

Mathematical Modelling and Optimization of Dynamic Transportation Networks Using Time-Dependent Graphs

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Abstract - Transportation networks are dynamic systems in which travel time, traffic demand, congestion, road capacity, operating conditions, and incident effects vary over time. A route that is attractive under free-flow conditions may therefore become inefficient when demand increases or an incident reduces capacity. This paper develops an expanded mathematical framework for representing and optimizing such networks using time-dependent weighted directed graphs. Each transportation link is associated with a time-dependent traversal function and capacity, while route selection is formulated as a multi-objective optimization problem involving travel time, operating cost, and congestion exposure. Flow conservation, capacity, temporal propagation, and FIFO-compatible conditions are incorporated to maintain feasible route evaluation. A structured architecture is proposed in which network data, demand information, temporal conditions, a travel-time estimation layer, a route-generation engine, and a multi-objective decision layer operate as an integrated pipeline. A corresponding data-flow model describes the movement of static network data and dynamic observations through the system. A synthetic example illustrates how departure time changes can modify route evaluation. The numerical values are illustrative and are not presented as empirical transportation observations. The expanded framework provides a foundation for theoretical analysis, real-world validation, uncertainty modelling, resilience analysis, real-time re-optimization, and large-scale computational experimentation.

Key Words: dynamic transportation network; time-dependent graph; network optimization; shortest path; congestion; mathematical modelling; multi-objective optimization; real-time routing.

1.INTRODUCTION

Transportation systems are characterized by temporal variation. Traffic volumes change throughout the day, incidents may temporarily reduce link capacity, weather can influence travel speed, and recurrent congestion can create substantial differences between peak and off-peak travel conditions. Consequently, transportation planning and route guidance cannot always be represented adequately by a static network in which every edge has one fixed cost. In a dynamic setting, the effective cost of traversing an edge depends on when the vehicle enters that edge and, in many models, on the prevailing flow.

Graph theory provides a natural abstraction for transportation infrastructure. Intersections, terminals, stations, or decision points are represented as vertices, while directed road or transit connections are represented as edges. Let $G(t)$ denote the network state at time t . The time-dependent representation extends the conventional graph by allowing edge weights to evolve with temporal conditions. This makes it possible to evaluate routes according to the actual departure time rather than relying only on nominal free-flow values.

An additional challenge is that transportation decisions are rarely governed by travel time alone. Operators may consider fuel or operating cost, while travelers and network managers may also be concerned with congestion exposure and reliability. This motivates a multi-objective formulation. The present work therefore combines time-dependent travel time with operating cost and congestion exposure within a transparent scalarized framework.

The contribution of this manuscript is methodological. It organizes graph representation, dynamic travel-time modelling, route feasibility, temporal propagation, multi-objective evaluation, and repeated re-optimization into a single framework. The architecture and data-flow representation included in this expanded version also

clarify how the mathematical model can be connected to a future real-time transportation information system.

1.1 Research Objectives

- To represent a transportation network as a time-dependent directed weighted graph.
- To model link travel time as a function of entry time and congestion level.
- To formulate route selection as a multi-objective optimization problem involving travel time, operating cost, and congestion exposure.
- To incorporate flow conservation, link capacity, and temporal propagation constraints.
- To develop a structured procedure for repeated time-dependent route evaluation.
- To define an architecture that can integrate network, demand, and real-time traffic information.
- To provide a basis for future empirical validation, uncertainty analysis, resilience modelling, and large-scale experimentation.

2. Literature Background and Research Gap

Time-dependent routing has been studied through shortest-path algorithms, vehicle-routing problems, dynamic network-flow formulations, and mathematical programming. A central feature of these models is that traversal time is a function of departure time. The FIFO property is particularly important in road-network models because it prevents an earlier departure on the same link from arriving later solely as a consequence of the departure time.

Research in this area has increasingly considered travel-time prediction, real-time re-optimization, dynamic discretization, and data-driven optimization. These developments create opportunities to combine mathematical network models with continuously updated traffic observations. Nevertheless, a transparent formulation that simultaneously exposes temporal edge weights, congestion, operating cost, and interpretable route selection remains useful as a modelling foundation.

The research gap addressed here is primarily methodological rather than empirical. The framework does not claim that the illustrative network or parameter values represent a particular city. Instead, it provides a reusable structure that can subsequently be calibrated with validated transportation data.

2.1 Selected Literature

Reference	Relevance to the present framework
Gendreau, Ghiani & Guerriero (2015)	Review of time-dependent routing problems and their algorithmic characteristics.
Adamo et al. (2024)	Review of recent advances in time-dependent vehicle routing and dynamic decision-making.
Wu et al. (2019)	Definitions, applications, and algorithms for time-dependent graphs.
Bookbinder (1980)	Survey of dynamic transportation problems and network-flow perspectives.
Rakhmangulov, Osintsev & Mishkurov (2025)	Discussion of spatio-temporal transportation graphs, optimization challenges, and prospects.

3. Proposed System Architecture

The proposed architecture separates the dynamic transportation problem into logically connected layers. The first layer receives the road-network structure and operational parameters. The second layer receives dynamic observations such as traffic flow and time-period conditions. The travel-time estimation layer converts these inputs into effective time-dependent edge weights. The routing engine generates feasible origin-to-destination paths. The optimization layer evaluates the alternatives against the selected objectives. Finally, the decision and monitoring layer returns the selected route and can trigger re-optimization when network conditions change.

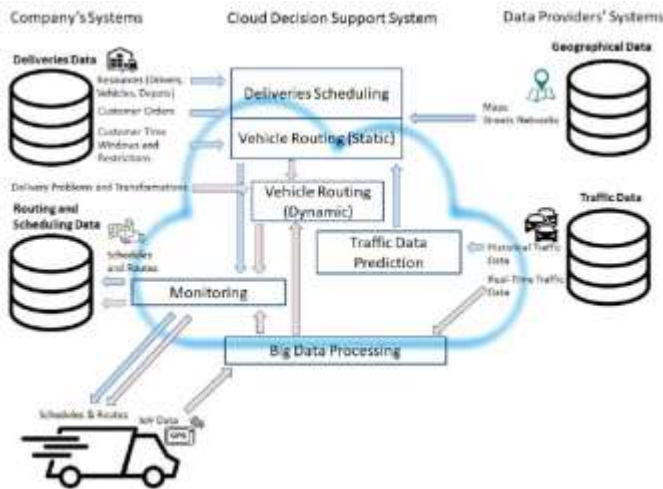


Figure 1. High-level architecture of the proposed dynamic transportation network optimization framework.

3.1. Data Flow Diagram

The data-flow model describes how information moves through the proposed system. Static network information enters the model through the network-data store, while dynamic traffic observations and travel demand enter through the dynamic-data interface. These inputs are merged to compute time-dependent edge weights. The route engine then produces feasible candidate paths. Candidate routes are evaluated by the objective module, after which the decision module produces the selected route and associated performance measures. The resulting state can be stored for analysis and used as input to the next optimization cycle.

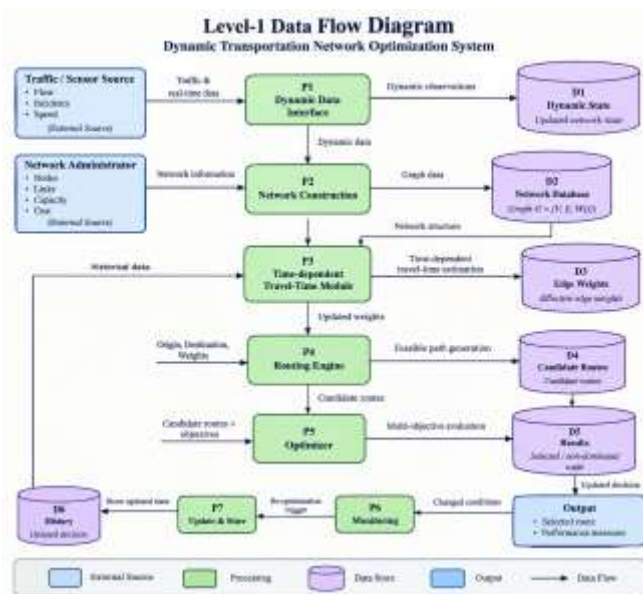


Figure 2. Level-1 data-flow representation of the proposed system.

3.2. Mathematical Representation of the Dynamic Network

Let the transportation network at time t be represented by the directed graph $G(t) = (V, E, W(t))$, where V is the set of nodes, E is the set of directed links, and $W(t)$ is the set of time-dependent link weights. For each directed edge $(i,j) \in E$, let $\tau_{ij}(t, q_{ij})$ denote the traversal time when a vehicle enters the link at time t under flow q_{ij} .

$$G(t) = (V, E, W(t))$$

$$\tau_{ij}(t, q_{ij}) = \tau_{ij}^0 [1 + \alpha_{ij} (q_{ij}(t)/c_{ij})^{\beta_{ij}}]$$

Here, τ_{ij}^0 is the free-flow travel time, $q_{ij}(t)$ is the flow on link (i,j) , c_{ij} is the link capacity, and α_{ij} and β_{ij} are calibration parameters. The expression is a modelling template; its parameters must be estimated or justified for a real-world case study.

3.3. Multi-Objective Optimization Model

Consider an origin node o and destination node d . Let x_{ij} be a binary decision variable that takes value 1 when edge (i,j) is selected and 0 otherwise. Three principal objectives are defined.

$$T(x) = \sum_{(i,j) \in E} \tau_{ij}(t, q_{ij}) x_{ij}$$

$$C(x) = \sum_{(i,j) \in E} c_{ij}^{op} x_{ij}$$

$$D(x) = \sum_{(i,j) \in E} d_{ij} (q_{ij}/c_{ij}) x_{ij}$$

$$\text{Minimize } Z(x) = \lambda_1 T(x) + \lambda_2 C(x) + \lambda_3 D(x)$$

$$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \text{ and } \lambda_1 + \lambda_2 + \lambda_3 = 1$$

The coefficients λ_1 , λ_2 , and λ_3 express the relative importance assigned to travel time, operating cost, and congestion exposure. Since these objectives may have different units and magnitudes, a practical implementation should consider normalization before applying a weighted-sum formulation. Repeated scalarization can be used to explore different trade-offs, while Pareto-dominance analysis can identify solutions for which no objective can be improved without worsening at least one other objective.

3.4. Network Constraints and Temporal Propagation

The selected route must satisfy flow conservation. Define $b_k = 1$ at the origin, $b_k = -1$ at the destination, and $b_k = 0$ at all other nodes.

$$\sum_i x_{ik} - \sum_j x_{kj} = b_k, \text{ for every node } k$$

$$0 \leq q_{ij}(t) \leq c_{ij}, \text{ for every } (i,j) \in E$$

$$t_j = t_i + \tau_{ij}(t_i, q_{ij})$$

The final relation propagates time through the route. A FIFO-compatible travel-time function should preserve the appropriate monotonicity relationship between departure and arrival times. This is essential because a route evaluation algorithm that ignores temporal propagation can assign incorrect weights to downstream links.

4. Proposed Algorithmic Procedure

1. Initialize the directed transportation graph $G=(V,E)$.
2. Load free-flow travel times, capacities, operating costs, and calibration parameters.
3. Load or estimate the current dynamic state, including flow and other available traffic information.
4. Select the origin, destination, and departure time.
5. Compute effective time-dependent edge weights.
6. Generate feasible origin-to-destination paths using a time-dependent shortest-path or k-shortest-path procedure.
7. Propagate arrival time along every candidate route.
8. Calculate total travel time, operating cost, and congestion exposure for each route.
9. Normalize objective values where necessary and apply the weighted objective or Pareto-dominance criterion.
10. Return the selected route or the non-dominated solution set.
11. Store the decision and performance measures.

12. Monitor for network-state changes and repeat the optimization cycle when re-optimization is required.

4.1. Pseudocode of the Proposed Method

INPUT: $G=(V,E)$, origin o , destination d , departure time t_o
 INPUT: capacities, free-flow times, operating costs, congestion parameters
 INPUT: dynamic flow state $q(t)$
 1. Update edge travel-time functions using current $q(t)$
 2. Generate feasible candidate paths $P(o,d,t_o)$
 3. FOR each path $p \in P$:
 set current time = t_o
 FOR each edge e in p :
 evaluate $\tau_e(\text{current time}, q_e)$
 current time = current time + τ_e
 END FOR
 compute $T(p)$, $C(p)$, $D(p)$
 4. Normalize objective measures if required
 5. Evaluate weighted objective or Pareto dominance
 6. Return selected/non-dominated path(s)
 7. IF network state changes materially: update $q(t)$ and repeat
 OUTPUT: route decision and performance measures

4.2. Illustrative Numerical Example

A synthetic network consisting of origin O , destination D , and intermediate nodes A , B , and C is used to illustrate the framework. The values below are illustrative only and are not empirical observations.

Edge	Free-flow time	Capacity	Operating cost	Peak-time travel time
O-A	5	100	4	8
O-B	7	120	3	9
A-C	6	80	3	10
B-C	4	100	2	5
C-D	5	90	3	7
B-D	10	70	5	14

Under free-flow conditions, route $O-B-C-D$ has a nominal travel time of $7 + 4 + 5 = 16$ -time units, while

O-A-C-D also has $5 + 6 + 5 = 16$ -time units. Under the illustrative peak-time values, O-B-C-D has $9 + 5 + 7 = 21$ time units, whereas O-A-C-D has $8 + 10 + 7 = 25$ time units. Thus, the relative evaluation of routes changes when the temporal edge weights change. The example demonstrates the limitation of relying exclusively on static free-flow weights.

4.3. Computational Experiment Design

For empirical evaluation, a real or benchmark transportation network should be divided into representative time intervals such as off-peak, morning peak, afternoon peak, and evening peak. For each interval, link travel times and flows should be estimated from an appropriate validated data source. The proposed method should be compared with at least one conventional static shortest-path baseline and, where appropriate, another time-dependent optimization method.

4.3.1. Experimental Variables

- Departure time and time interval.
- Traffic flow and demand intensity.
- Link capacity and capacity reduction scenarios.
- Congestion parameters α and β .
- Objective weights λ_1 , λ_2 , and λ_3 .
- Incident or disruption scenarios.
- Network size and number of candidate paths.

4.3.2. Performance Measures

- Total travel time.
- Operating cost.
- Congestion exposure.
- Computational time.
- Route stability across consecutive time intervals.
- Objective-function value.
- Number of re-optimization cycles.
- Solution robustness under demand and capacity changes.

4.4 Sensitivity and Scenario Analysis

Sensitivity analysis can be used to determine how route decisions respond to changes in model parameters. For example, increasing demand may increase q_{ij}/c_{ij} and therefore increase congestion-related travel time. Reducing link capacity can produce a similar

effect. Changes in the objective weights can reveal the trade-off between fast routes and routes with lower operating cost or congestion exposure.

A useful experimental design is to construct low-, medium-, and high-demand scenarios and combine these with off-peak and peak temporal profiles. Incident scenarios can then be introduced by reducing the capacity of selected links. The resulting route choices and objective values can be compared across scenarios. Such experiments would provide evidence for the sensitivity of the model while preserving the distinction between illustrative simulation and real-world observation.

4.4.1. Limitations

The current work is a mathematical framework and expanded journal-style manuscript rather than a completed empirical validation study. The numerical example uses synthetic values and therefore cannot establish real-world performance. The travel-time function is presented as a modelling template, and its parameters require calibration in an actual case study. The weighted-sum formulation also requires normalization and defensible selection of objective weights.

Before submission to a peer-reviewed journal, the manuscript should be strengthened with an explicit original research contribution, rigorous derivations or proofs where theoretical claims are made, reproducible code, validated transportation data, systematic baseline comparisons, sensitivity experiments, and appropriate statistical analysis where applicable.

5. Future Research Directions

- Derive sufficient conditions for FIFO-compatible time-dependent travel-time functions.
- Investigate the existence and feasibility of time-dependent paths under dynamic capacity conditions.
- Develop robust and stochastic formulations for uncertain travel times and demand.
- Integrate real-time traffic data and predictive machine-learning models.
- Study resilience under incidents, road closures, and capacity reductions.
- Develop scalable algorithms for large urban networks.
- Compare scalarized solutions with Pareto-optimal solution sets.

- Establish approximation bounds and computational complexity results.
- Validate the model using benchmark and real transportation datasets.
- Develop an operational prototype for dynamic route guidance and network management.

6. Conclusion

This paper developed an expanded mathematical and architectural framework for modelling and optimizing dynamic transportation networks using time-dependent graphs. By allowing edge traversal time to vary with entry time and congestion, the framework captures a central characteristic of transportation systems that static network models may overlook. The multi-objective formulation provides a structured mechanism for considering travel time, operating cost, and congestion exposure simultaneously.

The proposed architecture and data-flow representation clarify how network data, dynamic observations, travel-time estimation, route generation, optimization, and monitoring can be connected in an integrated system. The synthetic example demonstrates the basic temporal effect on route evaluation without making empirical claims. Future research should focus on validated real-world datasets, uncertainty modelling, resilience, real-time data integration, scalable computation, and rigorous comparative evaluation.

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