on the element type.

Material Properties

Material properties are required for most element types. Depending on the application, material properties may be linear or nonlinear, isotropic, orthotropic or anisotropic, constant temperature or temperature dependent. As with element types and real constants, each set of material properties has a material reference number. The table of material reference numbers versus material property sets is called the material table. In one analysis there may be multiple material property sets corresponding with multiple materials used in the model. Each set is identified with a unique reference number. Although material properties can be defined separately for each finite-element analysis, the ANSYS program enables storing a material property set in an archival material library file, then retrieving the set and reusing it in manifold analyses. Each material property set has its own library file.
Applying Loads

Loads can be applied using either preprocessor or the solution processor. Regardless of the chosen strategy, it is necessary to define the analysis type and analysis options, apply loads, specify load step options, and initiate the finite-element solution. The analysis type to be used is based on the loading conditions and the response which is wished to calculate. For example, if natural frequencies and mode shapes are to be calculated, then a modal analysis ought to be chosen. The ANSYS program offers the following analysis types static (or steady-state), transient, harmonic, modal, spectrum, buckling, and sub structuring. Not all analysis types are valid for all disciplines. Modal analysis, for instance, is not valid for thermal models. Analysis options allow for customization of analysis type. The word loads used here includes boundary conditions, i.e. constraints, supports, or boundary field specifications. It also includes other externally and internally applied loads.

Loads in the ANSYS program are divided into six categories DOF constraints, forces, surface loads, body loads, inertia loads, and coupled field loads. Most of these loads can be applied either on the solid model (key points, lines, and areas) or the finite-element model (nodes and elements). There are two important load-related terms. A load step is simply a configuration of loads for which the solution is obtained. In a structural analysis, for instance, wind loads may be applied in one load step and gravity in a second load step. Load steps are also useful in dividing a transient load history curve into several segments. Sub steps are incremental steps taken within a load step. They are mainly used for accuracy and convergence purposes in transient and nonlinear analyses. Sub steps are also known as time steps which are taken over a period of time. Load step options are alternatives that can be changed from load step to load step, such as number of sub steps, time at the end of a load step, and output controls.

Obtain Solution

ANSYS solve the problem and gives solution for the required parameters. Present the results after the solution has been obtained, ANSYS results can be presented with many ways by choosing from options such as tables, graphs and contour plots.

3.4. Buckling Analysis Procedure in ANSYS Software

The finite element analysis software needs to define solution domain, the physical model, boundary conditions and the physical properties. The meshing divides the complex model into small elements that become solvable in a too complex situation. Afterward the problem is solved and results are presented. Below describes the processes in terminology slightly more attune to the software,

A. Eigen-value Buckling Analysis

Preprocessor

Element type:

- Preprocessor - Element Type- Add/Edit/Delete.
- Click Add.
- Define Real Constants.

Material properties:

- Enter in values for Young’s modulus, Poisson’s ratio
- Modeling:
- Modeling – Create model- Lines/Coordinates/Key points/Area/Volume.

Meshing the model:

- On the Preprocessor menu, click Mesh Tool.
- Meshing - Size Controls – Manual Size- Lines - All Lines
- Select the size of meshing.

Assigning Loads and Solving:

- Define Analysis Type
- Solution - Analysis Type - New Analysis-Static
- Activate Prestress Effects

To perform an eigen value buckling analysis, prestress effects must be activated. We first ensure that we are looking at the unabridged solution menu so that we can select analysis options in the analysis type submenu. The last option in the solution menu will either be ‘unabridged menu’ (which means we are currently looking at the abridged version) or abridged menu (which means you are looking at the unabridged menu). If we are looking at the abridged menu, select the unabridged version.

- Select Solution - Analysis Type - Analysis Options

In the following window, change the [SSTIF][PSTRES] item to prestress ON which ensures the stress stiffness matrix is calculated. This is required in eigen value buckling analysis.

- Apply Constraints
- Solution - Define Loads – Apply – Structural – Displacement.

- Apply Loads
- Solution - Define Loads - Apply - Structural – Pressure/Force/Moment.

- Solve the System:
- Solution > Solve > Current LS

- Exit the Solution Processor:
- Close the solution menu and click FINISH at the bottom of the Main Menu.

- Define Analysis Type:
- Solution - Analysis Type - New Analysis - Eigen Buckling

- Specify Buckling Analysis Options:
- Select Solution - Analysis Type - Analysis Options - Block Lanczos.

Set the mode extraction method to subspace and enter the number of modes to extract (we are only interested in the first mode in which it buckles, but the absence of any constraints on rotation about the Z axis will introduce an extra buckling mode of simple rotation at a load of approximately0). Subspace eigen-value buckling box shown in Figure 7.
Figure 7: ANSYS Window for Subspace Eigen-value Buckling

Solve the System:
- Solution - Solve - Current LS
Exit the Solution Processor:
- Close the solution menu and click FINISH at the bottom of the main menu.
Expand the solution:
- Select Solution - Analysis Type - Expansion Pass and ensure that it is on.
Figure 8 shows the number of modes to be expand.

Figure 8: ANSYS Window for Number of Modes to Expand

Solve the System:
- Solution - Solve - Current LS
Post Processor:
- Results:
  - Buckling Load - List Results - Detailed Summary.
  - Mode Shape - Read Results - Last Set.
  - Plot Results – Shape/Nodal Solution/Element Solution.

3.5. Non-Linear Buckling Analysis
Preprocessor
Element type:
- Preprocessor - Element Type- Add/Edit/Delete.
- Click Add.
- Define Real Constants.
Material properties:
- Enter in values for Young’s modulus, Poisson’s ratio
Modeling:
- Modeling – Create model- Lines/Coordinates/Key points/Area/Volume.
Meshing the model:
- On the Preprocessor menu, click Mesh Tool.
- Meshing - Size Controls –Manual Size- Lines - All Lines
- Select the size of meshing.
Assigning Loads and Solving:
- Define Analysis Type Solution - New Analysis – Static
- Set Solution Controls Solution - Analysis Type - Solution Control

Figure 9: ANSYS Window for Solution Controls

Apply Constraints:
- Solution - Define Loads - Apply - Structural – Displacement.
Apply Loads:
- Solution - Define Loads - Apply - Structural – Pressure/ Force/Moment
Solve the System:
- Solution - Solve - Current LS
Post Processor:
Results:
- View the Deformed Shape Plot Results - Deformed Shape - Def + un-deformed.
- Contour Plot
Plot Results - Contour Plot - Nodal Solution - DOF solution.

4. PERFORMANCE ANALYSIS

4.1. Introduction
Buckling is a mathematical flux, leading to a failure mode. Buckling analysis of thin-walled cylinder under the action of load is being carried out. In this chapter, the modeling and analysis of different cases is carried out by using ANSYS 11 software. ANSYS 11 is general purpose software for analysis of structure. In this analysis shell 63 element in elastic range is considered for validation of procedure. Subsequently shell43 element and shell181 element in plastic are used to find the buckling analysis of thin-walled cylinder.
4.2. Boundary Conditions
In this analysis, shell43 element and shell181 elements in plastic range are used to find the buckling load of the structure. For each of the two ends, two different types of boundary conditions are used. At the fixed end, displacement degrees of freedom in 1, 2, 3 directions as well as rotational degrees of freedom in 1, 2, 3 directions were restrained to be zero. At the movable end, load was exerted with an even stress distribution in the longitudinal direction U3.

4.3. Linear Buckling Analysis
Thin-walled cylinder subjected to axially compressive load is model using ANSYS 11 software as shown in Figure 4.3.1. The geometric and material properties of thin-walled cylinders used in the analyses are tabulated in Table 4.3.1. The thin-walled cylinder is discretization in 32 elements in axial direction and 70 elements circumferential direction resulting in element size 7.1875 mmх7.912 mm.

![Figure 10: Modeling of Cylinder](image)

![Figure 11: Maximum Deformation for Linear Buckling](image)

![Figure 12: Maximum Buckling Stress for Linear Buckling](image)

Table 4.3.1.: Geometric and Material Properties

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Components</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Radius</td>
<td>88.2mm</td>
</tr>
<tr>
<td>2</td>
<td>Thickness</td>
<td>0.22mm</td>
</tr>
<tr>
<td>3</td>
<td>Height</td>
<td>230mm</td>
</tr>
<tr>
<td>4</td>
<td>Young’s Modulus</td>
<td>205GPa</td>
</tr>
<tr>
<td>5</td>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>Density</td>
<td>78000N/m³</td>
</tr>
<tr>
<td>7</td>
<td>Yield strength</td>
<td>240 MPa</td>
</tr>
</tbody>
</table>

4.4. Validation
Figure 4.3.1. shows the deformation plot of linear buckling analysis. The Figure 11 shows maximum deformation of 0.00511mm for the given buckling load. Ceiling displacement can be observed at the free end and least displacement can be observed at the constraint end. The Figure 12 shows a buckling stress of around 19.87MPa due to eigen-Value buckling analysis. The result presented in Table 4.3.1.1 shows the convergences study and the results of meshing size 32x70, validated with Ref [1], and gave a superior concord. Thus it can be concluded that shell63 element predicts the buckling phenomenon within engineering accuracy.

Table 4.3.2.: Convergences Study and Validation

<table>
<thead>
<tr>
<th>No. of elements</th>
<th>Linear buckling analysis</th>
<th>Non linear buckling analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement (mm)</td>
<td>Stress (MPa)</td>
</tr>
<tr>
<td>8x18</td>
<td>0.002124</td>
<td>5.072</td>
</tr>
<tr>
<td>16x35</td>
<td>0.004809</td>
<td>13.457</td>
</tr>
<tr>
<td>32x70</td>
<td>0.005111</td>
<td>19.87</td>
</tr>
<tr>
<td>32x70 Ref[1]</td>
<td>--</td>
<td>23.17</td>
</tr>
</tbody>
</table>

5. EXPERIMENTAL ANALYSIS
To validate the buckling load, thin-walled cylinders were tested in the laboratory. The cylinders to be tested were chosen from the available sizes. And the dimensions are given in the Table 5.1.1. The experimental set-up and pressure gauge is as shown in the Figure 13. Figure 14. The cylinder is fixed at one end and the other end; load was applied to test the buckling load.
The cylinders to be tested were chosen at different thickness. Buckling load is recorded throughout the experiment using a pressure gauge. The experimental buckling load given in Table 4.8.2 is recorded at different thickness. Table 4.8.2 shows the experimental and analytical results from the von mises formula for the buckling load of thin-walled cylinders. On comparing the buckling load attained by experiment with those predicted by the analytical, admirable correlation is shown. For thickness 1mm, it is clearly seen that there is not much difference the results of experimental and analytical and there is very little difference at thickness 1.5mm. The study shows the good agreement between experimental and the analytical results. Figure 4.8.3 shows the deform shape of the cylinder.

Table 5.1.1.: Geometric and Material Properties

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Components</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Radius</td>
<td>43mm</td>
</tr>
<tr>
<td>2</td>
<td>Thickness</td>
<td>1mm</td>
</tr>
<tr>
<td>3</td>
<td>Height</td>
<td>204mm</td>
</tr>
<tr>
<td>4</td>
<td>Young’s Modulus</td>
<td>200GPa</td>
</tr>
<tr>
<td>5</td>
<td>Poisson’s ratio</td>
<td>0.27</td>
</tr>
<tr>
<td>6</td>
<td>Density</td>
<td>78500N/m²</td>
</tr>
<tr>
<td>7</td>
<td>Yield strength</td>
<td>200MPa</td>
</tr>
</tbody>
</table>

The cylinders to be tested were chosen at different thickness. Buckling load is recorded throughout the experiment using a pressure gauge. The experimental buckling load given in Table 4.8.2 is recorded at different thickness. Table 4.8.2 shows the experimental and analytical results from the von mises formula for the buckling load of thin-walled cylinders. On comparing the buckling load attained by experiment with those predicted by the analytical, admirable correlation is shown. For thickness 1mm, it is clearly seen that there is not much difference the results of experimental and analytical and there is very little difference at thickness 1.5mm. The study shows the good agreement between experimental and the analytical results. Figure 4.8.3 shows the deform shape of the cylinder.

6. CONCLUSION

6.1. Introduction
A finite element analysis is carried out to find buckling strength of thin-walled cylinders. The buckling may be due to compressive loads created due to self-weight or internal pressure or external forces on imperfections in the structure. Non-linear buckling analysis carried out in order to investigate the buckling behavior of thin-walled cylinder considering shell43 & shell 181 element. The elastic-plastic material model based on the classic flow theory employs the von mises yield criterion and isotropic hardening.

6.2. Conclusions
The buckling analysis of thin-walled cylinder has been carried out. For each of the two ends, two different types of boundary conditions are used. Also, another cylinder under internal pressure is considered and analyzed. The analysis has been carried out using ANSYS 11. The conclusions drawn from the analysis with buckling analysis indicates that;
- To visualize the actual buckling behavior of thin-walled cylinders ANSYS software is the best one as other software.
- Buckling is caused by a bifurcation in the solution to the equations of static equilibrium. At certain stage under an increasing load, further load is able to be sustained in one of two states of equilibrium, an un-warped state or a laterally-warped state.
- Convergences study shows the good consensus results of mesh size 32x70.
- The stresses are very high in eigen-value buckling analysis and for non-linear analysis the stresses are within the convenient working conditions.
- The linear buckling analysis assumes a membrane state of stress before buckling. It is well known that the linear buckling prediction of thin-walled cylinders is purely theoretical and it should be reduced in order to account

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>Experimental buckling load (kN)</th>
<th>Analytical bucking load (kN)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shell43 element</td>
<td>Shell181 element</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>54</td>
<td>53.943</td>
<td>0.10</td>
</tr>
<tr>
<td>1.5</td>
<td>75</td>
<td>80.966</td>
<td>7.46</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>93.943</td>
<td>0.20</td>
</tr>
</tbody>
</table>
for the influence of geometric imperfection and other inadequacy.

- Shell 63 element in the linear analysis and shell43, shell 181 are considered for plastic conditions. Shell 63 is a linear element and does not support non-linearity in the problem. And shell43 element is a nonlinear element supports the plasticity in the analysis.
- Non-linear buckling analysis is an iterative solution and failure loads can be predicted by running the problem with more steps. From parametric study, it is observed that the non-linear buckling analysis is important to find the actual buckling loads of thin-walled cylinders.
- Shell181 element is suitable for analyzing thin to moderately-thick shell structures. Shell181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. For initial lower pressure the behavior under linear and nonlinear analyses is nearly same, the nonlinearity appears when pressure approaches to yield stress. There is a linear relation between cylinder thickness and load carrying capacity.
- The rate independent plasticity algorithm incorporates the von mises criterion which defines the yielding of material.
- Variation of tangent modulus from 0%È to 80%È has given a significant effect. Increases the value of tangent modulus exerts a more pronounced effect, resulting in a significantly increases the stresses, relative pressure and load carrying capacity of thin-walled cylinder. It is useful observation that an increase the value of tangent modulus in the plastic region produces an increase in pressure carrying capacity of the thin-walled cylinder.
- The pattern of load-displacement and load-stress is similar which shows that material nonlinearity is contributing more in the buckling behavior and thus the result of shell181 element which cater large strain situation, predicts the correct buckling behavior. The study enhanced the impetus in the field of buckling of cylindrical shell.
- The plastic material model based on the classic flow theory employs the von mises yield criterion and isotropic strain hardening.
- The finite element formulation provides a solution to nonlinear problems with combined geometric and material nonlinearities, was derived by the standard discretization procedure.
- The difference in displacement increases with increasing internal pressure.
- From eigen-value buckling and non-linear buckling analyses, it is concluded that the spreading of plastic regions through the tangent modulus has a significantly greater effect on stresses and displacement.
- An experimental and analytical analysis of the buckling of cylinder is presented, this study shows the analytical analysis of buckling closely follows the experimental behavior.

7. FUTURE SCOPE

The work presented in this dissertation may be used as basis for future works as suggested below:
1. Different analyses considering different types of shell element and also different types of structure.
2. Time history analysis can be done to verify the buckling behavior of cylinders.
3. Buckling analysis of hemisphere in contact with a rigid flat can be done.

8. APPLICATION

Thin-walled cylinders are mainly used in storage tanks, pressure vessels, pipelines, offshore platforms, liquid storage tanks, and other industrial applications. Shell buckling is usually a major failure mode of thin-walled cylinders due to compressive loads or internal pressure. Buckling of thin-walled cylinders is highly dependent to imperfections, large strain, large displacement, dynamics of loading, shell geometry, material plasticity.

REFERENCES
11. W. Rust and K. Schweizerhof “Finite Element Limit Load Analysis of Thin-walled Structures by ANSYS (implicit), LS-DYNA (explicit) and in Combination”, Thin-Walled Structures, 2023, pp. 227–244.
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