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Neutrosophic Generalized Semi Pre-Regular and Normal Space

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Abstract -In this manuscript, we inaugurate Neutrosophic generalized semi-pre regular and normal space. We investigate its properties. Also, we add some improvisation of Neutrosophic generalized semi pre regular and normal space.

Keywords: Neutrosophic generalized semi pre closed sets; Neutrosophic generalized semi pre regular space, Neutrosophic generalized semi pre normal space.

1.INTRODUCTION

In 2014, the pioneering work of Salama, Smarandache, and Valeri [10] introduced the concept of Neutrosophic closed sets and Neutrosophic continuous functions. Subsequent advancements by Salama and Alblowi [10] led to the development of generalized Neutrosophic sets and generalized Neutrosophic topological spaces. Notably, the idea of NOS (Neutrosophic Open Sets) gained prominence through the contributions of Wadel and Smarandache [14]. Furthermore, Ishwarya and Bageerathi [8] offered insights into the perspective of NSO (Neutrosophic Sets) within the framework of Neutrosophic topological spaces. In their publication [25], Rajeshwaran N and Chandramathi N presented the novel idea of Neutrosophic generalized semi pre closed sets within the realm of Neutrosophic topological spaces. Similarly, in another work [26], they introduced the concept of Neutrosophic generalized semi pre Homeomorphisms in the same context. This manuscript seeks to define and investigate the concept of Neutrosophic generalized semi pre-connected space, delving into its inherent properties. The study encompasses an exploration of various related notions and introduces a collection of noteworthy theorems within this domain.

2. Preliminaries

Definition 2.1: [10] A neutrosophic topology (NT for short) a non-empty set X is a family τ_N of neutrosophic subsets in X adheres the following axioms

(NT1) $0_N, 1_N \in \tau_N$

(NT2) $G_1 \cap G_2 \in \tau_N$

(NT3) $\cup G_i \in \tau_N, \forall\{G_i : i \in J\} \subseteq \tau_N$

Here (X, τ_N) is called a neutrosophic topological space (NTS for short).

Definition 2.2: [10] Let A_1 *and* A_2 be two Neutrosophic Sets (NS for Short) of the form

$A_1 = \{\langle X, \mu_{A_1}(X), \sigma_{A_1}(X), \gamma_{A_1}(X)\rangle : x\epsilon X\}$, $A_2 = \{\langle X, \mu_{A_2}(X), \sigma_{A_2}(X), \gamma_{A_2}(X)\rangle : x\epsilon X\}$.

(a) $A_1 \subseteq A_2$ if and only if

$\mu_{A_1}(X) \le \mu_{A_2}(X), \sigma_{A_1}(X) \le \sigma_{A_2}(X)$ and $\gamma_{A_1}(X) \ge \gamma_{A_2}(X)$ for all $x \in X$

(b) $A_1{}^C = \{\langle X, \gamma_{A_1}(X), 1 - \sigma_{A_1}(X), \mu_{A_1}(X)\rangle : x\epsilon X\}$

(c) $A_1 \cap A_2 = \{\langle X, \mu_{A_1}(X) \wedge \mu_{A_2}(X), \sigma_{A_1}(X) \wedge \sigma_{A_2}(X), \gamma_{A_1}(X) \vee \gamma_{A_2}(X)\rangle : x\epsilon X\}$

(d) $A_1 \cup A_2 = \{\langle X, \mu_{A_1}(X) \vee \mu_{A_2}(X), \sigma_{A_1}(X) \vee \sigma_{A_2}(X), \gamma_{A_1}(X) \wedge \gamma_{A_2}(X)\rangle : x\epsilon X\}$

We can use the symbol $A_1 = \{\langle X, \mu_A(X), \sigma_A(X), \gamma_A(X)\rangle : x\epsilon X\}$

Definition 2.3: [25] Let $(X, \tau_{\mathcal{N}})$ be a neutrosophic topological space. A subset Ą of $(X, \tau_{\mathcal{N}})$ is called Neutrosophic generalized semi pre closed [ƝǴŚƤ -closed] set if $spcl_{\mathcal{N}}$(Ą) ⊆ Ʊ, whenever Ą ⊆ Ʊ and Ʊ is Neutrosophic open set.

3.Generalized Semi Pre Regular and Normal Space in Neutrosophic Topological Spaces

In this paper we introduce the new concept namely Neutrosophic generalized semi pre regular space in neutrosophic topological spaces. We delve into the foundations of Neutrosophic generalized semi-pre regular space.

Definition 3.1.1: A topological space $(X, \tau_{\mathcal{N}})$ is said to be ƝǴŚƤ regular if for each ƝǴŚƤ closed set M of $(X, \tau_{\mathcal{N}})$ and each point $x \in X - M$, there exist disjoint open sets P and Q of $(X, \tau_{\mathcal{N}})$ such that $x \in P$ and $M \subseteq Q$. Since every ƝǴ-closed set is ƝǴŚƤ-closed set so every ƝǴŚƤ regular space is ƝǴ regular space.

Definition: 3.1.2 A Neutrosophic topology $(X, \tau_{\mathcal{N}})$ is said to be (ƝǴŚƤ, ƝǴŚ) regular if for each ƝǴŚƤ closed set of

$(X, \tau_{\mathcal{N}})$ and each point $x \in X - M$, there exists disjoint ƝǴŚ open sets P and Q of $(X, \tau_{\mathcal{N}})$ such that $x \in P$ and $M \subseteq Q$.

Theorem 3.1.3: A Neutrosophic topological space $(X, \tau_{\mathcal{N}})$ is a ƝǴŚƤ regular if and only if for each ƝǴŚƤ closed set M of $(X, \tau_{\mathcal{N}})$ and each point $x \in X - M$, there exist open sets P and Q of $(X, \tau_{\mathcal{N}})$ such that $x \in P : M \subseteq Q$ and $cl_{\mathcal{N}}(P) \cap cl_{\mathcal{N}}(Q) = \phi$.

Proof: Necessity: Let M be a ƝǴŚƤ closed set of $(X, \tau_{\mathcal{N}})$ and $x \in X - M$. There exist Neutrosophic open sets P_0 and Q of $(X, \tau_{\mathcal{N}})$ such that $x \in P_0$, $M \subseteq Q$ and $P_0 \cap Q = \phi$, hence $P_0 \cap cl_{\mathcal{N}}(Q) = \phi$. Since $(X, \tau_{\mathcal{N}})$ is ƝǴŚƤ regular, there exist Neutrosophic open sets G and H of $(X, \tau_{\mathcal{N}})$ such that $x \in G$, $cl_{\mathcal{N}}(Q) \subseteq H$ and $G \cap H = \phi$, hence $cl_{\mathcal{N}}(G) \cap H = \phi$. Now put $P = P_0 \cap G$, then P and Q are Neutrosophic open sets of $(X, \tau_{\mathcal{N}})$ such that $x \in P$, $M \subseteq Q$ and $cl_{\mathcal{N}}(P) \cap cl_{\mathcal{N}}(Q) = \phi$.
Sufficiency: This is obvious.

Theorem 3.1.4: Let $(X, \tau_{\mathcal{N}})$ be a Neutrosophic topological space then the following statements are equivalent:

(i) Let $(X, \tau_{\mathcal{N}})$ is ƝǴŚƤ regular space.

(ii) For each point $x \in (X, \tau_{\mathcal{N}})$ and for each ƝǴŚƤ open neighbourhood W of x, there exists a Neutrosophic open set of x, such that $cl_{\mathcal{N}}(Q) \subseteq W$

(iii) For each point of $x \in (X, \tau_{\mathcal{N}})$ and for each ƝǴŚƤ closed not containing x, then there exists a Neutrosophic open set Q of X, such that $cl_{\mathcal{N}}(Q) \cap M = \phi$.

Proof: (i)⇒(ii) Let W be a ƝǴŚƤ open neighbourhood of x. Then there exists a ƝǴŚƤ open set G such that $x \in X \subseteq W$. Since $X - G$ is ƝǴŚƤ closed set and $x \notin X - G$, by hypothesis there exist Neutrosophic open sets P and Q such that $X - G \subseteq P$, $x \in Q$ and $P \cap Q = \phi$ and so $Q \subseteq (X - P)$. Now $cl_{\mathcal{N}}(Q) \subseteq cl_{\mathcal{N}}(X - P) = (X - P)$ and $(X - G) \subseteq P$ implies $(X - P) \subseteq G \subseteq W$. Therefore $cl_{\mathcal{N}}(Q) \subseteq W$.

(ii)⇒(i) : Let M be any ƝǴŚƤ closed set of $x \notin M$. Then $x \in X - M$ and $X - M$ is ƝǴŚƤ open and so $X - M$ is a ƝǴŚƤ open neighbourhood of x. By hypothesis there exists a Neutrosophic open Q of x such that $x \in Q$ and $cl_{\mathcal{N}}(Q) \subseteq (X - M)$ which implies $M \subseteq (X - cl_{\mathcal{N}}(Q))$. Then $(X - cl_{\mathcal{N}}(Q))$ is Neutrosophic open set containing M and $Q \cap (X - cl_{\mathcal{N}}(Q)) = \phi$. Therefore $(X, \tau_{\mathcal{N}})$ is ƝǴŚƤ regular space.

(ii)⇒(iii) : Let $x \in X$ and M be a ƝǴŚƤ closed set such that $x \notin M$. Then $(X - M)$ is a ƝǴŚƤ open neighbourhood of x and by hypothesis there exists a Neutrosophic open set Q of x such that $cl_{\mathcal{N}}(Q) \subseteq (X - M)$ and therefore $cl_{\mathcal{N}}(Q) \cap M = \phi$

(iii)⇒(ii) : Let $x \in X$ and W be a ƝǴŚƤ open neighbourhood of x then there exists a Neutrosophic ǴŚƤ open set G such that $x \in G \subseteq W$. Since $(X - G)$ is ƝǴŚƤ closed and $x \notin (X - G)$ by hypothesis there exists a Neutrosophic open set Q of x such that $cl_{\mathcal{N}}(Q) \cap (X - G) = \phi$. Therefore $cl_{\mathcal{N}}(Q) \subseteq G \subseteq W$.

Theorem 3.1.5: A Neutrosophic topological space $(X, \tau_{\mathcal{N}})$ is a ƝǴŚƤ regular space if and only if given any $x \in P$ and any Neutrosophic open set P of $(X, \tau_{\mathcal{N}})$ there is a ƝǴŚƤ open set Q such that $x \in Q \subseteq gspcl_{\mathcal{N}}(Q) \subseteq P$.

Proof: Let P be a Neutrosophic open set, $x \in P$. So $X - P$ is closed set such that $x \notin P$. Since X is a ƝǴŚƤ regular space then there exist a Neutrosophic ǴŚƤ open sets Q_1 and Q_2 such that $Q_1 \cap Q_2 = \phi$, $X - P \subseteq Q_2$, $x \in Q_1$. Since $Q_1 \cap Q_2 = \phi$, we have $gspcl_{\mathcal{N}}(Q_1) \subseteq gspcl_{\mathcal{N}}(X - Q_2) = X - Q_2$. Since $X - P \subseteq Q_2$, we have $X - Q_2 \subseteq P$. Hence we have $x \in Q_1 \subseteq gspcl_{\mathcal{N}}(Q_1) \subseteq X - Q_2 \subseteq P$.
Conversely, let M be a Neutrosophic closed set in X and $x \in X - M$. So $X - M$ is a Neutrosophic open set such that $x \in X - M$. Hence there exists a ƝǴŚƤ open set P such that $x \in P \subseteq gspcl_{\mathcal{N}}(P) \subseteq (X - M)$. Let $Q = X - gspcl_{\mathcal{N}}(Q)$. So Q is a ƝǴŚƤ open set which contains M and $P \cap Q = \phi$. Hence X is a ƝǴŚƤ regular space.

Theorem 3.1.6: Let $(X, \tau_{\mathcal{N}})$ and $(Y, \sigma_{\mathcal{N}})$ be a Neutrosophic topological space and $(Y, \sigma_{\mathcal{N}})$ is a regular. If $\varphi_{\mathcal{N}} : (X, \tau_{\mathcal{N}}) \to (Y, \sigma_{\mathcal{N}})$ is Neutrosophic closed ǴŚƤ irresolute and one to one then X is a ƝǴŚƤ regular space.

Proof: Let M be a closed set in X, $x \notin M$. Since $\varphi_{\mathcal{N}}$ is closed mapping, then $\varphi_{\mathcal{N}}(M)$ is closed set in $(Y, \sigma_{\mathcal{N}})$, $\varphi_{\mathcal{N}}(x) = y \notin \varphi_{\mathcal{N}}(M)$. But $(Y, \sigma_{\mathcal{N}})$ is ƝǴŚƤ regular space then there are two Neutrosophic open sets P and Q in $(Y, \sigma_{\mathcal{N}})$ such that $\varphi_{\mathcal{N}}(M) \subseteq Q$, $y \in P$, $P \cap Q = \phi$. Since $\varphi_{\mathcal{N}}$ is ƝǴŚƤ-irresolute mapping and one to one so ${\varphi_{\mathcal{N}}}^{-1}(P), {\varphi_{\mathcal{N}}}^{-1}(Q)$ are two Neutrosophic open sets in X and $x \in {\varphi_{\mathcal{N}}}^{-1}(P)$, $M \in {\varphi_{\mathcal{N}}}^{-1}(Q)$, ${\varphi_{\mathcal{N}}}^{-1}(P) \in {\varphi_{\mathcal{N}}}^{-1}(Q) = \phi$. Hence X is ƝǴŚƤ regular space.

Theorem 3.1.7: A Neutrosophic topological space $(X, \tau_{\mathcal{N}})$ is a (ƝǴŚƤ, ƝǴŚ) regular space if and only if given ƝǴŚƤ open set P with $x \in P$, there exists ƝǴŚ open sets Q such that $x \in Q \subseteq scl_{\mathcal{N}}(Q) \subseteq P$.

Proof: Let Ƥ be a ƝǴŚƤ open set, $x \in Ƥ$. So $X - Ƥ$ is a ƝǴŚƤ closed set such $x \notin X - Ƥ$. Since $(X, \tau_{\mathcal{N}})$ is (ƝǴŚƤ, ƝǴŚ) regular space then there exist ƝǴŚ open sets Q_1 and Q_2 such that $Q_1 \cap Q_2 = \phi$, $X - Ƥ \subseteq Q_2$, $x \in Q_1$. Since $Q_1 \cap Q_2 = \phi$, we have $scl_{\mathcal{N}}(Q_1) \subseteq scl_{\mathcal{N}}(X - Q_2) = X - Q_2$. Since $X - Ƥ \subseteq Q_2$ we have $X - Q_2 \subseteq Ƥ$. Hence we have $x \in Q_1 \subseteq scl_{\mathcal{N}}(Q_1) \subseteq X - Q_2 \subseteq Ƥ$. Conversely, let M be a ƝǴŚƤ closed set in $(X, \tau_{\mathcal{N}})$ and $x \in X - M$. So $X - M$ is a ƝǴŚ open set such that $x \in X - M$. Hence there exists a ƝǴŚ open set Ƥ such that $x \in Ƥ \subseteq scl_{\mathcal{N}}(Q) \subseteq X - M$. Let $Q = X - gspcl_{\mathcal{N}}(Q)$. So Q is a ƝǴŚ open set which contains M and $Ƥ \cap Q = \phi$. Hence X is a (ƝǴŚƤ, ƝǴŚ) regular space.

3.2 ƝŚƤ, ƝǴŚƤ normal spaces

In this section, we delve into the fundamental concepts and properties of generalized semi pre normal space in Neutrosophic topology, exploring their significance in Neutrosophic topology.

Definition 3.2.1: A Neutrosophic topological space $(X, \tau_{\mathcal{N}})$ is said to be ƝǴŚƤ normal if for any pair of disjoint ƝǴŚƤ closed sets Ą and Ɓ, there exist disjoint Neutrosophic open sets Ʊ and Ṿ such that $Ą \subset Ʊ, Ɓ \subset Ṿ$.
Since every ƝǴ-closed set is ƝǴŚƤ-closed set so every ƝǴŚƤ normal space is ƝǴ normal space.

Theorem 3.2.2: A Neutrosophic topological space $(X, \tau_{\mathcal{N}})$ is a ƝǴŚƤ normal space if and only if any disjoint ƝǴŚƤ closed sets P and Q of $(X, \tau_{\mathcal{N}})$, there exist Neutrosophic open sets M and N of $(X, \tau_{\mathcal{N}})$ such that $P \subset M, Q \subset N$ and $cl_{\mathcal{N}}(M) \cap cl_{\mathcal{N}}(N) = \varphi$.

Proof: Necessity: Let P and Q be any disjoint $\mathcal{N}G\acute{S}\mathcal{P}$-closed sets of $(X,\tau_{\mathcal{N}})$. There exist Neutrosophic open sets M_0 and N of $(X,\tau_{\mathcal{N}})$ such that $P \subset M_0$, $Q \subset N$ and $M_0 \cap N = \varphi$ hence $M_0 \cap cl_{\mathcal{N}}(N) = \varphi$. Since $(X,\tau_{\mathcal{N}})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ normal there exist Neutrosophic open sets G and H of $(X,\tau_{\mathcal{N}})$ such that $P \subset G$, $cl_{\mathcal{N}}(N) \subset H$ and $G \cap H = \varphi$, hence $cl_{\mathcal{N}}(G) \cap H = \varphi$. Now put $M = M_0 \cap G$, then M and N are Neutrosophic open sets of $(X,\tau_{\mathcal{N}})$ such that $P \subset M, Q \subset N$ and $cl_{\mathcal{N}}(M) \cap cl_{\mathcal{N}}(N) = \varphi$.
Sufficiency: Obvious.

Theorem 3.2.3: A Neutrosophic topological space $(X,\tau_{\mathcal{N}})$ is said to be a $\mathcal{N}G\acute{S}\mathcal{P}$ normal space if and only for every Neutrosophi closed set F and for every Neutrosophic open set G contain F there exist $\mathcal{N}G\acute{S}\mathcal{P}$ open set M such that $F \subset M \subset gspcl_{\mathcal{N}}(M) \subset G$.

Proof: Let F be a Neutrosophic closed set in $(X,\tau_{\mathcal{N}})$ and G be a Neutrosophic open set in $(X,\tau_{\mathcal{N}})$ such that $F \subset M$, $X - G$ is a Neutrosophic closed set and $(X - G) \cap F = \varphi$. Since $(X,\tau_{\mathcal{N}})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ normal space then there exist open sets M and N of $(X,\tau_{\mathcal{N}})$ such that $M \cap N = \varphi$, $X - G \subset N$ and $F \subset M$, $M \subset (X - N)$. Since every Neutrosophic open set is $\mathcal{N}G\acute{S}\mathcal{P}$ open set and hence M and N are $\mathcal{N}G\acute{S}\mathcal{P}$ open sets of $(X,\tau_{\mathcal{N}})$ such that $gspcl_{\mathcal{N}}(M) \subset gspcl_{\mathcal{N}}(X - N) = X - N$. Hence $F \subset M \subset gspcl_{\mathcal{N}}(N) \subset (X - N) \subset G$.

Theorem 3.2.4: If $\varphi_{\mathcal{N}} : (X,\tau_{\mathcal{N}}) \rightarrow (Y,\sigma_{\mathcal{N}})$ is a open $\mathcal{N}G\acute{S}\mathcal{P}$-irresolute bijection and $(X,\tau_{\mathcal{N}})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ normal, then $(Y,\sigma_{\mathcal{N}})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ normal.

Proof: Let P and Q be any disjoint $\mathcal{N}G\acute{S}\mathcal{P}$ closed sets of $(Y,\sigma_{\mathcal{N}})$. Since $\varphi_{\mathcal{N}}$ is $\mathcal{N}G\acute{S}\mathcal{P}$-irresolute, ${\varphi_{\mathcal{N}}}^{-1}(P)$ and ${\varphi_{\mathcal{N}}}^{-1}(Q)$ are disjoint $\mathcal{N}G\acute{S}\mathcal{P}$ closed sets $(X,\tau_{\mathcal{N}})$. Since $(X,\tau_{\mathcal{N}})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ normal then there exists disjoint Neutrosophic open sets M and N such that ${\varphi_{\mathcal{N}}}^{-1}(P) \subset M$ and ${\varphi_{\mathcal{N}}}^{-1}(Q) \subset N$. Since $\varphi_{\mathcal{N}}$ is Neutrosophic open and bijectivity, we obtain $P \subset \varphi_{\mathcal{N}}(M)$, $Q \subset \varphi_{\mathcal{N}}(N)$, $\varphi_{\mathcal{N}}(M) \cap \varphi_{\mathcal{N}}(N) = \varphi$ and also $\varphi_{\mathcal{N}}(M)$ and $\varphi_{\mathcal{N}}(N)$ are Neutrosophic open sets of $(Y,\sigma_{\mathcal{N}})$. This shows that $(Y,\sigma_{\mathcal{N}})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ normal.

Theorem 3.2.5: The following properties are equivalent for a space $(X,\tau_{\mathcal{N}})$.

(i) (i). $(X,\tau_{\mathcal{N}})$ is $(\mathcal{N}\acute{S}\mathcal{P}, \mathcal{N}G\acute{S}\mathcal{P})$- normal

(ii) (ii). For any pair of disjoint Neutrosophic semi pre closed sets P and Q of $(X,\tau_{\mathcal{N}})$, there exist disjoint $\mathcal{N}G\acute{S}\mathcal{P}$ open sets M and N such that $P \subset M$ and $Q \subset N$.

(iii) (iii). For any Neutrosophic semi pre closed set P and Neutrosophic semi pre open set N containing P, there exists $\mathcal{N}G\acute{S}\mathcal{P}$ open set M such that $P \subset M \subset spcl_{\mathcal{N}}(M) \subset N$.

Proof: (i)⇒(ii) This proof is obvious since every $\mathcal{N}\acute{S}\mathcal{P}$ open set is $\mathcal{N}G\acute{S}\mathcal{P}$ open set.

(ii)⇒(iii) Let P be any $\mathcal{N}\acute{S}\mathcal{P}$ closed set and N be a $\mathcal{N}\acute{S}\mathcal{P}$ open set containing P. Since P and X-N are disjoint $\mathcal{N}\acute{S}\mathcal{P}$ closed set of $(X,\tau_{\mathcal{N}})$, since P and X-N are disjoint $\mathcal{N}\acute{S}\mathcal{P}$ closed sets of $(X,\tau_{\mathcal{N}})$, then there exist $\mathcal{N}G\acute{S}\mathcal{P}$ open sets M, W of $(X,\tau_{\mathcal{N}})$ such that $P \subset M$, $X - N \subset W$ and $M \cap N = \varphi$,since $M \cap spint_{\mathcal{N}}(W) = \varphi$. We have $spcl_{\mathcal{N}}(M) \cap spint_{\mathcal{N}}(W) = \varphi$ and hence $spcl_{\mathcal{N}}(M) \subset X - spint_{\mathcal{N}}(W) \subset N$. Therefore, we obtain $P \subset M \subset spcl(M) \subset N$.

(iii)⇒(i) Let P and Q be any disjoint $\mathcal{N}\acute{S}\mathcal{P}$ closed sets of $(X,\tau_{\mathcal{N}})$. Since X-Q is a $\mathcal{N}\acute{S}\mathcal{P}$ open set containing P, there exist a $\mathcal{N}G\acute{S}\mathcal{P}$ open set G, such that $P \subset G \subset spcl_{\mathcal{N}}(G) \subset X - Q$, we have $P \subset spcl_{\mathcal{N}}(G)$. Put $M = spint_{\mathcal{N}}(G)$ and $N = X - spcl_{\mathcal{N}}(G)$. Then M and N are disjoint $\mathcal{N}\acute{S}\mathcal{P}$ open sets and hence are disjoint $\mathcal{N}G\acute{S}\mathcal{P}$ open sets such that $P \subset M$ and $Q \subset N$. Therefore $(X,\tau_{\mathcal{N}})$ is $(\mathcal{N}\acute{S}\mathcal{P}, \mathcal{N}G\acute{S}\mathcal{P})$- normal.

Definition: 3.2.6 A function $\varphi_{\mathcal{N}} : (X,\tau_{\mathcal{N}}) \rightarrow (Y,\sigma_{\mathcal{N}})$ is called Neutrosophic pre generalized semi pre closed (brifly, $\mathcal{N}\mathcal{P}$ $G\acute{S}\mathcal{P}$ closed) if for each Neutrosophic semi pre closed set $\mathcal{D}$ of $(X,\tau_{\mathcal{N}})$, $\varphi_{\mathcal{N}}(\mathcal{D})$ is $\mathcal{N}G\acute{S}\mathcal{P}$ closed set in $(Y,\sigma_{\mathcal{N}})$.

Theorem 3.2.7: A surjective function $\varphi_{\mathcal{N}} : (X,\tau_{\mathcal{N}}) \rightarrow (Y,\sigma_{\mathcal{N}})$ is a $\mathcal{N}\mathcal{P}$ $G\acute{S}\mathcal{P}$-closed if and only if for each subset $\mathcal{D}$ of $(Y,\sigma_{\mathcal{N}})$, and $\mathcal{N}\acute{S}\mathcal{P}$ open set M of $(X,\tau_{\mathcal{N}})$ containing ${\varphi_{\mathcal{N}}}^{-1}(\mathcal{D})$, there exists a $\mathcal{N}G\acute{S}\mathcal{P}$ open set N of $(Y,\sigma_{\mathcal{N}})$ such that $\mathcal{D} \subset N$ and ${\varphi_{\mathcal{N}}}^{-1}(N) \subset M$.

Proof: Necessity: Suppose that $\varphi_{\mathcal{N}}$ is $\mathcal{N}\mathcal{P}$ $G\acute{S}\mathcal{P}$ closed. Let $\mathcal{D}$ be any subset of $(Y,\sigma_{\mathcal{N}})$ and M be $\mathcal{N}\acute{S}\mathcal{P}$ open set of $(X,\tau_{\mathcal{N}})$ containing ${\varphi_{\mathcal{N}}}^{-1}(\mathcal{D})$. Put $N = Y - \varphi_{\mathcal{N}}(X - M)$. Then N is $\mathcal{N}G\acute{S}\mathcal{P}$ open in $(Y,\sigma_{\mathcal{N}})$, $\mathcal{D} \subset N$ and ${\varphi_{\mathcal{N}}}^{-1}(N) \subset M$.
Sufficiency: Let W be any $\mathcal{N}\acute{S}\mathcal{P}$ closed set of $(X,\tau_{\mathcal{N}})$. Put $\mathcal{D} = Y - \varphi_{\mathcal{N}}(W)$, then we have ${\varphi_{\mathcal{N}}}^{-1}(\mathcal{D}) \subset X - W$ and X-W is $\mathcal{N}\acute{S}\mathcal{P}$ open in $(X,\tau_{\mathcal{N}})$. There exists a $\mathcal{N}G\acute{S}\mathcal{P}$ open set N of $(Y,\sigma_{\mathcal{N}})$ such that $\mathcal{D} = Y - \varphi_{\mathcal{N}}(W)) \subset N$ and ${\varphi_{\mathcal{N}}}^{-1}(N) \subset X - W$. Therefore, we obtain $\varphi_{\mathcal{N}}(W) = Y - N$ and hence $\varphi_{\mathcal{N}}(W)$ is $\mathcal{N}\mathcal{P}$ $G\acute{S}\mathcal{P}$ closed in $(Y,\sigma_{\mathcal{N}})$. This show that $\varphi_{\mathcal{N}}$ is $\mathcal{N}\mathcal{P}$ $G\acute{S}\mathcal{P}$ closed.

Theorem 3.2.8: If $\varphi_{\mathcal{N}} : (X,\tau_{\mathcal{N}}) \rightarrow (Y,\sigma_{\mathcal{N}})$ is a Neutrosophic semi pre-irresolute pre $G\acute{S}\mathcal{P}$-closed surjection and $(X,\tau_{\mathcal{N}})$ is semi pre normal. Then $(Y,\sigma_{\mathcal{N}})$ is $(\mathcal{N}\acute{S}\mathcal{P}, \mathcal{N}G\acute{S}\mathcal{P})$- normal.

Proof: Let P and Q be any disjoint $\mathcal{N}G\acute{S}\mathcal{P}$ closed sets of $(Y,\sigma_{\mathcal{N}})$. Then ${\varphi_{\mathcal{N}}}^{-1}(P)$ and ${\varphi_{\mathcal{N}}}^{-1}(Q)$ are disjoint Neutrosophic semi pre closed sets of $(X,\tau_{\mathcal{N}})$, as $\varphi_{\mathcal{N}}$ is Neutrosophic semi pre-irresolute. Since $(X,\tau_{\mathcal{N}})$ is Neutrosophic semi pre normal exist disjoint Neutrosophic semi pre open sets M and N of $(X,\tau_{\mathcal{N}})$ such that ${\varphi_{\mathcal{N}}}^{-1}(P) \subset M$ and ${\varphi_{\mathcal{N}}}^{-1}(Q) \subset N$. Since $\varphi_{\mathcal{N}}$ is Neutrosophic pre $G\acute{S}\mathcal{P}$-closed. By theorem 3.2.7 there exists $\mathcal{N}G\acute{S}\mathcal{P}$ open sets G and H such that $P \subset G$, $Q \subset H$, ${\varphi_{\mathcal{N}}}^{-1}(G) \subset M$ and ${\varphi_{\mathcal{N}}}^{-1}(H) \subset N$. Since M and N are disjoint, we have $G \cap H = \varphi$. This show that $(Y,\sigma_{\mathcal{N}})$ is $(\mathcal{N}\acute{S}\mathcal{P}, \mathcal{N}G\acute{S}\mathcal{P})$- normal.

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