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On a Mode-I Crack Configuration Analyzed Through 2-D in Generalized Thermoelasticity

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ABSTRACT

This paper is concerned with the dynamical problem of an infinite type-III which occurs a finite linear mode-I crack due to load inside the homogeneous and isotropic medium of thermoelastic space. The temperature distribution and stress leads to the crack in the boundary. The basic governing equation developed by Green and Naghdi have been solved by using integral transform and reduces to four dual integral equation by employing boundary conditions which is equivalent to Fredholm's integral equation of first kind. For numerical solution inversion of Laplace transform has been used.

Keywords: *Dynamical problem, Thermoelasticity, type-III, mode-I crack, Fredholm's integral equation, Inversion of Laplace transform etc.*

1.1 Introduction

Biot [2] investigated the coupled thermoelasticity to remove the paradox inherent in the classical theory that elastic changes have no effect on the temperature. This theory, based on firm grounds of irreversible thermo-dynamics, had been widely assuming to study the coupling effects of elastic and thermal effects. The detailed discussions and applications of Biot's theory have been discussed by some researchers, such as Chadwick [8], Carlson [5], Nowinski [1], Nowacki [6], Dhaliwal and Singh [3], Parkus [7], Boley and Wiener [4]. One of the earliest development of a second sound theory for thermoelasticity was reported by Fox [9]. The generalized theories were specially formulated to Lord and Shulman [10] developed the theory of generalized themoelasticity with one relaxation time parameter for the special case of an isotropic medium. Two thermal relaxation time were introduced in the theory elaborated by Green and Lindsay[11]. The book "thermoelasticity with finite wave speeds" by Ignaczak and Ostoja-Starzewski[12] addressed a detailed analysis of the generalized thermoelasticity theory. During the period (1986-1999), Chandrashekhariah [13, 14] and Hetnarski and Ignaczak [15] also investigated review articles.

Subsequently, Green and Naghdi [16-18] described theories of thermoelasticity in different types like GN-I, G-II and GN-III. GN-I model corresponds to the classical thermoelastic model. In GN-II model, the internal rate of production of entropy is considered to be identically zero i.e., there is no decadence of thermal energy. It admits undamped thermoelastic waves in a thermoelastic material which is referred as the theory of thermoelasticity without energy dissipation. In GN-III model, Fourier law of heat conduction is generalized in the form of equation

$$q(P,t) + [K \nabla T(P,t) + K^{*} \nabla \upsilon(P,t)] = 0,$$

where υ the thermal displacement gradient is satisfying $\dot{\upsilon} = T$ and the two material constants K and K^{*} are the thermal conductivity and the rate of thermal conductivity respectively.

The thermoelasticity theories reported by Green and Naghdi have drawn the attention of several researchers during last few years. Puri and Jordon [14], Kothari and Mukhopadhyay [42], Kovalev and Radayev [43] have investigated the harmonic plane wave propagating in thermoelastic medium of type III in a detailed way. The variational reciprocity theorems in the contexts of linear theory of thermoelasticity of type-II and type-III are developed by Mukhopadhyay and Prasad [44] and Chirita and Ciarletta [45]. Quintanilla [46] and Quintanilla and Straughan [47] have established the growth of solutions and uniqueness theorem in the contexts of both the thermoelasticity type-II and III theories. The nature of discontinuity waves propagating in type-III thermoelastic media has been reported by Quintanilla and Straughan [48].

Over the years, significant efforts are taken to study the cracks and failures in solid which has great deal in various engineering industries like aerospace, aircraft fuselage, wings, earthquake engineering, fabrication of electronic components and geophysics. Mostly, dynamical problem creates non-homogeneity of the body i.e. crack are done using the equations of coupled theories of thermoelasticity. The application of mathematical theory of homogeneous elastic solids to real substances may lead to error, unless the smallest material load involved. Considering two-dimensional crack problems constituted by a line segment was developed by Griffith [19]. In two dimension, there are three types of cracks in three different modes like Mode-I, II and III. In Mode-I, Griffith studied crack in a solid medium of the length 2a due to tensile force in the direction perpendicular to the line of the crack. In case of opening crack, Mode-1 represents a symmetric opening the displacement of the surface medium being normal to the crack region, Irwin [20]. Florence and Goodier [21] discussed the flow-induced thermal stresses in the infinite isotropic solids. Several researchers Choudhuri and Ray [22], Prasad and Aliabadi [23], Sih [24], Raveendra and Banerjee [25], Kassir and Bergman [26] have investigated crack problems in thermoelastic medium. Mallik and Kanoria [27] investigated a unified way generalized thermoelasticity problem formulation to the a penny-shaped crack analysis. Recently, Sherief and El-Maghraby [28], Prasad and Mukhopadhyay [29], and S. Kant et al.[30] have enumerated the opening mode crack problem of infinite thermoelasticitic medium in the context of Lord-Shulman's [11] and Green-Naghdi's [18] theory respectively. Furthermore, Lotfy [31] studied on plane waves for Mode-I crack problem in generalized thermoelasticity.

1.2 Formulation of the problem:

In this present work, we construct a dynamical problem for an infinite elastic medium $-\infty < x < \infty, -\infty < y < \infty$, with a crack on the $|x| \leq a, y = 0$. The crack region is subjected to the temperature and normal stress distributions. The equations of motion are

$$(\lambda + \mu)\frac{\partial \Upsilon}{\partial x} + \mu\nabla^2 u - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial^2 t} \quad (1)$$

$$(\lambda + \mu)\frac{\partial \Upsilon}{\partial y} + \mu\nabla^2 v - \gamma\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial^2 t} \quad (2)$$

The heat conduction equation, (Green and Naghdi) type-III [18]

$$\left(K^* + K\frac{\partial}{\partial t}\right)\nabla^2 T = \rho c_v \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 \Upsilon}{\partial t^2} \quad (3)$$

The following constitutive relations supplement the above mention equations

$$\sigma_{xx} = 2\mu u_x + \lambda\Upsilon - \gamma(T - T_0) \quad (4)$$

$$\sigma_{y\ y} = 2\mu v_y + \lambda\Upsilon - \gamma(T - T_0) \quad (5)$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad (6)$$

where λ and μ are Lame's elastic constants, α_t is the coefficient of linear thermal expansion, $\gamma = (3\lambda + 2\mu)\alpha_t$, ρ is the density of the material, T is the absolute temperature, T_0 is the reference temperature, K is the thermal conductivity, K^* is the rate of thermal conductivity, c_v is the specific heat at constant strain or volume, u is horizontal displacement along x direction, v is the vertical displacement along y direction, σ_{ij} are the stress components, ∇^2 is the Laplacian operator, t is the time and Υ is the cubical dilatation given by :

$$\Upsilon = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (7)$$

For the sake of simplicity, we use the following non-dimensional quantities/ variables as Sherief and El-Maghraby [28]

$$x' = c_1\eta x, y' = c_1\eta y, u' = c_1\eta u, v' = c_1\eta v, t' = c_1^2\eta t, \sigma_{ij}' = \frac{\sigma_{ij}}{\mu}, \theta = \frac{T-T_0}{T_0}$$

With $= \frac{\rho c_v}{K}, c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$, where c_1 is the speed of propagation of longitudinal elastic waves. Now from above mention non-dimensional quantities, then Eqs. (1)-(6) reduce to the following forms

$$(\alpha^2 - 1)\frac{\partial \Upsilon}{\partial x} + \nabla^2 u = b_1\frac{\partial \theta}{\partial x} + \alpha^2\frac{\partial^2 u}{\partial t^2} \quad (8)$$

$$(\alpha^2 - 1)\frac{\partial \Upsilon}{\partial y} + \nabla^2 v = b_1\frac{\partial \theta}{\partial y} + \alpha^2\frac{\partial^2 v}{\partial t^2} \quad (9)$$

$$\left(a_0 + \frac{\partial}{\partial t}\right)\nabla^2\theta = \frac{\partial^2}{\partial t^2}(\theta + b_2\Upsilon) \quad (10)$$

$$\sigma_{xx} = 2u_x + (\alpha^2 - 2)\Upsilon - b_1\theta \quad (11)$$

$$\sigma_{yy} = 2v_x + (\alpha^2 - 2)\Upsilon - b_1\theta \quad (12)$$

$$\sigma_{xy} = u_y + v_x \quad (13)$$

where, $a_0 = \frac{K^*}{Kc_1^2\eta}, b_1 = \frac{\gamma T_0}{\mu},\ b_2 = \frac{\gamma}{K\eta}, \alpha^2 = \frac{\lambda+2\mu}{\mu}$.

Using Eq.(7), eliminating v and u from Eqs.(8) and (9), we finally obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\Upsilon = c\nabla^2\theta, \quad (14)$$

where, $c = \frac{b_1}{\alpha^2}$.

1.2 Solution in the Laplace transform domain:

Applying the Laplace transform to both sides of Eqs. (7)-(10) and (14), we obtain

$$\breve{\Upsilon} = \frac{\partial \breve{u}}{\partial x} + \frac{\partial \breve{v}}{\partial y} \quad (15)$$

$$b_1\frac{\partial \breve{\theta}}{\partial x} - (\alpha^2 - 1)\frac{\partial \breve{\Upsilon}}{\partial x} = (\nabla^2 - \alpha^2 p^2)\breve{u} \quad (16)$$

$$b_1\frac{\partial \breve{\theta}}{\partial y} - (\alpha^2 - 1)\frac{\partial \breve{\Upsilon}}{\partial y} = (\nabla^2 - \alpha^2 p^2)\breve{v} \quad (17)$$

$$[(a_0 + p)\nabla^2 - p^2]\breve{\theta} = p^2 b_2\breve{\Upsilon} \quad (18)$$

$$(\nabla^2 - p^2)\breve{\Upsilon} = c\nabla^2\breve{\theta} \quad (19)$$

Now eliminating $\breve{\Upsilon}$ from Eqs.(18) and (19), we obtain the differential equation satisfied by $\breve{\theta}$ as

$$\left((\nabla^2 - m_1^2)\right)(\nabla^2 - m_2^2)\breve{\theta} = 0 \quad (20)$$

where m_1^2 and m_2^2 are thr roots with real parts of the following bi- quadratic equation

$$(a_0 + p)m^4 - p^2(1 + a_0 + p + \epsilon)m^2 + p^4 = 0 \quad (21)$$

where, $\epsilon = cb_2$.

We can write $\breve{\theta}$, the solution of Eq. (20), in the following standard form

$$\breve{\theta} = \breve{\theta}_1 + \breve{\theta}_2$$

Where $\breve{\theta}_i$ is the solution of the equation given as

$$(\nabla^2 - m_i^2)\breve{\theta}_i = 0,\ i = 1,2 \tag{22}$$

1.3 Solution in the Fourier transform domain:

Applying the exponentional Fourier transform to both sides of Eq. (22), we get

$$[D^2 - (m_i^2 + q^2)]\breve{\theta}_i^* = 0, \tag{23}$$

where, $D = \frac{\partial}{\partial y}$.

The solution of Eq. (23) is bounded at infinity can be obtained in the following form

$$\breve{\theta}_i^* = G_i(q,p)(m_i^2 - p^2)e^{-q_i|y|} \tag{24a}$$

Where, $q_i^2 = q^2 + m_i^2$ and $G_i(q,p)$ is the parameter which depends on of q and p for $i = 1,2$. For the case of symmetry, we take the $y > 0$. Then above Eq. (24a) can be expressed as :

$$\breve{\theta}_i^* = G_i(q,p)(m_i^2 - p^2)e^{-q_i y} \tag{24b}$$

In a similar way, now eliminating $\breve{\theta}$ from Eqs.(18) and (19), we get $\breve{\Upsilon}^* = \breve{\Upsilon}_1^* + \breve{\Upsilon}_2^*$,

$$\breve{\Upsilon}_i^* = G_i'(q,p)e^{-q_i y}, \qquad i = 1,2. \tag{25}$$

$G_i'(q,p), i = 1,2$ are also which depend only on q and p.

Therefore, substituting from Eqs. (24b) and (25) into Eq. (19), we get the equation which relates the $G_i(q,p)$ and $G_i'(q,p)$ for $i = 1,2$ in the following expression;

$$G_i'(q,p) = cm_i^2 G_i(q,p) \tag{26}$$

Therefore, putting Eq. (26) into Eq. (25), we find

$$\breve{\Upsilon}_i^* = cG_i(q,p)m_i^2 e^{-q_i y} \tag{27}$$

Now, we use exponential Fourier transform to Eqs. (16) and (17) to obtain

$$(D^2 - q^2 - \alpha^2 p^2)\breve{u}^* = qb_1\breve{\theta}^* - (\alpha^2 - 1)iq\breve{\Upsilon}_i^* \tag{28}$$

$$(D^2 - q^2 - \alpha^2 p^2)\breve{v}^* = b_1 D\breve{\theta}^* - (\alpha^2 - 1)D\breve{\Upsilon}_i^* \tag{29}$$

Taking Eqs. (24) and (27), Eqs. (28) and (29) reduces to

$$(D^2 - q^2 - \alpha^2 p^2)\breve{u}^* = iqc\sum_{i=1}^{2}(m_i^2 - \alpha^2 p^2)\, G_i(q,p)e^{-q_i y} \tag{30}$$

$$(D^2 - q^2 - \alpha^2 p^2)\breve{v}^* = -c\sum_{i=1}^{2}(m_i^2 - \alpha^2 p^2)\, G_i(q,p)q_i e^{-q_i y} \tag{31}$$

The solution $\breve{u}^*$ of equation (30) has the form

$$\breve{u}^* = iqc\left(\sum_{i=1}^{2} G_i e^{-q_i y} + H_1 e^{-\delta y}\right), \tag{32}$$

where, $\delta = \sqrt{q^2 + \alpha^2 p^2}$ and $H_1 = H_1(q,p)$ is a parameter depending on q and p.

Applying the exponential Fourier transform w.r.t. x to both sides of above Eq.(15),

We obtain

$$\frac{\partial \breve{v}^*}{\partial y} = \breve{\Upsilon}^* - iq\breve{u}^* \tag{33}$$

With help of Eqs.(27) and (32) along with the integration w.r.t. y Eq.(33) is rewritten as :

$$\breve{v}^* = -c\left[\sum_{i=1}^{2} G_i(q,p)q_i e^{-q_i y} + \frac{q^2 H_1(q,p)}{\delta} e^{-\delta y}\right] \tag{34}$$

Taking the Laplace transform and then exponential Fourier transforms to both sides of Eqs.

(11)-(13) and using the main results of Eqs. (24), (27), (32) and (34), we can write the stress

Components in the Laplace and Fourier transform domain in the following expression

$$\breve{\sigma}_{xx}^* = c\left[\sum_{j=1}^{2} G_j\left(\alpha^2 p^2 - 2q_j^2\right)e^{-q_j y} - 2H_1 q^2 e^{-\delta y}\right] \tag{35}$$

$$\breve{\sigma}_{yy}^* = c\left[\left(\alpha^2 p^2 + 2q^2\right)\sum_{j=1}^{2} G_j e^{-q_j y} + 2H_1 q^2 e^{-\delta y}\right] \tag{36}$$

$$\breve{\sigma}_{xy}^* = -icq\left[2\sum_{j=1}^{2} G_j q_j e^{-q_j y} + \frac{q^2+\delta^2}{\delta} H_1 e^{-\delta y}\right] \tag{37}$$

Taking the inverse Fourier transform of Eqs. (24), (27), (32) and (34) –(37), we Obtain solution in the Laplace transform domain.

$$\breve{\theta} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sum_{j=1}^{2} G_j\left(m_j^2 - p^2\right)e^{-q_j y}\right] e^{iqx} dq \tag{38}$$

$$\breve{\Upsilon} = \frac{c}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sum_{j=1}^{2} G_j m_j^2 e^{-q_j y}\right] e^{iqx} dq \tag{39}$$

$$\breve{u} = \frac{ic}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sum_{j=1}^{2} G_j e^{-q_j y} + H_1 e^{-\delta y}\right] q e^{iqx} dq \tag{40}$$

$$\breve{v} = \frac{-c}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sum_{j=1}^{2} G_j q_j e^{-q_j y} + \frac{H_1 q^2}{\delta} e^{-\delta y}\right] e^{iqx} dq \tag{41}$$

$$\breve{\sigma}_{xx} = \frac{c}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sum_{j=1}^{2} G_j\left(\alpha^2 p^2 - 2q_j^2\right)e^{-q_j y} + 2H_1 q^2 e^{-\delta y}\right] e^{iqx} dq \tag{42}$$

$$\breve{\sigma}_{yy} = \frac{c}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\left(\alpha^2 p^2 + 2q^2\right)\sum_{j=1}^{2} G_j e^{-q_j y} + 2H_1 q^2 e^{-\delta y}\right] e^{iqx} dq \tag{43}$$

$$\breve{\sigma}_{xy} = \frac{-ic}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[2\sum_{j=1}^{2} G_j q_j e^{-q_j y} + \frac{\left(q^2+\delta^2\right)}{\delta} H_1 e^{-\delta y}\right] q\, e^{iqx} dq \tag{44}$$

1.5 Boundary condition:

Now, we consider the boundary conditions for heat conduction problem at $y = 0$ as

$$\frac{\partial\theta}{\partial y} = 0, \ |x| > a \tag{45}$$

$$v = 0, \ |x| > a \tag{46}$$

$$\theta = g(x)H(t), \ |x| < a \tag{47}$$

$$\sigma_{yy} = -\sigma_1(x)H(t), \quad |x| < a \tag{48}$$

$$\sigma_{yy} = 0, \ -\infty < x < \infty \tag{49}$$

where, $H(\cdot)$ is the Heaviside unit step function.

1.6 Dual integral equation formulation:

Now, using the boundary conditions given by above Eqs. (45) and (47), Eq.(38) is rewritten as below :

$$\int_{-\infty}^{\infty}\sum_{j=1}^{2} G_j\left(m_j^2 - p^2\right)e^{iqx} dq = \frac{\sqrt{2\pi}g(x)}{p}, \ |x| < a \tag{50}$$

$$\int_{-\infty}^{\infty}\sum_{j=1}^{2} G_j q_j\left(m_j^2 - p^2\right)e^{iqx} dq = 0, \ |x| > a \tag{51}$$

and using the boundary conditions Eqs.(46), (48) and (49). Eqs. (41),(43) and (44) are written as below :

$$\int_{-\infty}^{\infty}\left[G_1q_1+G_2q_2+\frac{H_1q^2}{\delta}\right]e^{iqx}dq=0, \quad |x|>a \tag{52}$$

$$\int_{-\infty}^{\infty}[(\alpha^2p^2+2q^2)(G_1+G_2)+2q^2H_1]e^{iqx}dq=-\frac{\sqrt{2\pi}p_1(x)}{pc}, \ |x|<a \tag{53}$$

$$\int_{-\infty}^{\infty}\left[2(G_1q_1+G_2q_2)+\frac{(q^2+\delta^2)}{\delta}H_1\right]qe^{iqx}dq=0, \qquad -\infty<x<\infty \tag{54}$$

Now from Eq. (54), we obtain :

$$H_1=\frac{2\delta(G_1q_1+G_2q_2)}{q^2+\delta^2} \tag{55}$$

Then, using Eq.(55) and for the case symmetry of the problem to consider x only intervals $[0,a]$ and $[a,\infty[$ (see Ref.[28]), Eqs.(50)-(53) are written as below :

$$\sum_{i=1}^{2}(m_i^2-p^2)\int_{-\infty}^{\infty}G_i\cos(qx)\,dq=\sqrt{\frac{\pi}{2}}\frac{g(x)}{p}, \quad 0<x<a \tag{56}$$

$$\sum_{i=1}^{2}(m_i^2-p^2)\int_{-\infty}^{\infty}G_i\cos(qx)\,dq=0, \ x>a \tag{57}$$

$$\sum_{i=1}^{2}(m_i^2-p^2)\int_{0}^{\infty}\frac{G_iq_i}{\alpha^2p^2+2q^2}\cos(qx)\,dq=0, \ x>a \tag{58}$$

$$\sum_{i=1}^{2}\int_{0}^{\infty}G_i\left[\frac{(\alpha^2p^2+2q^{22})-4q^2q_i\delta}{\alpha^2p^2+2q^2}\right]\cos(qx)\,dq=-\sqrt{\frac{\pi}{2}}\frac{p_1(x)}{pc}, \ 0<x<a \tag{59}$$

Eqs.(56) –(59) from a set of four dual integral equations. From above equations, we can find the unknown parameters G_1 and G_2, and the solve these dual integral equation, we follow the (Sherief and El-Maghraby [28]), from which we assume the following

$$G_i(q,p)=\int_0^a h_i(v,p)J_0(qv)dv, \qquad x<a, \ i=1,2, \tag{60}$$

Where $J_0(qw)$ is the Bessel function of the first kind with order zero and h_i are functions of

Parameter v and p only. Now substituting the value $G_i(q,p)$ from Eq. (60) into Eq.(56),

After changing order of integration, we get the following equation

$$\sum_{i=1}^{2}(m_i^2-p^2)\int_0^a h_i(v,p)dv\int_0^{\infty}cos(qx)J_0(qv)dq=\sqrt{\frac{\pi}{2}}\frac{g(x)}{p}, 0<x<a \tag{61}$$

Eq. (61) reduces to

$$\int_0^{\infty}cos(tx)J_0(tv)dt=\begin{cases}\frac{1}{\sqrt{v^2-x^2}} & when\ x<v,\\ 0 & when\ x>v,\end{cases} \tag{62}$$

Eq. (61) reduces to

$$\sum_{i=1}^{2}(m_i^2-p^2)\int_x^{\infty}\frac{h_i(u,p)du}{\sqrt{u^2-x^2}}=\sqrt{\frac{\pi}{2}}\frac{g(x)}{p}, \quad 0<x<a$$

Multiply the above equation with $\frac{x}{\sqrt{x^2-v^2}}$ and integrate w.r.t. x from v to a. after changing the order of integration and differentiating the final equation, we obtain :

$$(m_1^2-p^2)h_1(v,p)+(m_2^2-p^2)h_2(v,p)=-\frac{A(v)}{p}, \qquad 0<x<a \tag{63}$$

where, $A(v)=\sqrt{\frac{2}{\pi}}\frac{d}{dv}\int_v^a\frac{xg(x)dx}{\sqrt{x^2-v^2}}$ (64)

Multiply both sides of above Eq. (63) by $J_0(qv)$ and integrate w.r.t. v from 0 to a,

We finally get

$$G_2 = \frac{-1}{m_2^2 - p^2}\left[\frac{J(q)}{p} + (m_1^2 - p^2)G_1\right], \quad 0 < x < a \tag{65}$$

Where, $J(q) = \int_0^a A(v) J_0(qv)dv$ (66)

For obtaining the similar relation Eq. (65) between G_1 and G_2 for the case when $x > a$, we finally obtain

$$q_i G_i(q,p) = \int_a^\infty h_i(v,p) J_0(qv)dv, \quad x > a, \; i = 1,2 \tag{67}$$

Now, using Eq.(62) into Eq. (57) and after changing the order of integration, we get

$$\sum_{i=1}^2 (m_i^2 - p^2) \int_x^\infty \frac{h_i(v,p)du}{\sqrt{u^2 - x^2}} = 0, \quad x > a$$

In similar manner follows, multiply both sides of the above relation by $\frac{x}{\sqrt{x^2 - v^2}}$ and integrate w.r.t. x from v to ∞. After changing the order of integration and with the help of Eq. (67), we finally obtain

$$G_2 = -\frac{(m_1^2 - p^2)q_1}{(m_2^2 - p^2)2} G_1, \quad x > a \tag{68}$$

using Eq. (65) into Eq. (59), we obtain

$$\int_0^\infty \frac{G_1 q_1 M(q,p)}{2q^2 + \alpha^2 p^2} cos(qx)dq = \bar{R}(x,p), \quad x < a, \tag{69}$$

Where, $M(q,p) = \frac{(m_2^2 - m_1^2)(2q^2 + \alpha^2 p^2)^2 - 4q^2\delta[q_1(m_2^2 - p^2) - q_2(m_1^2 - p^2)]}{q_1}$,

$$\bar{R}(x,p) = \sqrt{\frac{2}{\pi}} \frac{(m_2^2 - p^2)p_1(x)}{pc} + \frac{1}{p}\int_0^\infty J(q)\left[\frac{(2q^2 + \alpha^2 p^2) - 4q^2\delta q_2}{2q^2 + \alpha^2 p^2}\right] cos(qx)dq, \quad x < a \tag{70}$$

Substituting from Eq.(68) into Eq.(58), we get

$$\int_0^\infty \frac{G_1 q_1 cos(qx)}{2q^2 + \alpha^2 p^2} dq = 0, \quad x > a \tag{71}$$

We have replaced the four dual integral Eqs.(56)- (59) in the parameters G_1 and G_2 are Obtained to only two dual integral from Eqs. (69) and (71) in the parameter G_1.

1.7 **Solution of the dual integral equations:**

For solving the above mention two integral Eqs. (69) and (71), we take the substitution [28]:

$$G_1(q,p) = \frac{(2q^2 + \alpha^2 p^2)}{q_1} \Psi(q,p) \tag{72}$$

Therefore, from Eqs. (69) and (71) are reduced into the following expressions

$$\int_0^\infty M(q,p)\, \Psi(q,p)\, cos(qx)dq = \bar{R}(x,p), \; 0 < x < a \tag{73}$$

$$\int_0^\infty \Psi(q,p)\, cos(qx)dq = 0, \quad x > a \tag{74}$$

In order to mention for all values of x, we are extending the definition of the integral which is in Eq. (74) manner:

$$\int_0^\infty \Psi(q,p)\, cos(qx)dq = \begin{cases} \sqrt{2\pi}\frac{d}{dx}\left[x\int_x^a \frac{\Psi(z,p)dz}{\sqrt{z^2-x^2}}\right], & 0 < x < a \\ 0, & x > a \end{cases} \quad (75)$$

Where, $\Psi(z,p)$ is a function which has to be determined.

We see that left side of Eq.(75) is just the Fourier cosine transform of the function $\Psi(q,p)$,

Therefore, by using the Fourier transform formula [26,35,36] we obtain

$$\Psi(q,p) = \int_0^a \frac{d}{dx}\left(x\int_x^a \frac{\Psi(z,p)dz}{\sqrt{z^2-x^2}}\right) cos(qx)dx \quad (76)$$

Now using integration by parts and followed by changing the order of integration to solve above Eq. (76), we have

$$\Psi(q,p) = q\int_0^a \Psi(q,p)dz \int_0^z \frac{x\sin(qx)dx}{\sqrt{z^2-x^2}} \quad (77)$$

Using the formula [35, 36]

$$\int_0^z \frac{x\sin(qx)dx}{\sqrt{z^2-x^2}} = \frac{\pi}{2} zJ_1(qz),$$

We can write $\Psi(q,p)$ in the form

$$\Psi(q,p) = \frac{\pi q}{2}\int_0^a z\Psi(z,p)J_1(qz)dz, \quad (78)$$

Now, substituting from equation (78) into equation (73), we get

$$\int_0^a \bar{M}(z,x,p)\Psi(z,p)dz = \bar{R}(x,p), \qquad x < a \quad (79)$$

where, $\bar{M}(z,x,p) = \frac{\pi z}{2}\int_0^\infty q\mathrm{M}(z,p)J_1(qz)cos(qx)dq,$

To solve the integral equation Eq.(79) numerically, we follow the regularization method [37]. For inverting the Laplace transforms, we employ a numerical method used by Bellman et. al. [32].

1.8 Numerical results:

We have considered the copper material having the opening mode crack (Mode-I) with unit length. The material constants are taken as follows Sherief and El-Maghraby[28]:

$\alpha = 2, \alpha_t = 1.78\times10^{-5}, c_1 = 4.158\times10^3 ms^{-1}, c = 0.01, \rho = 8954 kgm^{-3},$

$\eta = 88886.73 ms^{-2}, a = 1, c_v = 383.1 Jkg^{-1}k^{-1}, \lambda = 7.76\times10^{10} Nm^{-2},$

$\mu = 3.86\times10^{10} Nm^{-2}, \theta_0 = 1K, T_0 = 293K, b_1 = 0.042.$

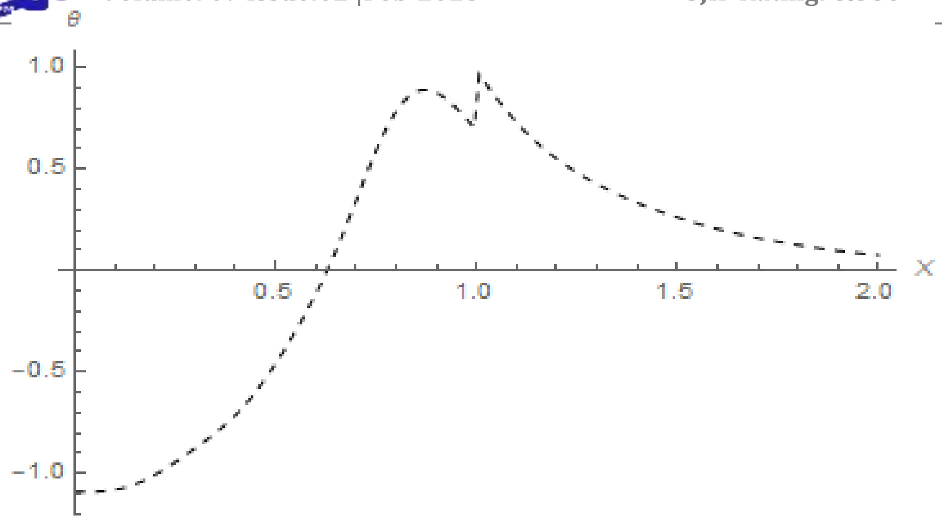


Fig 1.1 Temperature distributions at the vertical distance $y = 0.3$

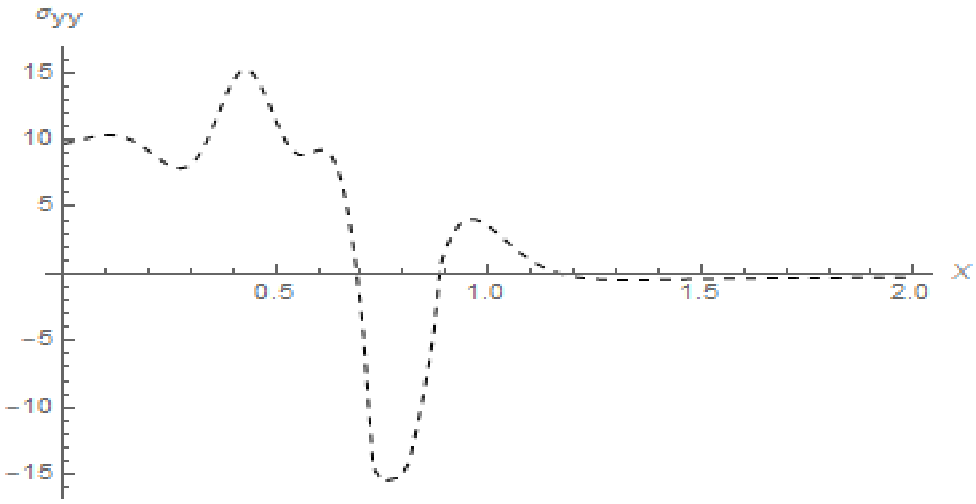


Fig 1.3 Vertical stress distributions at the vertical distance $y = 0.3$

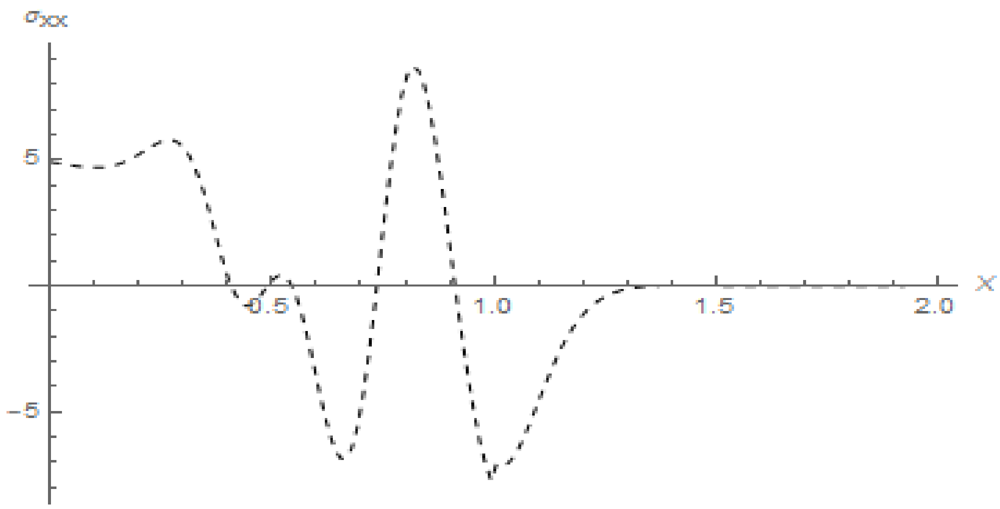


Fig 1.2 Stress distributions at the vertical distance $y = 0.3$

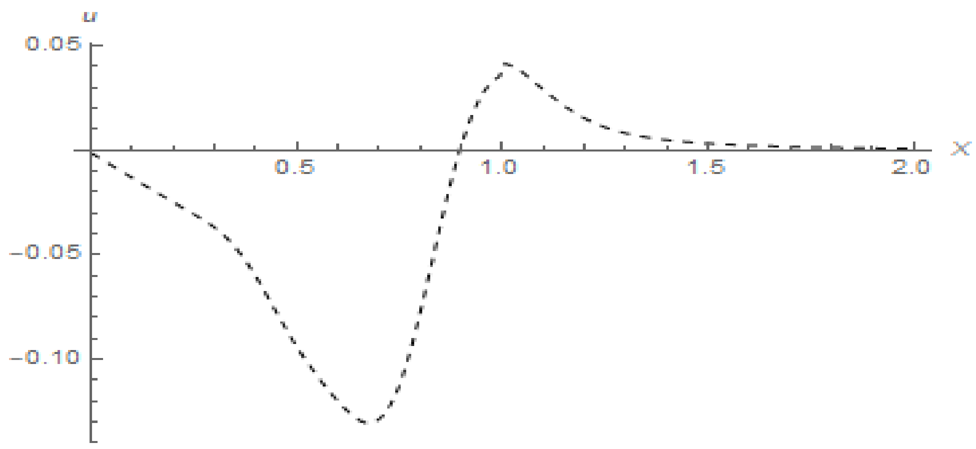
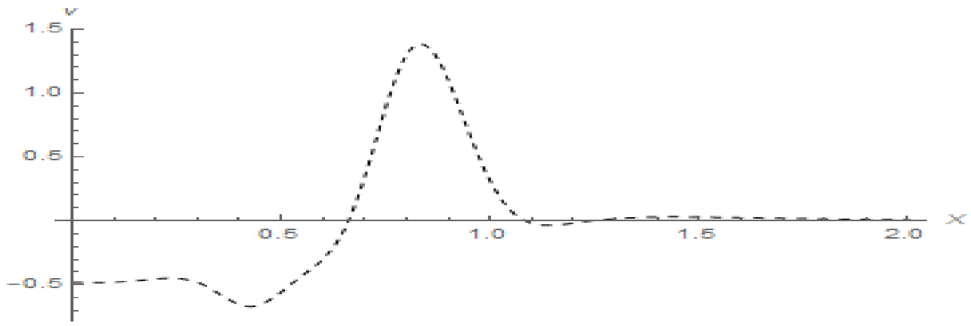


Fig 1.4 Horizontal displacement distributions at the vertical distance y = 0 .3



1.5 Vertical displacement distributions at the vertical distance y = 0 .3

1.9 Conclusion:

In this work, we investigated a dynamical problem of an infinite two-dimensional elastic medium with a crack of Mode-I type in the contexts of thermo elasticity theory, namely Green and Naghdi [28]. The temperature and impact loading are considered at the boundary of the crack region inside the medium. Laplace and Fourier transform techniques are used to solved above the problem. We obtain the four dual integral equations which are reduced into two dual integral equations. The dual integral equations are solved by using the regularization method and a Bellman method is used to inverted the Laplace transform numerically to obtain the final solution of the above mention problem. The most important part of the analysis is the study of behavioral changes of the horizontal and vertical stresses in the vicinity of the crack. Therefore, it may be concluded that the study of thermoelastic interaction in the elastic medium in the presence of a crack will benefit the scientist working in the area of thermoelasticity.

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