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N-SERIES EQUATIONS INVOLVING GENERALISED BATEMAN –K FUNCTIONS

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Abstract

In this paper, we have considered the N- Fourier series equations involving generalised Bateman- K function of the first kind and solve the set of series equations.

Keywords: Integral equation, Series equation, Bateman –k polynomials.

1. Introduction

If the review the literature then we observe that the existing solutions on series equations are derived only from dual to six fourier series equations. No further generalizations are available till date. This tempted us to find the solution of N- fourier series equations involving some special functions and in this paper we have obtained certain results. By considering the special values of n= 2,3,4,5,6 we shall be able to derive solutions of dual , triple, quadruple, 5- tuple and 6- tuple fourier series equations involving respective special functions.

2. N- series equations of the first kind

N- Series equations involving generalised Bateman K– Functions:

$$\sum_{m=0}^{\infty} \frac{A_m}{\Gamma(2\beta+\sigma+m+1)} K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x) = f_i(x), \quad a_{i-1} < x < a_i \tag{1.1}$$

Where i = 1, 3, 5…….., n-1 & $a_0 = 0$

$$\sum_{m=0}^{\infty} \frac{A_m}{\Gamma(2\nu+\sigma+m+1)} K_{2(m+\alpha)}^{2(\beta+\sigma)}(x) = f_j(x), \quad a_{j-1} < x < a_j \tag{1.2}$$

Where j = 2, 4, 6,…….., n & $a_n = \infty$

$K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)$ is the generalised Bateman K- function, A_n is an unknown coefficient $f_1(x), f_2(x), \dots \dots \dots f_n(x)$ are known functions in the given interval and α, β, σ, $\boldsymbol{\nu}$ all are parameters > -1. Here n is taken as an even number.

If n is odd then the above equations will be

$$\sum_{m=0}^{\infty}\frac{B_m}{\Gamma(2\beta+\sigma+m+1)}K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)=f_i(x),\quad a_{i-1}<x<a_i \tag{1.3}$$

Where i = 1, 3, 5……., n & $a_0=0,\quad a_n=\infty$

$$\sum_{m=0}^{\infty}\frac{B_m}{\Gamma(2\nu+\sigma+m+1)}K_{2(m+\alpha)}^{2(\beta+\sigma)}(x)=f_j(x),\quad a_{j-1}<x<a_j \tag{1.4}$$

Where j = 2, 4, 6,…….., n-1

Here we solve only equations (1.1) and (1.2) solution of equations (1.3) and (1.4) can be obtained easily by following the same procedure.

3. Preliminary Results

1- The orthogonality relation is

$$\int_0^{\infty}x^{-(2\alpha+2\sigma+1)}K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)K_{2(n+\alpha)}^{2(\alpha+\sigma)}(x)dx=\frac{2^{2\alpha+2\sigma}\Gamma(n-\sigma)\delta_{mn}}{\Gamma(2\alpha+\sigma+n+1)} \tag{2.1}$$

Where δ_{mn} is kronecker delta and $\alpha+\sigma+1>0$, $\alpha+1\le 0$ for $\alpha+\sigma>-1$, $\beta>0$

2- $$\int_0^{\xi}e^{x}(\xi-x)^{\beta-1}K_{2(n+\alpha)}^{2(\alpha+\sigma)}(x)dx=\Gamma\beta\quad e^{\xi}\,K_{2(n+\alpha+\beta)}^{2(\alpha+\sigma+\beta)}(\xi) \tag{2.2}$$

$and\ for\ \ 2\alpha+\sigma+n+1>\beta>0$ the equation is

$$\int_{\xi}^{\infty}e^{-x}x^{-(2\alpha+2\sigma+1)}(x-\xi)^{\beta-1}K_{2(n+\alpha)}^{2(\alpha+\sigma)}(x)dx=\frac{\Gamma\beta\Gamma(2\alpha-\beta+\sigma+n-1)}{\xi^{(2\alpha-\beta+2\sigma+1)}\Gamma(2\alpha+\sigma+n+1)}$$

$$e^{-\xi}\ K_{2(n+\alpha-\beta)}^{2(\alpha+\sigma-\beta)}(\xi) \tag{2.3}$$

3- $$S(r,x)=\sum_{n=0}^{\infty}\frac{\Gamma(2\nu+\sigma+n+1)}{2^{2\beta+2\sigma}\Gamma(n-\sigma)}K_{2(n+\beta)}^{2(\beta+\sigma)}(x)K_{2(n+\alpha)}^{2(\alpha+\sigma)}(x) \tag{2.4}$$

4. The Solution

Let us assume

$$\sum_{n=0}^{\infty}\frac{A_m}{\Gamma(2\beta+\sigma+m+1)}K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)=\emptyset_i(x),\ a_{i-1}<x<a_i \tag{3.1}$$

Using Orthogonality relation, we get from equations (1.1) & (3.1)

$$A_m = \frac{\Gamma(2\alpha+\sigma+m+1)\Gamma(2\beta+\sigma+m+1)}{2^{2\sigma+2\sigma}\,\Gamma(m-\sigma)}\left[\sum_{i=0}^{(n-2)/2}\int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(x)x^{-(2\alpha+2\sigma+1)}K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)dx + \sum_{i=0}^{(n-2)/2}\int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(x)x^{-(2\alpha+2\sigma+1)}K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)dx\right]$$

Substituting the value of A_m in equation (1.2), we get

$$\sum_{m=0}^{\infty}\frac{\Gamma(2\alpha+\sigma+m+1)\Gamma(2\beta+\sigma+m+1)}{2^{2\alpha+2\sigma}\Gamma(m-\sigma)\Gamma(2\nu+\sigma+m+1)}\sum_{i=0}^{(n-2)/2}\left[\int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(r)r^{-(2\alpha+2\sigma+1)} + \int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}\right]K_{2(m+\alpha)}^{2(\alpha+\sigma)}(x)K_{2(m+\beta)}^{2(\beta+\sigma)}(x)\,dr = f_j(x)$$

$$a_{j-1} < x < a_j \;\&\; j = 2,4,6,\ldots\ldots n-2, n \qquad (3.2)$$

Interchanging the order of integration and summation, we get

$$\sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}S(r,x)dr = M_j(x),$$

$$a_{j-1} < x < a_j \;\&\; j = 2,4,6,\ldots\ldots n-2, n$$

Where,

$$M_j(x) = \frac{\Gamma(2\nu+\sigma+n+1)^2}{\Gamma(2\alpha+\sigma+n+1)\Gamma(2\beta+\sigma+n+1)}f_j(x) - \sum_{i=0}^{\frac{n-2}{2}}\int_{a_{2i}}^{a_{2i+1}} f_{2i+1}(r)r^{-(2\alpha+2\sigma+1)}S(r,x)dr$$

For all j = 2, 4 ,........, n-2, n. (3.3)

Now in the equation (3.2) if j = k where k is an even number and $2 \le k \le n$ we have

$$\sum_{i=0}^{\frac{k-4}{2}}\int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}S_r(r,x)dr + \int_{a_{k-1}}^{x} \emptyset_k(r)\,r^{-(2\alpha+2\sigma+1)}S_r(r,x)dr$$

$$+\int_{x}^{a_k} \emptyset_k(r)\,r^{-(2\alpha+2\sigma+1)}S_r(r,x)dr + \sum_{i=k/2}^{\frac{n-2}{2}}\int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}S_r(r,x)dr =$$

$$\frac{e^x\Gamma(2\alpha-2\nu)\Gamma(2\beta-2\nu)}{2^{2\alpha-2\nu}}M_k(x), \quad a_{k-1} < x < a_k$$

Or

$$\int_{a_{k-1}}^{x} \emptyset_k(r)\, r^{-(2\alpha+2\sigma+1)} \int_0^r \frac{E(y)dy\, dr}{(x-y)^{1-2\alpha+2\nu}(r-y)^{1-2\beta+2\nu}} + \int_x^{a_k} \emptyset_k(r)\, r^{-(2\alpha+2\sigma+1)}$$

$$\int_0^x \frac{E(y)dy\, dr}{(x-y)^{1-2\alpha+2\nu}(r-y)^{1-2\beta+2\nu}} = \frac{e^x\Gamma(2\alpha-2\nu)\Gamma(2\beta-2\nu)}{2^{2\alpha-2\nu}} M_k(x) -$$

$$\sum_{i=0}^{(k-4)/2} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(r)\, r^{-(2\alpha+2\sigma+1)} \int_0^r \frac{E(y)dy\, dr}{(x-y)^{1-2\alpha+2\nu}(r-y)^{1-2\beta+2\nu}} -$$

$$\sum_{i=k/2}^{(n-2)/2} \int_{a_{2i+1}}^{a_{2i+2}} \emptyset_{2i+2}(r)\, r^{-(2\alpha+2\sigma+1)} \int_0^x \frac{E(y)dy\, dr}{(x-y)^{1-2\alpha+2\nu}(r-y)^{1-2\beta+2\nu}} \qquad (3.4)$$

Changing the order of integration in this equation, we get

$$\int_{a_{k-1}}^{x} \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_y^{a_k} \frac{\emptyset_k(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} = \frac{e^x\Gamma(2\alpha-2\nu)\Gamma(2\beta-2\nu)}{2^{2\alpha-2\nu}} M_k(x) -$$

$$\int_0^{a_{k-1}} \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_{a_{k-1}}^{a_k} \frac{\emptyset_k(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}}$$

$$-\sum_{i=0}^{\frac{k-4}{2}} \left\{ \int_0^{a_{2i+1}} \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} + \int_{a_{2i+1}}^{a_{2i+2}} \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_y^{a_{2i+2}} \frac{\emptyset_{2i+2}(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} \right\}$$

$$- \sum_{i=k/2}^{(n-2)/2} \int_0^x \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} \qquad (3.5)$$

Assuming ,

$$\int_y^{a_k} \frac{\emptyset_k(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} = \overline{\emptyset_k(y)} \quad \text{, for all k = 2,4,........., n-2,n} \qquad (3.6)$$

With the help of equation (3.6) , the equation (3.5) takes the form

$$\int_{a_{k-1}}^{x} \frac{E(y)\overline{\emptyset_k(y)}dy}{(x-y)^{1-2\beta+2\nu}} = \frac{e^x\Gamma(2\alpha-2\nu)\Gamma(2\beta-2\nu)}{2^{2\alpha-2\nu}} M_k(x) - \int_0^{a_{k-1}} \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_{a_{k-1}}^{a_k} \frac{\emptyset_k(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} -$$

$$\sum_{i=0}^{\frac{k-4}{2}} \left\{ \int_0^{a_{2i+1}} \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} + \int_{a_{2i+1}}^{a_{2i+2}} \frac{\overline{\emptyset_{2i+2}(y)}E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \right\}$$

$$- \sum_{i=k/2}^{(n-2)/2} \int_0^x \frac{E(y)dy}{(x-y)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r) r^{-(2\alpha+2\sigma+1)} dr}{(r-y)^{1-2\beta+2\nu}} \qquad (3.7)$$

This is an Abel type integral equation; hence its solution is given by,

$$E(y)\overline{\emptyset_k(y)} = A_k(y) - \frac{\sin(1-2\alpha+2\nu)\pi}{\pi} \Big[\int_0^{a_{k-1}} E(\xi)d\xi \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{k-1}}^{a_k} \frac{\emptyset_k(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}} + \sum_{i=0}^{(k-4)/2}\Big\{\int_0^{a_{2i+1}} E(\xi)d\xi \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}} + \int_{a_{2i+1}}^{a_{2i+2}} E(\xi)\overline{\emptyset_{2i+2}(\xi)}d\xi \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}}\Big\} + \sum_{i=k/2}^{(n-2)/2} \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}} \int_0^x \frac{E(\xi)d\xi}{(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}}\Big] \tag{3.8}$$

where,

$$A_k(y) = \frac{\sin(1-2\alpha+2\nu)\pi\Gamma(2\alpha-2\nu)\Gamma(2\beta-2\nu)}{\pi\, 2^{2\alpha-2\nu}} \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{e^x M_k(x)dx}{(y-x)^{2\alpha-2\nu}}$$

$$for\ all\ k = 2,4,6,\ldots..n-2,n \tag{3.9}$$

Inverting the order of integration in the last integral of equation (3.8)

$$E(y)\overline{\emptyset_k(y)} = A_k(y) - \frac{\sin(1-2\alpha+2\nu)\pi}{\pi} \Bigg[\int_0^{a_{k-1}} E(\xi)d\xi \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{k-1}}^{a_k} \frac{\emptyset_k(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}} + \sum_{i=0}^{(k-4)/2}\Bigg\{\int_0^{a_{2i+1}} E(\xi)d\xi \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}} + \int_{a_{2i+1}}^{a_{2i+2}} E(\xi)\overline{\emptyset_{2i+2}(\xi)}d\xi \frac{d}{dy}\int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}}\Bigg\} + \sum_{i=k/2}^{(n-2)/2} \frac{d}{dy}\Bigg\{\int_0^{a_{k-1}} E(\xi)d\xi \int_{a_{k-1}}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}} + \frac{d}{dy}\int_{a_{k-1}}^{y} E(\xi)d\xi \int_{\xi}^{y} \frac{dx}{(y-x)^{2\alpha-2\nu}(x-\xi)^{1-2\alpha+2\nu}} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}}\Bigg\}\Bigg] \tag{3.10}$$

Using the results

$$\frac{d}{dy}\int_a^y \frac{dx}{(y-x)^{1-m}(x-t)^m} = \frac{(a-t)^{1-m}}{(y-t)(y-a)^{1-m}} \tag{3.11}$$

$$and \int_t^\xi \frac{dx}{(\xi-x)^{2\alpha-2\nu}(x-t)^{1-2\alpha+2\nu}} = \frac{\pi}{\sin(1-2\alpha+2\nu)\pi} \tag{3.12}$$

$Equation (3.10) will\ take\ the\ form$

$$E(y)\overline{\emptyset_k(y)} = A_k(y) - \frac{\sin(1-2\alpha+2\nu)\pi}{\pi(y-a_{k-1})^{2\alpha-2\nu}}$$

$$\left[\int_0^{a_{k-1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}d\xi}{(y-\xi)} \int_{a_{k-1}}^{a_k} \frac{\emptyset_k(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}} \right.$$
$$+ \sum_{i=0}^{(k-4)/2}\left\{\int_0^{a_{2i+1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}d\xi}{(y-\xi)} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}}\right.$$
$$\left. + \int_{a_{2i+1}}^{a_{2i+2}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}\overline{\emptyset_{2i+2}(\xi)}d\xi}{(y-\xi)}\right\}$$
$$+ \sum_{i=k/2}^{(n-2)/2}\int_0^{a_{k-1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}d\xi}{(y-\xi)} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}}$$
$$\left. - \sum_{i=k/2}^{n-2/2} \frac{d}{dy}\int_{a_{k-1}}^{y} E(\xi)d\xi \int_{a_{2i+1}}^{a_{2i+2}} \frac{\emptyset_{2i+2}(r)r^{-(2\alpha+2\sigma+1)}dr}{(r-\xi)^{1-2\beta+2\nu}}\right]$$

Equation (3.7) is also an Abel type integral equation hence its solution will be

$$\emptyset_k(r)r^{-(2\alpha-2\sigma-1)} = -\frac{\sin(1-2\beta+2\nu)\pi}{\pi}\frac{d}{dr}\int_r^{a_k} \frac{\overline{\emptyset_k(y)}dy}{(y-r)^{2\beta-2\nu}}$$

For all k = 2, 4, 6, …….n. (3.13)

Therefore,

$$\int_{a_{k-1}}^{a_k} \frac{\emptyset_k(r)r^{-(2\alpha-2\sigma-1)}}{(r-\xi)^{1-2\beta+2\nu}} = \frac{\sin(1-2\beta+2\nu)\pi}{\pi(a_{k-1}-\xi)^{-2\beta+2\nu}} \int_{a_{k-1}}^{a_k} \frac{\overline{\emptyset_k(y)}dy}{(y-\xi)(y-a_{k-1})^{2\beta-2\nu}}$$

For all k = 2, 4, 6, …….n. (3.14)

Applying the result (3.14) in equation (3.12) and also applying the Leibnitz theorem, we get

$$E(y)\overline{\emptyset_k(y)} = A_k(y) - \frac{\sin(1-2\alpha+2\nu)\pi\sin(1-2\beta+2\nu)\pi}{\pi(y-a_{k-1})^{2\alpha-2\nu}}$$

$$\left[\int_0^{a_{k-1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}(a_{k-1}-\xi)^{2\beta-2\nu}d\xi}{(y-\xi)} \int_{a_{k-1}}^{a_k} \frac{\overline{\emptyset_k(t)}dt}{(t-\xi)(t-a_{k-1})^{2\beta-2\nu}}\right.$$

$$+\sum_{i=0}^{(k-4)/2}\left\{\int_0^{a_{2i+1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}(a_{2i+1}-\xi)^{2\beta-2\nu}d\xi}{(y-\xi)} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\overline{\emptyset_{2i+2}(t)}dt}{(t-\xi)(t-a_{2i+1})^{2\beta-2\nu}} - (2\beta\right.$$

$$\left. - 2\nu)\int_{a_{2i+1}}^{a_{2i+2}} \overline{\emptyset_{2i+2}(t)}dt \int_{a_{2i+1}}^{t} \frac{dr}{(t-r)^{1+2\beta-2\nu}} \int_{a_{2i+1}}^{r} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}d\xi}{(y-\xi)(r-\xi)^{1-2\beta+2\nu}}\right\}$$

$$\left. + \sum_{i=k/2}^{(n-2)/2}\int_0^{a_{k-1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}(a_{2i+1}-\xi)^{2\beta-2\nu}d\xi}{(y-\xi)} \int_{a_{2i+1}}^{a_{2i+2}} \frac{\overline{\emptyset_{2i+2}(t)}dt}{(t-\xi)(t-a_{2i+1})^{2\beta-2\nu}}\right]$$

$$-\sum_{i=k/2}^{n-2/2} \frac{\sin(1-2\alpha+2\nu)\pi}{\pi}\frac{d}{dy}\int_{a_{k-1}}^{y} E(\xi)(a_{2i+1}-\xi)^{2\beta-2\nu}d\xi \int_{a_{2i+1}}^{a_{2i+2}} \frac{\overline{\emptyset_{2i+2}(t)}dt}{(t-\xi)(t-a_{2i+1})^{2\beta-2\nu}}$$

This equation can be written as

$$E(y)\overline{\emptyset_k(y)} = A_k(y) - \int_{a_{k-1}}^{a_k} \overline{\emptyset_k(y)}L_k(t,y)dt - \sum_{i=0}^{(k-4)/2}\int_{a_{2i+1}}^{a_{2i+2}} \overline{\emptyset_{2i+2}(t)}R_{2i+2}(t,y)dt$$

$$-\sum_{j=k/2}^{(n-2)/2}\int_{a_{2j+1}}^{a_{2j+2}} \overline{\emptyset_{2j+2}(t)}T_{2j+2}(t,y)dt \quad (3.15)$$

Where

$$L_k(t,y) = \frac{\sin(1-2\alpha+2\nu)\pi\sin(1-2\beta+2\nu)\pi}{\pi^2(y-a_{k-1})^{2\alpha-2\nu}}\alpha\frac{1}{(t-a_{k-1})^{2\beta-2\nu}}$$

$$\int_0^{a_{k-1}} \frac{E(\xi)(a_{k-1}-\xi)^{2\alpha+2\beta-4\nu}d\xi}{(y-\xi)(t-\xi)} \quad (3.16)$$

$$R_{2i+2}(t,y) = \frac{\sin(1-2\alpha+2\nu)\pi\sin(1-2\beta+2\nu)\pi}{\pi^2(y-a_{k-1})^{2\alpha-2\nu}}$$

$$\left[\frac{1}{(t-a_{2i-1})^{2\beta-2\nu}}\int_{0}^{a_{2i+1}}\frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}(a_{2i+1}-\xi)^{2\beta-2\nu}d\xi}{(y-\xi)(t-\xi)}-(2\beta-2\nu)\int_{a_{2i+1}}^{t}\frac{dr}{(t-r)^{1+2\beta-2\nu}}\int_{a_{2i+1}}^{r}\frac{E(\xi)(a_{k-1}-\xi)^{2\alpha-2\nu}d\xi}{(y-\xi)(r-\xi)^{1-2\beta+2\nu}}\right]$$

Where I = 0, 1, 2,…………(k-4)/2 (3.17)

$$T_{2j+2}(t,y)=\frac{\sin(1-2\alpha+2\nu)\,\pi}{\pi}\frac{1}{\left(t-a_{2j+1}\right)^{2\beta-2\nu}}$$

$$\left[\frac{\sin(1-2\alpha+2\nu)\pi}{\pi(y-a_{k-1})^{2-2\nu}}\int_{0}^{a_{k-1}}\frac{E(\xi)(a_{k-1}-\xi)^{2\alpha+2\nu}(a_{2j+1}-\xi)^{2\beta-2\nu}d\xi}{(y-\xi)(t-\xi)}+\frac{d}{dy}\int_{a_{k-1}}^{y}\frac{E(\xi)(a_{2j+1}-\xi)^{2\beta-2\nu}d\xi}{(t-\xi)}\right]$$

Now substituting k = 2, 4, 6, ……..n-2, n in equation (3.15) we will get n/2 simultaneous Fredholm integral equations with the help of these n/2 simultaneous equations we can determine the values $\overline{\emptyset_2(y)},\overline{\emptyset_4(y)},\ldots\ldots\ldots..\overline{\emptyset_n(y)}$ and then the values $\emptyset_2(x),\emptyset_4(x),\ldots\ldots\ldots\ldots\ldots..\emptyset_n(x)$.

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