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Nonlinear Programming Techniques in Optimization: An exhaustive comparative study understanding Quasi – Newton and Gauss – Newton methods

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Abstract

Non – linear Programming Applications are becoming increasingly important as managers and operations researchers become more sophisticated in implementing decision – oriented mathematical models, as well as computer routines capable of solving large – scale nonlinear problems become more widely available. By means of this paper, we investigate two of the very widely and varied NLP techniques, namely the Gauss – Newton method and the Quasi – Newton method. When computation or iteration is expensive, Quasi –Newton methods are an effective method for function optimization. Even if their precise approaches differ, when the issues are complicated, they can all determine the optimum more quickly and effectively than Newton's Method. Gauss – Newton can be used to locate a single point or, as it is most frequently use, to evaluate how well a theoretical model matches a collection of experimental data points. We get the most accurate estimates of the unknown variables in a theoretical model by solving the system of nonlinear equations. In this review we present an overview of the methods mentioned earlier, discuss the scope of them, and advocate a comparison between the two.

1. Introduction

We have been taught how to solve problems using various algebraic techniques in elementary school. Some of the basic tactics which were taught to us were replacement and elimination procedures. In linear algebra, row reduction is one of the alternatives to solve systems of linear equations. No analytical methods are present which can be helpful for solving nonlinear systems of practical importance, such as engineering design optimization, solving equilibrium models, which are treated as complicated ODEs and PDEs. Many additional challenges for analytical techniques are presented to us by these non – linear situations. Iterative algorithms must be developed when using a single value decomposition method, initial estimates for solutions are improved until the function's value at such estimations converges to something. This increases the numerical mistakes in the context of nonlinear equations, where it is not practical to know how many possible solutions the system has. Additionally, only locally unique solutions may be successfully found using numerical approaches (explained later). The open methodology known as Newton's method (comprising of a single initial condition), is used to solve systems of equations of the type. There are many iterative techniques, including Bisection and false position method, but these call for two basic guesses in order to achieve the estimations. In this paper, we are giving a brief introduction and comparison between two of the most widely used methods of NLP: the Quasi – Newton method and Gauss – Newton method. In quasi – Newton techniques, we only estimate the Hessian using a positive – definite matrix B, which is updated from iteration to iteration using data derived from prior stages, rather than computing the actual Hessian. We can see right away that this technique would result in a significantly less expensive algorithm than Newton's method since we are mostly using already computed numbers rather than calculating a significant number of new quantities at each iteration. A common iterative technique for resolving nonlinear least squares issues is the Gauss – Newton algorithm. Issues with the assimilation of enormous diverging data that arise in weather and marine projections is particularly well suited to it. The approach consists of a series of "inner" direct or iterative processes that approximate the nonlinear issue using linear least squares in each step. The technique is intriguing because, unlike Newton's method and its adaptations, it does not necessitate the evaluation of descendant derivatives in the Matrix of the objective function.

2. Method Description

2.1 Quasi – Newton method

2.1.1 Overview

In Nonlinear Programming, Quasi – Newton Methods (QNMs) are a family of optimization techniques that are typically utilized when complete Newton's Methods are either too time – consuming or challenging to use. These techniques are more explicitly utilized to locate the global minimum of a twice – differentiable function f(x). For large – scale and intricate non – linear situations, Quasi – Newton Methods provide clear benefits over the complete Newton's Method. However, depending on the specific Quasi – Newton Method utilized and the issue being addressed, these approaches may have various shortcomings. Despite this, except for straightforward issues, Quasi – Newton Methods are often worthwhile to use.
Newton's approach has drawbacks that Quasi – Newton methods avoid. Newton's law fails if H (k) is not positive definite, second derivatives are required, and a linear system must be solved at each iteration. Learn about limited-memory quasi-Newton methods and more recent Quasi – Newton techniques. Overcome Newton's computational pitfalls preserve quick local convergence.

2.1.2 Description

Due to the rarity of work, focusing on determining the square matrix of second – order partial derivatives of a scalar – valued function numerically using a function that is used to control the location is of utmost importance. However, the expense of computing the same is relatively high., hence efforts were undertaken to develop a more efficient technique to calculate the Hessian. Broyden (1969) provided a crucial idea that led to the development of the present quasi-Newton techniques: compute the new Hessian using data from the current iteration.
Let

$$s_k = x_{m+1} - x_m = \alpha_m \delta_m$$

be the parameter modification in the present cycle, and

$$\eta_m = g_{m+1} - g_{mm}$$

be the slopes changing. Then, in the subsequent iteration, a natural estimate of the functional determinants the answer to the set of affine equations would be H_{m+1}.

$$H_{m+1}\delta_m = eta_m$$

that is, H_{m+1} is the proportion of the gradient's change to the parameters' change. The Quasi – Newton condition refers to this. This set of equations has several solutions. A second update was the approach that Broyden proposed.

$$H_{m+1} = H_m + uv^t$$

Other types of secant updates have been produced via further research, with the DFP and the BFGS being the most significant. The best performing approach is typically thought to be the BFGS:

$$H_{m+1} = H_m + \frac{n_m\, n_m^t}{n_m^t s_m} - \frac{H_m s_m\, s_m^t H_m}{s_m^t H_m s_m}$$

$$= H_m + \frac{n_m\, n_m^t}{n_m^t s_m} - \frac{g_m\, g_m^t}{\delta_m^t g_m}$$

the fact that $H_m s_m = \alpha_m H_m \delta_m = \alpha_m g_m$. The GAUSS function QNEWTON employs the BFGS technique. Instead of the Hessian itself, or R where $H = R^t R$, the triangular decomposition of H is revised. H is not calculated anywhere in the iterations of QNEWTON. CHOLSOL is used to calculate the direction δ_m as a solution to

$$R^t{}_m R_m \delta_m = g_m$$

where R_m and g_m are its arguments, and R_m is the decomposition devised by H_m. Then R_{m+1} is computed as an evolution and devolution to R_m using the hypergeometric function Cholesky Update and Cholesky Downdate.

2.2 Gauss – Newton

2.2.1 Overview

A common iterative technique for resolving nonlinear least squares issues is the Gauss – Newton algorithm. Particularly well adapted to it is the handling of outsized dynamic data assimilation problems that arise in climatic and marine trend analysis. The approach consists of a series of "inner" direct or iterative processes that approximate the nonlinear issue using linear least squares in each step. The method is intriguing as it doesn't demand the evaluation of the derivative of the first derivative of the given function in the Matrix of the objective function, unlike Newton's method and its variations. In order to lower calculation costs and provide real-time solutions, several approximations are used in place of the precise Gauss – Newton technique in meteorological forecasting since it is too expensive to use. Here, we investigate how the Gauss – Newton approximation approach, which is frequently employed in data assimilation, affects its convergence. We look at Gauss – Newton techniques that "truncate" the inner linear least squares issue. In addition to deriving rates of convergence for the iterations, we provide criteria guaranteeing that the shortened Gauss – Newton technique will converge.

2.2.2 Truncated Gauss – Newton method

The simultaneous least squares problem is too pricey theoretically to answer accurately in the first run of the method, which poses a significant challenge for the employment in wide range of uses like data gathering of the gradient descent method.

The first foundation for regular matrices computations frequently has dimensions that are large even in factored form, the design values are incompatible with central storage space. Therefore, approximations inside the Gauss – Newton process are required in order to effectively address the entire complex problem in the anticipated period. There are two typical approximation kinds. First, an "inner" iteration approach that is trimmed before solving the linearized least squares results in a rough solution.

We answer the standard problems at each step k of the Marquardt method.

$$J(x_k)^T J(x_k)_s = - J(x_k)^T f(x_k)$$

when solving the rectilinear Linear regression equation iteratively. Intuitively, If the function f is non – linear and x_k is distant from x^*, it is not feasible to solve it with high precision. When the relative residual satisfies, the iterative procedure naturally comes to an end

$$|| J(x_k)^T J(x_k)s_k + J(x_k)^T f(x_k)||_2/|| J(x_k)^T f(x_k)||_2 \leq \beta_k.$$

Here, the variable stands for the most recent estimate of the answer, and k is a defined tolerance. Due to this, the Truncated Gauss – Newton method is defined as follows.

Truncated Gauss – Newton Algorithm (TGN)

Step 0: Choose an initial $x_0 \in R^n$

Step 1: Repeat until convergence:

Step 1.1: Find s_k such that

$$(J(x_k)^T J(x_k))s_k = - J(x_k)^T f(x_k) + r_k,$$

$$with \ ||r_k||_2 \leq \beta_k \ ||J(x_k)^T f(x_k)\ ||_2$$

Step 1.2: Alter the previous action

The tolerances variable where, $k = 0, 1, 2\ldots$ must be selected to guarantee overall convergence of the method to the nonlinear OLS's ideal x star.

Since they are beyond the purview of this paper's investigation, conditions ensuring the convergence approach are not discussed.

3. Comparison between Quasi – Newton and Gauss – Newton methods

Smoothness – constrained least – squares is a common method for the rotation of impedance data points in two dimensions and three dimensions. The Gauss – Newton method, which dynamically adjusts the Mathematical model of derivative for each iteration, is widely used to solve the minimum squares problem.

The computer time has also been decreased by using the Quasi – Newton approach. In this approach, the first iteration uses the Hessian matrix, and the matrices for following runs computed using an ever – dynamic mechanism. The GN approach should converge faster since it employs accurate partial derivatives. However, the Quasi – Newton approach has the potential to be much quicker than the GN algorithm for a variety of data sets. The Gauss – Newton approach greatly outperforms the Quasi – Newton method in regions with high resistivity contrasts.

4. Conclusion

After an exhaustive rumination we presented a study of the two extensively used methods of Optimization in Nonlinear Programming. It has been observed that both the methods discussed in this paper are effective when tested only against experimental data and when they are applied to industrially beneficial commercial applications. The true extent and further implications of these methods are still under scrutiny and further studies in the methods can give an extent of the widespread implications of these methods.

5. References

1. An Overview of Some Practical Quasi-Newton Methods for Unconstrained Optimization | Sultan Qaboos University Journal for Science [SQUJS]
2. Quasi-Newton Method - an overview | ScienceDirect Topics
3. Quasi-Newton Methods: A New Direction
4. Quasi-Newton Methods, Motivation and Theory
5. Quasi-Newton methods - optimization
6. Newton and Quasi-Newton Methods - GIAN Short Course on Optimization: Applications, Algorithms, and Computation
7. Newton's Method on a System of Nonlinear Equations
8. Lecture 13 Nonlinear Systems - Newton's Method
9. Newton and Quasi-Newton Methods
10. A comparison of the Gauss-Newton and quasi-Newton methods in resistivity imaging inversion Loke, MH; Dahlin, Torleif
11. Difference between Gauss-Newton method and quasi-Newton method for optimization - Computational Science Stack Exchange
12. (PDF) Approximate Gauss–Newton Methods for Nonlinear Least Squares Problems
13. Gauss-Newton Method - an overview | ScienceDirect Topics
14. (PDF) Approximate Gauss–Newton Methods for Nonlinear Least Squares Problems
15. 4.5.3 Variants of Newton's Method 4.5.4 Gauss Newton Approximation
16. https://philstat.org.ph/index.php/MSEA/article/view/767/424
17. https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&as_ylo=2022&as_rr=1&q=Non++%22Linear+Programming%22+and+mathematical+modelling&btnG=