Multi News Boy Vendor Model with Exponentional Demand Rate

* **Dr.Maruti Bhosale** February 11, 2026

Abstract

Now a days, there is huge stock of products are rotten which leads to loss of entire world. Some products have a very small life period. It can be modeled by using single cycle inventory models. Here, inven- tory models is presented with constant deterioration rate and and peak demand rate is exponential. With the help of inventory theory, inven- tory levels obtained. This models is solved by using Newton Raphson method. Sales revenue, order cost, holding cost, and profit is obtained . At the end of the article, real life example is illustrated to support the model.

Keywords:- Newsboy problem, Supply Chain,inventory technique, Fuzzy number,Triangular number, exponential rate.

1 Introduction:

Supply chain is the process of supplying the finished or semi-finished products from the manufacturer to the customer. In this process, there are number of stake holders are taking part such as manufacturer, wholesaler, customer retailer etc. The main objective of supply chain is to satisfy the customer demand with high level service and maximum profit. If the demand distribu- tion is known then we can find trend and able to forecast by using statistical tools and techniques. To satisfy the customers demand and increase the ser- vice level, it is essential to maintain the inventory level. While optimizing the problem, how much inventory will maintain, what are the various cost, what are the changes in the costs these types of problems arises. In supply chain, inventory technique plays an important role to maximize the profit. Manufacturer gives the various discounts to the buyer such as price discounts, quantity discounts, cash back discounts to increase the sell as well as profit. Manufacturer adopts the various ideas to increase the profit of their business. Demand of people rapidly increases and changes time to time and it is not homogeneous in their utilization. Price discounts, quantity discounts are the common and effective policies for suppliers to promote their products. The problem is to find order quantities for the perishable items having the age dependent demand rate. Manufacturer makes the various decision such that the product will sold out within short time. If the manufacturer produces perishable items, then it is necessary to sale the product before deteriorate. Medical experts frequently gives the importance of best quality food. Also, the importance of cow milk is very high in the health issues. Therefore, we focused to study the inventory models for fresh cow milk vendor with age dependent demand rate. We are assuming that D as a peak demand rate and each customer has a threshold and if the perishable item is older than the threshold will not purchase the item.

Early in the morning, vendor purchase the Cow fresh milk from the farmer and sells to the customer. At time $t = 0$ there is very high peak demand. As time passes then the demand decreases rapidly because milk is the perishable therefore it is necessary to sale before deteriorate. If time t is very high then peak demand will becomes zero. Hence, fresh cow milk has the age depen- dent demand. The perishability of milk is purely depends upon the time. In this situation vendor purchase the fresh cow milk early in the morning from the farmer and sold out to the customer within the short period. This is the single cycle process which is called as newsboy vendor problem. Therefore we are defining the age dependent demand function as exponential demand rate, which is given below, Demand rate function is:

$$D(t) = \theta e^{-\theta t} , \quad 0 \leq t \leq \tau \qquad (1)$$
$$= 0, \quad \text{otherwise}$$

For simplification, we are assuming that customers are uniformly distributed with their threshold on $(0, \tau)$, where τ is life length of product. Thus, on a day t in the cycle and which is in $(0, \tau)$, $D\left(1-\frac{t}{\tau}\right)$customer will buy. For simplification, we are assuming that both time and demand continuous. The proposed model is solved by using Newton-Raphson method for the supply chain of fresh cow milk vendor with age dependent demand rate. The article is organized as: Section 2 explains the exhaustive review of literature on supply chain. Section 3 gives problem description . Section 4 includes mathematical model, assumptions and notations. Section 5 presents numerical examples and discussion based on the results. Lastly Section 6 presents the concluding remarks of the study. In this paper, more realistic parameters such as peak demand, selling price, purchase cost holding cost, deterioration rate are collected from Delhi Gotha, Vadgaon Bk, Pune, India.

2 Review of Literature:

Padmanabhan et al. (1995) developed a EOQ model for perishable items un- der stock dependent selling rate with deterioration rate is constant. He ob- tained economic order quantity for three case viz, without shortages, partially shortages and full shortages. Xu et al.(2004) proposed a supply chain model under fuzzy demand for single cycle. Lie et al.(2006) proposed a supply chain model for single stage and they also extended work for supply chain coordination by revenue sharing contract under uncertain environment. Xu et al.(2010) developed a newsboy vendor model for imperfect quality under both the system centralized as well as decentralized and obtained optimal or- der quantity, profit for retailers, manufacturer and total supply chain. Chen et al. (2011) proposed a newsboy vendor model for quality discount for fuzzy demand. Chen & Bell (2011) presented a newsboy vendor model for single stage by assuming fuzzy demand. Zhang et al. (2014) proposed a supply chain model under fuzzy demand for centralized and decentralized system. They showed that performance in centralized system is more better than de- centralized system. Kannan et al. (2014) proposed model for reverse supply chain for multi-stage. In this study, they considered that at the end of life of product, retailer collects obsolete product by giving reward. Retailer transfer these products to the manufacturer and manufacturer reproduce the product and sold it in the market by assuming remanufacturing capacity is infinite and normally distributed. Xia & Hu (2014) presented a model for reverse supply chain and discussed the impact of reward on remanufacturing process to control the carbon emission. Zahara & Jafar (2017) presented two stage supply chain model for non-traditional and new sustainable green product for centralized and decentralized system. Jafar et al.(2018) proposed reverse supply chain coordination model under stochastic remanufacturing capacity is limited. They defined new reward function and which is deterministic and obtained manufacturers, retailers and total supply chain profit and they shown that by increasing the number of returned obsolete product is suitable for enough capacity in the remanufacturing process. Bhosale & Latpate (2021) developed a supply chain model for milk product by assuming demand follows weibull distribution. In this article, author obtained retailers, manufacturers and total supply chain and they shown that centralized system gives better solution than decentralized system. Latpate & Bhosale (2020) proposed a supply chain model for perishable items by assuming demand has lognormal distribution and they obtained retailers, manufacturers and total supply chain profit and they shown that centralized system gives better solution than decentralized system. In this research article, we developed age dependent demand function which is given in equation (1). We developed two models by assuming age dependent demand rate constant in model-1 and linear function in model-2 and obtained total profit per unit for both the models and shown that model-2 is superior than model-1.

3 Problem Description:

A supply chain is the process of supplying the finished products from the manufacturer to the customer. The main objective of the supply chain is to satisfy the customer demand, maximize the supply chain profit, minimize the cost, optimize the economic order quantity. The supply chain includes not only flow of material from the supplier to the customer but also the exchange of funds and information. The efficiency of supply chain depends upon proper management of goods and services across in all the stages. In supply chain, inventory technique plays an important role to increase the profit, service level, optimize order quantity etc. Manufacturer produce multiple products and supply to the customers. To increase the profit of supply chain manufacturer applies different policies such as price discount, quantity discount, cashback, reward, gift vouchers etc. In this article, we developed inventory model for fresh cow milk vendor with age dependent demand rate. The demands for all products are assumed to be linear which is defined in equation (1). The objective of this research is to analyze the effect of EOQ for perishable items with age dependent demand rate.

4 Mathematical Model:

4.1 Assumption:

1. Demand rate is constant in model.
2. Manufacturer tries to satisfy the customers demand and shortages are not allowed.
3. Inventory policy is a continuous of EOQ type.
4. Distribution of deterioration items follows uniform distribution.
5. Inventory policy is a continuous review policy of EOQ type.

4.2 Notations:

Following notations are used to develop the model:
Q= Economic Order quantity.
D= Demand.
T= Total Cycle time.
τ= Life length of product.
t= Time period.
S=Selling price per unit are known and constant. *C*= Purchase cost of the item is known and constant *h*=Holding cost is known and constant.
OC= Order cost is known and constant.
I(*t*)= Inventory level at time *t*.
θ (*t*) = Deterioration rate at time t
In Model deterioration rate is assumed to be constant θ

4.3 Model

The main objective of the model is to obtain optimum order quantity *Q* for perishable items having age dependent demand with peak demand is *D* and constant deterioration rate θ. The differential equation representing inventory level at time *t* can be written as

The solution of equation (2) with boundary condition $I(\tau) = 0$ is

$$\frac{d(I(t)}{dt} + \theta\, I(t) = -D\,\theta e^{-\theta\tau}\ , \qquad 0 \le t \le \tau \qquad (2)$$

$$e^{\int \theta dt}\frac{dI(t)}{dt} + e^{\theta dt}\theta I(t) = -D\theta e^{-\tau} e^{\int \theta\, dt}$$

$$\frac{d}{dt}\left[e^{\int \theta\, dt} I(t)\right] = -D\theta e^{-\theta\tau} e^{\theta t}$$

Integrating on both sides of above equation, we get

$$\left[e^{\int \theta dx} I(x)\right] = -D \int^{\tau} \theta e^{-\theta \tau} e^{\theta t} dx$$

By using boundary condition $I(\tau) = 0$, we get

$$I(t) = De^{\theta t}\left[1 - e^{-\theta(-\tau+t)}\right], \qquad 0 \leq t \leq \tau \qquad (3)$$

Various costs which are related to inventory are given below.

Sales Revenue Per Cycle

$$SR = S \int_0^{\tau} I(t)dt$$

$$SR = DS\left(-\frac{e^{-\theta t}-1}{\theta} - te^{-\theta t}\right) \qquad (4)$$

Material Cost (MC)-

Material Cost is given by

$$MC = C\, I(\theta)$$

$$MC = c \int_t^{\tau} I(\theta)$$

$$MC = c \int_t^{\tau} I(t)dt$$

$$MC = c \int_t^{\tau} De^{-\theta t}\left(1 - e^{\theta(t-\tau)}\right)dt$$

$$MC = cD\left(-\frac{e^{-\tau\theta} - e^{-t\theta}}{\theta} - \tau e^{-\theta t} + te^{-\theta t}\right) \qquad (5)$$

Holding Cost (HC)-

$$\boldsymbol{HC = h \int_0^{\tau} I(t)dt}$$

$$HC = hD \int_0^{\tau} e^{-\theta t}\left(1 - e^{\theta(t-\tau)}\right)dt$$

$$HC = hD\left(-\frac{e^{-\tau\theta} - e^{-t\theta}}{\theta} - te^{-\theta t}\right) \qquad (6)$$

Total Profit per unit is

$$P(\tau) = \frac{1}{\tau}[SR - OC - HC - MC]$$

$$P(\tau) = \frac{1}{\tau}[\, DS\left(\frac{e^{-\theta t} - 1}{\theta} - te^{-\theta t}\right) - OC - hD\left(-\frac{e^{-\tau\theta} - e^{-t\theta}}{\theta} - te^{-\theta t}\right) - cD\left(-\frac{e^{-\tau\theta} - e^{-t\theta}}{\theta} - \tau e^{-\theta t} + te^{-\theta t}\right)]$$

The necessary condition for maximum profit per unit time is

$$\frac{d\,P(\tau)}{dt} = \frac{D}{\tau}\, e^{-\theta t}(S\theta - h\theta t - c) = 0 \qquad (7)$$

$$\tau = De^{-\theta t}(S\theta - h\theta t - c) \qquad (8)$$

From equation (8) the optimum value of τ can be obtained by using Newton -Raphson method and equation (3) gives the optimum order quantity.

$$\frac{d^2P(\tau)}{dt^2} = -\frac{D}{\tau}\{e^{-\theta t}(-h\theta) - (S\theta - h\theta t - c)\frac{e^{-\theta t}}{\theta}\} \quad (9)$$

In equation (9) all the three terms are negative, therefore, second derivative is negative. Therefore

$$\frac{d^2P(\tau)}{d\tau^2} \leq 0 \quad (10)$$

Equation (10) showed that convexity condition satisfies. Thus, profit maximizes.

Table 1: Effect of deterioration rate on profit, and Economic Order Quantity

θ		$D=100$	$D=110$	$D=120$	$D=130$	$D=140$	$D=150$
0.05	τ	0.949408	0.905721	0.867578	0.833894	0.803863	0.776869
	Profit	645.2932	715.2129	785.3588	855.7031	926.2232	996.9007
	Q	48.23052	50.57523	52.81558	54.96436	57.03197	59.02696
0.06	τ	0.876883	0.836583	0.801392	0.770313	0.742603	0.717692
	Profit	636.7057	706.2123	775.9635	845.9292	916.0851	986.4111
	Q	44.62327	46.79167	48.86355	50.85076	52.76289	54.60787
0.07	τ	0.818609	0.78103	0.748211	0.719224	0.693377	0.67014
	Profit	628.7116	697.8341	767.2182	836.8319	906.6491	976.6482
	Q	41.7236	43.75029	45.68677	47.5441	49.33127	51.05567
0.09	τ	0.729806	0.69637	0.667165	0.641365	0.618357	0.597669
	Profit	614.0949	82.5161	751.2302	820.2011	889.3998	958.8023
	Q	37.30253	39.11318	40.84324	42.50258	44.09925	45.63983
0.1	τ	0.694887	0.663079	0.635294	0.610748	0.588856	0.56917
	Profit	607.333	675.4302	743.8347	812.5087	881.4217	950.5485
	Q	35.5633	37.28897	38.93783	40.5193	42.04102	43.5093
0.2	τ	0.498128	0.47549	0.455702	0.43821	0.422603	0.408562
	Profit	551.7236	617.1667	683.0354	749.2772	815.85	882.7187
	Q	25.75453	27.00102	28.19203	29.33436	30.43353	31.4941

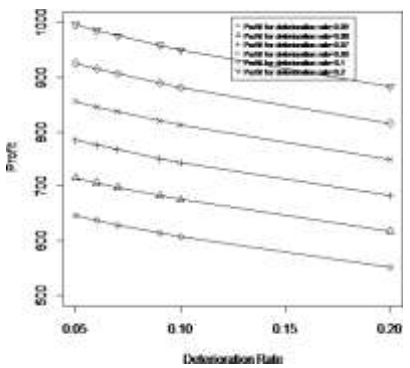

Figure 1: The effect of deterioration rate on Profit for Model

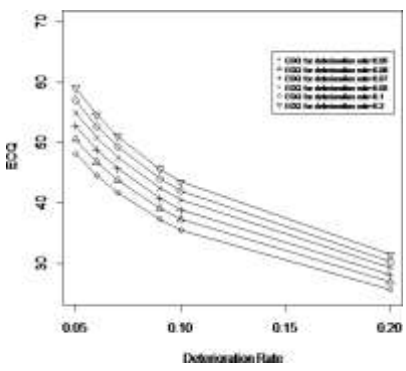

Figure 2: The effect of deterioration rate on EOQ for Model

5 Numerical Analysis and Discussion

For numerical analysis, we set the parameters $D = Rs.100$, $OC = Rs.50$, $h = Rs.3$, $c = Rs.55$, $S = Rs.70$. Deterioration rate varies from $\theta = 0.05$ to $\theta = 0.2$ and demand varies from $D = 100$ to $D = 150$. Table 1 showed that as deterioration rate increases from θ =0.05 to θ =0.2 then profit decreases for all the peak demands and peak demand varies from 100 to
150. As demand increases then profit rapidly increases from Rs.645.2932 to Rs. 996.9007 for the deterioration rate θ =0.05. Similarly, same situation happens in the other deterioration rates but order quantity increases from 48.23052 to 59.62696 for the deterioration rate θ =0.05. And for all other deterioration rates EOQ increases and profit decreases. Figure 1 shows that as deterioration rate increases from $\theta = 0.05$ to θ =0.2 then profit decreases for all the peak demands $D = 100$ to $D = 150$. Also, for all deterioration rates, time τ decreases. Figure 2 shows that as deterioration rate increases then EOQ decreases for all the deterioration rates. It shows that if demand is maximum then EOQ also maximum.

6.Conclusion

Inventory model is presented the newsboy vendor problem with age dependent demand rate for fresh cow milk. Two models with peak demand is D and deterioration rate constant and linear. For both the models, total profit per unit is obtained. After the numerical analysis, it showed that when peak demand D and deterioration rate is linear superior than the deterioration rate constant. For numerical analysis, more realistic parameters are considered for developing the model. The impact of age dependent selling rate, perishability, economic order quantity, profit and inventory level are reported. The application of age dependent model is given for fresh cow milk distribution. It showed that as time passes then there is deterioration of milk in such situations the profit and EOQ decreases. Thus, the presented model tackles the real world situations. It can extended to multi-item inventory models.

References

[1] Bhosale, M. R. and Latpate.R.V. Single stage fuzzy supply chain model with Weibull distributed demand for milk commodities. Granular Computing, 6 (2), 255-266 (2021).

[2] Bhosale, M.R. and Latpate, R., Gitte, S. Fuzzy Supply Chain Newsboy Problem Under Lognormal Distributed Demand for Bakery Products. Applied Statistical Methods. ISGES 2020. Springer Proceedings in Mathematics & Statistics, vol 380. Springer, Singapore (2022).

[3] Chen, S.P. and Bell, P. C. Coordinating a decentralized supply chain with customer returns and price dependent stochastic demand using a buyback policy. European Journal Operation Research, 212, 293-30 (2011).

[4] Govindan, K. and Popiuc, M.N., Reverse supply chain coordination by revenue sharing contract: A case for the personal computers industry. European Journal of Operational Research, 233(2), 326-336 (2014).

[5] Govindan, K., Soleimani, H. and Kannan, D. Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. European journal of operational research, 240(3), 603-626 (2015).

[6] Heydari, J., Govindan, K. and Sadeghi, R. Reverse supply chain coordination under stochastic remanufacturing capacity. International Journal of Production Economics, 202, 1-11 (2018).

[7] Jie, W, and Zhao. J. Reverse channel decisions for a fuzzy closed-loop supply chain. Applied Mathematical Modelling, 37(3), 1502-1513 (2013).

[8] Kurade, S.S. and Latpate.R.V. Demand and deterioration of items per unit time inventory models with shortages using genetic algorithm. Journal of Management Analytics, 8(3), 502-529 (2021) DOI: 10.1080/23270012.2020.1829113.

[9] Kurade, S., Latpate, R., and Hanagal, D. Probabilistic Supply Chain Models with Partial Backlogging for Deteriorating Items. Applied Statistical Methods. ISGES 2020. Springer Proceedings in Mathematics & Statistics, vol 380. Springer, Singapore (2022).

[10] Kurade S. S. An Evolutionary Based Optimization for Price Varying Demand Inventory Model with Linear Holding Cost. Journal of Scientific Research, 65 (9), 63-70 (2021). DOI: 10.37398/JSR.2021.650910.

[11] Kwakernaak, H. Fuzzy Random Variabhs-I. Definitions and Theorem. Information Sciences, 15, 1-29 (1978).

[12] Latpate, R. V. and Bajaj,V. H. Fuzzy programming for Multi-objective Transportation and inventory management problem with retailer storage. International Journal of Agricultural and Statistical Sciences, 7, 377-326 (2011).

[13] Latpate, R. V. and Bajaj,H.V. Fuzzy multi-objective, multi-product, production distribution problem with manufacturer storage. In Proceedings of International Congress on PQROM, 340-355 (2011).

[14] Latpate, R.V. and Kurade, S.S. Fuzzy MOGA for supply chain models with Pareto decision space at different α- cuts. International Journal of Advanced Manufacturing Technology, 91, 3861–3876 (2017). DOI: 10.1007/s00170-016-9966-5.

[15] Latpate, R.V. and Bhosale, M.R. Single cycle supply chain coordination model for fuzzy stochastic demand of perishable items. Iranian Journal of Fuzzy Systems, 17(2), 39-48 (2020).

[16] Latpate, R. V. and Kurade, S.S. Multi-objective multi-index transportation model for crude oil using fuzzy NSGA-II. IEEE Transactions on Intelligent Transportation Systems, 23(2), 1347-1356 (2022). DOI: 10.1109/TITS.2020.3024693.

[17] Oliver, R.K. and Webber, M.D. Supply-chain management: logistics catches up with strategy. Outlook, 5(1), 42-47 (1982).

[18] Padmanabhan, G. and Vrat, P., EOQ models for perishable items under stock dependent selling rate. European Journal of Operational Research, 86(2), pp.281-292 (1995).

[19] Shi, C. and Bian, D. Closed-loop supply chain coordination by contracts under government subsidy. IEEE In Control and Decision Conference, 2747-2750 (2011).

[20] Xia, L. and He, L. Game theoretic analysis of carbon emission reduction and sales promotion in dyadic supply chain in presence of consumers' low-carbon awareness. Discrete Dynamics in Nature and Society, 13

(2014). DOI:10.1155/2014/837376.

[21] Xu, R. and Zhai, X., Analysis of supply chain coordination under fuzzy demand in a two-stage supply chain. Applied Mathematical Modelling, 34(1), 129-139 (2010).

[22] Zahra, B. and Jafar, H., A mathematical model for green supply chain with coordination with substitutable products. Journal of Cleaner Production, 145, 232-249 (2017).

[23] Zhang, B., Lu. S., Zhang, D., and Wen, K. Supply chain coordination based on a buyback contract under fuzzy random variable demand. Fuzzy Sets and Systems, 255, 1-16 (2014).

[24] Zhang, Guoqing, Jianmai Shi, Sohail S. Chaudhry, and Xindan Li. "Multi-period multi-product acquisition planning with uncertain demands and supplier quantity discounts." Transportation Research Part E: Logistics and Transportation Review 132 (2019): 117-140.

[25] Zhang, G., 2010. The multi-product newsboy problem with supplier quantity discounts and a budget constraint. European Journal of Operational Research, 206(2), pp.350-360.