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On a Subclasses of generalized JanowskiType Functions using Salagean Operator

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ABSTRACT

In this paper, we present the class $D^{b,\beta}_{R}(\alpha,\beta,\delta,A,B)$ of generalized Janowski type functions of complex order defined by Salagean derivative Operator in the open unit disk. A few outcomes of our principlehypotheses are same as the out-comes got in the previous classes.

Keywords: Analytic functions, subordination, λ-spirallike functions, λ-Robertson function, λ-close-to-spirallike function, λ-close-to-Robertson function, Salagean derivative Operator

Introduction and definition

let A denote the class of functions of the form

$$f(z) = z + \sum_{k=1}^{\infty} a_k z^k, \tag{1}$$

and $k = 1,2,3,...$ which are analytic in the open unit disk. Let S denote the subclass of A which areunivalent in D. The Hadamard product or convolution of two functions $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ and $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$ denoted by, $f * g$ is defined by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k$$

for $z \in D$. In 1983, Salagean [10] introduced a differential and integral operator

$$D^n f(z) = z + \sum_{n=2}^{\infty} n^k a_n z^n \tag{2}$$

It is easy to see that the series $D^n f(z)$ is convergent in the unit disc for each $k \in N$.Further , we have the following differential operator.

$$D^1 f(z) = z f'(z).\ D^{-1} f(z) = \int_0^z \frac{f(\xi)}{\xi} d\xi = z + \sum_{n=2}^{\infty} n^{-1} a_n z^n$$

$$D^{-n} f(z) = D^{-1}(D^{-(n-1)} f(z)) = z + \sum_{n=2}^{\infty} n^{-k} a_n z^n$$

In recent years several authors obtained many interesting results for various subclasses of analyticfunctions defined by using the Salagean derivative operator.

Given two functions f and F which are analytic in unit disk D, we say that the function f issubordinated to F and write $f \prec F$ or $f(z) \prec F(z)$, if there exist a function ω analytic in D such that $|\omega(z)| < 1$ and $\omega(0) = 0$, with $f(z) = F(z)$ in D.
In particular, if F is univalent in D, then $f(z) \prec F(z)$ if and only if $f(0) = F(0)$ and $f(D) \subseteq F(D)$.
Let P denote the class of all functions of the form $P(z) = 1 + \sum_{n=1}^{\infty} P_n z^n$ that are analytic in D and for which $R_e(P(z) > 0)$ in D.
For arbitrary fixed numbers A and B with $-1 \leq B < A \leq 1$, Janowski introduced the class $\mathcal{P}(A,B)$, defined by the subordination principle as follows

$$\mathcal{P}(A,B) = \left\{ \mathcal{P} : \mathcal{P}(z) \prec \frac{1+Az}{1+Bz}, \mathcal{P}(z) = 1 + \mathcal{P}_1 z + \mathcal{P}_1 z^2 + \dots \right\}$$

Also, if we take $A = 1$ and $B = 1$, we obtain the well known class P of functions with positive realpart.
In 2006, Polatoglu [8] introduced the class $\mathcal{P}(A,B,\delta)$ of the generalization of Janowski functions as follows:

$$\mathcal{P}(A,B,\delta) = \mathcal{P} : \mathcal{P}(z) \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta, \mathcal{P}(z) = 1 + \mathcal{P}_1 z + \mathcal{P}_1 z^2 + \dots \quad (3)$$

for arbitrary fixed numbers A and B with $-1 \leq B < A \leq 1$, $0 \leq \delta < 1$, $z \in D$.
Let S^* and C be the subclasses of S of all starlike functions and convex functions of order α andthe classes of convex function of order α, where $0 \leq \alpha < 1$, respectively.
In particular, we note that $S^* := S^*(0)$ and $C^* = C^*(0)$.
In [9] Reade introduced the class CS^* of close-to-star function as follows

$$CS^* = R_e \left\{ f \in A : \frac{f(z)}{g(z)} > 0 \right\}$$

.
for all $z \in D$. Also, we denote by $CS^*(\beta)$ the class of close-to-star functions of order β where $0 \leq \beta < 1$. (See Goodman [3]).
In [6], Kaplan introduced the class CC of close-to-convex functions as follows:

$$CC = R_e \left\{ f \in A : \frac{f'(z)}{g'(z)} \right\} > 0$$

for all $z \in D$.Also, we denote by $CC(\beta)$ the class of close-to-convex functions of order β where $0 \leq \beta < 1$. (See Goodman [2]).
Clearly, we note that $CS^* := CS^*(0)$ and $CC := CC(0)$.
$f \in A$ is an λ-spirallike function, SP^λ, if and only if

$$R_e \left[e^{\iota\lambda} \frac{zf'(z)}{f(z)} \right] > 0$$

for some $|\lambda| < \frac{\pi}{2}$, $z \in D$. The class of λ-spirallike functions was introduced by Spacek in [11].
Also, $f \in SP^\lambda$ if and only if there exists a function $p \in P$ such that

$$f(z) = z \exp \left\{ \cos\lambda \, e^{-i\lambda} \int_0^z \frac{p(t) - 1}{t} dt \right\}$$

We note that the extremal function for the class of SP^λ

$$f(z) = \frac{z}{(1-z)^{2s}} \text{ where } s = e^{-i\lambda}\cos\lambda$$

the λ-spiral koebe function.
$f \in A$ is an λ-Robertson function, R^λ, if and only if

$$R_e \left[e^{\iota\lambda} \left(1 + \frac{zf''(z)}{f'(z)}\right) \right] > 0$$

for some $|\lambda| < \frac{\pi}{2}$, $z \in D$.
Lemma 1.1. $f \in R^\lambda$ if and only if there exist $p \in P$ such that

$$f(z) = \exp \left\{ e^{-i\lambda} \int_0^z \frac{p(t)\cos\lambda - e^{-i\lambda}}{t\cos\lambda} dt \right\}$$

for some $|\lambda| < \frac{\pi}{2}$, $z \in D$.
Proof. Suppose that $f \in R^\lambda$. Since it is λ-Robertson function, there exist a function $p \in P$ such that

$$e^{\iota\lambda}\left(1 + \frac{zf''(z)}{f'(z)}\right) = p(z)\cos\lambda$$

From this equality we can easily obtain the result 4.

Conversely, suppose that suppose result holds, then if we take logarithmic derivative then $f \in R^{\lambda}$. Sothat, the proof is completed.

We note that $f \in R^{\lambda}$ if and only if $zf' \in SP^{\lambda}$.

$f \in \{A$ is an λ-close-to-spirallike function, $\{CS\{P^{\lambda}$, if there exist a function $g \in SP^{\lambda}$ such that

$$\{R\left[\frac{f(z)}{g(z)}\right] > 0$$

for some $|\lambda| < \frac{\pi}{2}$, $z \in D$.

We note that the extremal function for the class CSP^{λ}

$$f(z) = \frac{z+z^2}{(1-z)^{2s+1}}, \text{ where } s = e^{-i\lambda}\cos\lambda.$$

the λ-close-to-spirallike koebe function.

$f \in A$ is an λ-close to Robertson function, CR^{λ}, if there exist a function $g \in R^{\lambda}$ such that

$$R\left[\frac{f'(z)}{g'(z)}\right] > 0$$

for some $|\lambda| < \frac{\pi}{2}$, $z \in D$.

We have introduced the class $g \in SP^{\lambda}(b)$ of λ-spirallike functions of complex order b as follows

$$SP^{\lambda}(b) = \left\{ f \in A : R\left\{1 + \frac{e^{i\lambda}}{b\cos\lambda}\left(\frac{z(D^n f)'}{D^n f} - 1\right)\right\} \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta \right\}$$

for some $b \in \{C - 0, z \in D$.

On the same way we have defined the class $R^{\lambda}(b)$ of λ- Robertson function of complex order b as follows

$$R^{\lambda}(b) = \left\{ f \in A : \{R\left\{1 + \frac{e^{i\lambda}}{b\cos\lambda}\left(\frac{z(D^n f)''}{(D^n f)'} - 1\right)\right\} \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta \right\}$$

.

Now, respectively, we introduce the classes of λ-close-to-spirallike functions of complex order b and λ-close-to-Robertson function of complex order b, denoted by $CSP^{\lambda}(b)$ and $CR^{\lambda}(b)$, as follows:

$$CSP^{\lambda}(b) = \left\{ f \in A : \{R\left\{1 + \frac{1}{b}\left(\frac{D^n f}{D^n g} - 1\right)\right\} \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta, \ g \in SP^{\lambda} \right\}$$

and

$$CR^{\lambda}(b) = \left\{ f \in A : \{R\left\{1 + \frac{1}{b}\left(\frac{(D^n f)'}{(D^n g)'} - 1\right)\right\} \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta, \ g \in R^{\lambda} \right\}$$

for some $|\lambda| < \frac{\pi}{2}$, $z \in D$.

Definition 1.2. The class of generalized Janowski functions which are defined by Salagean derivativeoperator in $z \in D$, denoted by $D_R^{b,\beta}(\alpha, \beta, \delta, A, B)$,is defined as

$$D_R^{b,\beta}(\alpha, \beta, \delta, A, B) = \left\{ f \in A : 1 + \frac{e^{i\beta}}{b\cos\beta}\left[\frac{D_R f(z)}{D_R g(z)} - 1\right] \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta, \ g \in SP^{\lambda} \right\}$$

for some for some $|\lambda| < \frac{\pi}{2}$, $z \in D$, $0 \le \delta < 1$, $-1 \le B < A \le 1$.

Nothing that the class $D_R^{b,\beta}(\alpha, \beta, \delta, A, B)$, includes several subclasses which have important role inthe analytic and geometric function theory.

By specializing the parameters $\alpha, \beta, \delta, A, B$ we obtained the following subclasses studied earlier:

(1) $CS_b^*(\delta, A, B) := D_b^0(0, 0, \delta, A, B)$ is the class of the generalized Janowski type close-to-star functions of complex order b,

(2) $CS_b^*(A, B) := D_b^0(0,0,0, A, B)$ is the class of the generalized Janowski type close-to-star functions of complex order b,

(3) $CS_b^*(A, B) := D_1^0(0,0,0, A, B)$ is the class of the generalized Janowski type close-to-star functions,

(4) $CS_b^*(\delta, A, B) := D_1^0(0, 0, 0, 1 - 2\eta, -1)$ is the class of the close-to-star functions of order η,

(5) $CS_b^*(\delta, A, B) := D_1^0(0, 0, 0, 1, -1)$ is the class of the close-to-star functions

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(6) $CC_b(\delta, A, B) := D^0(1, 0, \delta, A, B)$ is the class of the generalized Janowski type close-to-convexfunctions of complex order b,

(7) $CC_b(\delta, A, B) := D^0(1, 0, 0, A, B)$ is the class of the generalized Janowski type close-to-convexfunctions of complex order b,

(8) $CC_1(A, B) := D^0(1, 0, 0, A, B)$ is the class of the generalized Janowski type close-to-convex functions,

(9) $CC(\eta) := D^0(0, 0, 0, 1-2\eta, -1)$ is the class of the close-to-star functions of order η,

(10) $CC, B) := D^0_1(0, 0, 0, 1, -1)$ is the class of the close-to-convex functions

Lemma 1.3. [1] if the function $p(z)$ of the form

$$P(z) = 1 + \sum_{k=1}^{\infty} P_k z^k$$

is analytic in D and

$$P(z) \prec \frac{1+Az}{1+Bz}$$

then $|p_k| \leq A - B$, for $k \in \mathrm{N}$ $-1 \leq B < A \leq 1$.

Theorem 1.4. [3] $f \in SP^{\lambda}$, then

$$|a_n| \leq \prod_{k=1}^{n-1} \frac{|k+2s-1|}{k}$$

where $s = e^{-i\lambda}\cos\lambda, |\lambda| < \frac{\pi}{2}, z \in D$.

Subordination result and their Consequences

Theorem 2.1. $f(z) \in D_R^{b,\lambda}(\alpha, \beta, \delta, A, B)$ if and only if

$$\frac{D_R^{\alpha} f(z)}{D_R^{\beta} g(z)} - 1 \prec \frac{(1-\delta)(A-B)be^{-i\lambda}\cos\lambda\, z}{1+Bz} \tag{4}$$

Proof. Suppose that $f(z) \in S^* D_R^{b,\lambda}(\alpha, \beta, \delta, A, B)$.Using Subordination principle, we write

$$1 + \frac{e^{i\lambda}}{b\cos\lambda}\left(\frac{D_R^{\alpha} f(z)}{D_R^{\beta} g(z)} - 1\right) = (1-\delta)\frac{1+A\omega(z)}{1+B\omega(z)} + \delta \tag{5}$$

After simple calculations, we get

$$\frac{e^{i\lambda}}{b\cos\lambda}\left(\frac{D_R^{\alpha} f(z)}{D_R^{\beta} g(z)} - 1\right) = \frac{(1-\delta)(A-B)\omega(z)}{1+Bz}.$$

Thus, this equality completes the proof. Similarly, the other side is proved.
In this Theorem, if we choice special values $\alpha, \beta, \delta, \lambda, b$ and A, B we get following corollaries.

Corollary 2.2. $f \in CSP^{\lambda}(b)$ if and only if

$$\frac{f(z)}{g(z)} - 1 \prec \frac{2be^{-i\lambda}\cos\lambda\, z}{1-z}$$

and this result is as sharp as the function

$$\frac{2be^{-i\lambda}\cos\lambda\, z}{(1-z)^{2s+1}}, \text{ where } s = e^{-i\lambda}\cos\lambda$$

Proof. We let $\alpha = \beta = \delta = 0$ and $A = 1, B = -1$ in the Theorem 2.1.

Corollary 2.3. $f(z) \in CS^*(A, B)$ if and only if

$$\frac{f(z)}{g(z)} \frac{(A \quad B)\, z}{1 + B\, z} \quad 1 -$$

and this result is as sharp as the function

$$\frac{1+Az}{1+Bz} \cdot \frac{z}{(1-z)^2}.$$

Proof. We let $\lambda = \alpha = \beta = \delta = 0$ and $b = 1$ in Theorem 2.1. **Corollary 2.4.** $f(z) \in CS^*$ if and only if

$$\frac{f(z)}{g(z)} - 1 \prec \frac{2z}{1-z}$$

and the result is as sharp as the function

$$\frac{1+z}{1-z}.$$

Proof. We let $\lambda = \alpha = \beta = \delta = 0$ and $A = 1$, $B = -1$ in Theorem 2.1.

Corollary 2.5. $f(z) \in R^{\lambda}(b)$ if and only if

$$\frac{zf'(z)}{g(z)} - 1 \prec \frac{2be^{-i\lambda}\cos\lambda\, z}{1-z}.$$

Proof. We let $\alpha = 1$, $\beta = \delta = 0$ and $A = 1$, $B = -1$ in Theorem 2.1.

Corollary 2.6. $f(z) \in CC(A, B)$ if and only if

$$\frac{zf'(z)}{g(z)} - 1 \prec \frac{(A-B)z}{1+Bz}$$

Proof. We let $\lambda = \beta = \delta = 0$ and $\alpha = 1$, $b = 1$ in Theorem 2.1.

Corollary 2.7. $f(z) \in CC$ if and only if

$$\frac{zf'(z)}{g(z)} - 1 \prec \frac{2z}{1-z}$$

and this result is as sharp as the function

$$\frac{1+z}{1-z}.$$

Proof. We let $\lambda = \beta = \delta = 0$ and $\alpha = 1$, $b = 1$, $A = 1$, $B = -1$.

Coefficient estimates and their Consequences

Lemma 3.1. If the function $\varphi(z)$ of the form

$$\varphi(z) = 1 + \sum_{k=1}^{\infty} \varphi_k z^k$$

is analytic in D and

$$\varphi(z) \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta$$

then

$$|\varphi_k| \le (A-B)(1-\delta) \tag{6}$$

for $0 \le \delta < 1, -1 \le B < A \le 1, z \in D$.

Proof. Suppose that $\varphi(z) \prec (1-\delta)\frac{1+Az}{1+Bz} + \delta$ for $\varphi(z) = 1 + \sum_{k=1}^{\infty} \varphi_k z^k$. Using subordinationprinciple, we write

$$\varphi(z) = (1-\delta)\frac{1+A\omega(z)}{1+B\omega(z)} + \delta \tag{7}$$

From (3.2.), we get

$$\kappa(z) = \frac{\varphi(z)-\delta}{(1-\delta)} = \frac{1+A\omega(z)}{1+B\omega(z)}$$

By using Lemma 1.3. for the above $\kappa(z)$, we get

$$\left|\frac{\varphi_k}{1-\delta}\right| \le A - B$$

This inequality is equivalent to required result.

Theorem 3.2. If the function $f \in A$ be in the class $D^{b,\lambda}_R(\alpha,\beta,\delta,A,B)$, then

$$|a_n| \le \frac{1}{|b|} \times \left[|b| n^k(\beta) \prod_{k=1}^{n-1} \frac{|k+2s-1|}{k} + (A-B)(1-\delta) \sum_{m=1}^{n-1} n^k(\beta) \prod_{k=1}^{n-(m+1)} \frac{|k+2s-1|}{k} \right] \quad (8)$$

where $s = e^{-i\lambda}\cos\lambda$, $|\lambda| < \frac{\pi}{2}$, $b \in \mathbb{C} - 0$, $\alpha, \beta > -1$, $0 \le \delta < 1$ and $-1 \le B < A \le 1$.

Proof. Since $f(z) \in S^*_k D^{b,\lambda}_R(\alpha,\beta,\delta,A,B)$ there are analytic functions $g, \varphi : \mathbb{D} \longrightarrow \mathbb{D}$ such that $g(z) = z + \sum_{k=2}^{\infty} b_k z^k \in SP^{\lambda}$, $\varphi(z) = 1 + \sum_{k=1}^{\infty} \varphi_k z^k$ and $\omega(z)$ is a schawarz function as in Lemma3.1 such that

$$1 + \frac{e^{i\lambda}}{b\cos\lambda}\left(\frac{D^{\alpha}_R f(z)}{D^{\beta}_R g(z)} - 1\right) = (1-\delta)\frac{1+A\omega(z)}{1+B\omega(z)} + \delta = \varphi(z) \quad (9)$$

Now, As in subordination principle for $\varphi(z)$

$$(1-\delta)\frac{1+A\omega(z)}{1+B\omega(z)} + \delta = \varphi(z)$$

So,

$$1 + \frac{e^{i\lambda}}{b\cos\lambda}\left(\frac{D^{\alpha}_R f(z)}{D^{\beta}_R g(z)} - 1\right) = \varphi(z)$$

$D^{\alpha}_R f(z) / D^{\beta}_R g(z) - 1 = be^{-i\lambda}\cos\lambda(\varphi(z)-1)$ as $s = e^{-i\lambda}\cos\lambda$, So we can write

$$\frac{D^{\alpha}_R f(z)}{D^{\beta}_R g(z)} - 1 = sb(\varphi(z)-1)$$

$$D^{\alpha}_R f(z) = \{1 + sb(\varphi(z)-1)\} D^{\beta}_R g(z)$$

Now for Salagean operator, we can write

$$z + \sum_{k=2}^{\infty} n^k a_k z^k = \{1 + sb(\varphi(z)-1)\}\{z + \sum_{k=2}^{\infty} n^k b_k z^k\}$$

$$z + \sum_{k=2}^{\infty} n^k a_k z^k = z + \sum_{k=2}^{\infty} n^k b_k z^k + sb \sum_{k=2}^{\infty} (\varphi(z) n^k b_k z^k)$$

$$z + \sum_{k=2}^{\infty} n^k a_k z^k = z + \sum_{k=2}^{\infty} n^k b_k z^k + sb \sum_{k=2}^{\infty} \sum_{m=1}^{\infty} \varphi_m z^m n^k b_n z^n$$

equating the co-efficient for the power of z,we get

$$n^2(\alpha)a_2 = n^2(\beta)b_2 + sb\varphi_1$$

$$n^3(\alpha)a_3 = n^3(\beta)b_3 + sb\,(\varphi_1 b_2 n^2(\beta) + \varphi_2)$$

Similarly

$$n^n(\alpha)a_n = n^n(\beta)b_n + sb\,(n^{n-1}(\beta).b_{n-1}\varphi_1 + n^{n-2}(\beta).b_{n-2}\varphi_2 + \dots + \varphi_{n-1})$$

by using Lemma 3.1. and Theorem 1.4 we get

$$|a_n| \le \frac{1}{|b|} \times \left[|b| n^k(\beta) \prod_{k=1}^{n-1} \frac{|k+2s-1|}{k} + (A-B)(1-\delta) \sum_{m=1}^{n-1} n^k(\beta) \prod_{k=1}^{n-(m+1)} \frac{|k+2s-1|}{k} \right]$$

This completes the proof.

Corollary 3.3. Let $f \in A$ be in the class CSP^{λ}, then

$$|a_n| \le \frac{1}{|b|} \left[|b| \prod_{k=1}^{n-1} \frac{|k+2s-1|}{k} + 2 \sum_{m=1}^{n-1} \prod_{k=1}^{n-(m+1)} \frac{|k+2s-1|}{k} \right]$$

where $s = e^{-i\lambda}\cos\lambda$, $|\lambda| < \frac{\pi}{2}$, $b \in \mathbb{C} - 0$. **Proof**. We let $\alpha = \beta = \delta = 0$ and $A = 1, B = -1$ in Theorem 3.2.

Corollary 3.4. [7] Let $f \in A$ be in the class $CS^*(A, B)$, then

$$|a_n| \leq n + \frac{(A-B)(n-1)n}{2}$$

where $-1 \leq B < A \leq 1$. **Proof.** We let $\alpha = \beta = \delta = \lambda = 0$ and $b = 1$ in Theorem 3.2. **Corollary 3.5.** [7] Let $f \in A$ be in the class CS^*, then

$$|a_n| \leq n^2.$$

Proof.We let $\alpha = \beta = \delta = \lambda = 0$ and $b = 1$ in Theorem 3.2.

Corollary 3.6. Let $f \in A$ be in the class R^{λ}, then

$$|a_n| \leq \frac{1}{|b|n}\left[|b| \prod_{k=1}^{n-1} \frac{|k+2s-1|}{k} + 2 \sum_{m=1}^{n-1} \prod_{k=1}^{n-(m+1)} \frac{|k+2s-1|}{k}\right].$$

where $s = e^{-i\lambda}\cos\lambda$, $|\lambda| < \frac{\pi}{2}$, $b \in \mathbb{C} - 0$.

Proof. We let $\alpha = 1, \beta = \delta = 0$ and $A = 1, B = -1$ in Theorem 3.2.

Corollary 3.7. Let $f \in A$ be in the class $CC(A,B)$, *then*

$|a_n| \leq 1 + \frac{(A-B)(n-1)}{2}$, where $-1 \leq B < A \leq 1$.

Proof. We let $\alpha = 1, \beta = \delta = 0$ and $b = 1$ in Theorem 3.2.

Corollary 3.8. Let $f \in A$ be in the class CC, *then*

$$|a_n| \leq n.$$

Proof. We let $\alpha = 1, \beta = \delta = 0$ *and* $A = 1, B = -1$, b=1 *in Theorem* 3.2.

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