OPTIMAL SEQUENCE FOR TRAVELLING SALESMAN USING GRAPH STRUCTURE

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Abstract: The design of Travelling Salesman model to finding the optimal sequence for salesman is very complexity in its applications process. The objective of the model is to find the optimal sequence for salesman. The general mathematical design of travelling salesman is discussed briefly. A novel model for travelling salesman is designed as a graph structure and by applying the methodology of cycle path to the modeled graph we have many sequences for salesman. To get optimal sequence, Salesman paths from each node were evaluated using open software and solutions are presented as output.

Keywords: Optimal sequence, Travelling Salesman model, Complete graph, Non-Complete graph, Vertex and Weighted edges.

1. Introduction:

In our day today life the need of travelling is a necessary need in all the places of work. This necessity need was developed as a travelling salesman design from earlier days and the objective of the model is to find a good way to solve the design. The model of travelling salesman has being applied in many commercial and economic world, so this design was consider as optimization model. In general the model was stated as travelling salesman problem and different methods was applied to solve the problem. Due to application and presiding constraint present in the problem, this type of model is considered as very complexity models. To overcome the complexity present in the design many methods are used with the mathematical model and applied to find the optimal sequence for the salesman and optimal travelling route is identified.

In this research work, a design of the Travelling salesman model of mathematics from the general formation is stated with its parameters and varying constraints equations. The application of graph theory is applied and the design of the Travelling salesman is modeled as a Weighted edge simple graphs and according to the constraint present in the models the structure of the graphs are divided in to two categories as Complete graph and Non-Complete graph. The designed graphical models are simulated using the open source software and all possible sequences are found and the results are tabulated.

2. General Mathematical Model of Travelling Salesman Problem:

The normal objective of salesman is to start the work at the starting spot, reach different places to complete the work and should reach the starting spot at the end. In these processes the main objective of salesman is to complete the whole work with the optimal time or cost. The distance (time or cost) between every pair of cities are assumed to be known. The problem of finding the shortest distance (time or cost) if the salesman starts from his headquarters and passes through each city under his constraints exactly once and returns to the headquarters is called the Travelling salesman problem. The general mathematical formulation of the Travelling salesman problem is stated as

$$\text{Optimize} \quad Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$$

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j = 1,2,3,\ldots,n$$

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$$Xij = 1 \text{ or } 0 \quad \forall i, j$$

Where C_{ij} is the time or cost parameters.

2.1. Graph Theory Definitions:

Graph - linear graph (or simply a graph) G=(V,E) consists of a set of objects V={$v_1,v_2,\ldots$} called vertices, and another set E={$e_1,e_2,\ldots$},whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_1,v_2) of vertices, the vertices v_i, v_j associated with edge e_k are called the end vertices of e_k. Simple graph - A graph that has does not self-loop nor are parallel edges called a simple graph. Simple path - An open walk in which no vertex appears more than once is called a path (or a simple path or an elementary path). Simple circle - A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a circuit. Complete graph - A complete graph is a graph that has an edge between every single vertex in the graph, we represent a complete graph with n vertices using the symbol K_n. Non-Complete graph – A simple graph which is not complete is stated as Non – Complete graph.

3. Proposed Design of the Travelling Salesman Model:

The general model of the Travelling salesman is considered, and applied with the following proposed method to get an optimal sequence for the salesman.

Step 1: The Mathematical model of Travelling Salesman Problem is constructed as a Weighted edges simple graph design, here the weight represent the travel cost or travel distance.

Step 2: The designed Weighted edge graph is classified according to the given data as Complete graph and Non-Complete graph.

Step 3: Compiling the Weighted graph in the open source software and feeding the dates according to the designed model.

Step 4: Simulations are done to get the all possible travelling sequence from all the vertex of the graph.

Step 5: From the results derived from the simulation is tabulated and the optimal sequence of each vertex is obtained.

3.1. Design of Travelling Salesman Model as Complete Graph:

A Travelling Salesman model consist of seven place with the corresponding work should be completed is designed as the weighted complete graph , since each and every place are connected with the edges and here the weight represents the average expanse of the salesman in the corresponding place one unit represents thousand rupee. The Mathematical model of Travelling Salesman Problem, constructed with the following weights is represented in the table 3.1.1.

Table 3.1.1: Representation of the Average Expanse

Place	1	2	3	4	5	6	7
1	0	5	6	8	11	12	19
2	5	0	8	10	13	14	16
3	6	8	0	12	14	16	18
4	8	10	12	0	13	15	16
5	11	13	14	13	0	16	18
6	12	14	16	15	16	0	16
7	19	16	18	16	18	16	0

The designed Weighted graph as Complete graph is represented in the Fig. 3.1.1.

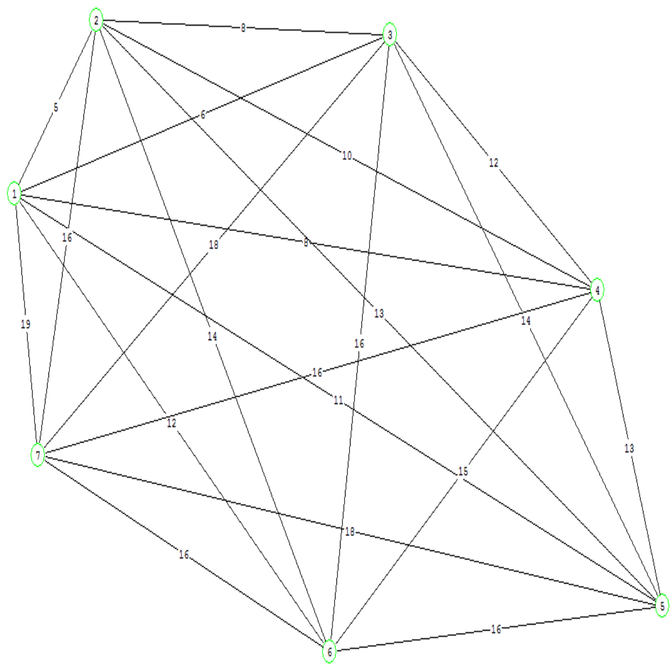

Fig. 3.1.1: Weighted Complete Graph Model

Compiling the Weighted graph in the open source software and feeding the dates according to the designed model. Simulations are done to get the all possible travelling sequence from all the vertex of the graph is tabulated as the table 3.1.2.

Table 3.1.2: All Possible Cycle Path

Place	S.No	Cycle – Path	Weight
1	1	1, 2, 3, 4, 5, 6, 7, 1	89
	2	1, 2, 3, 4, 5, 7, 6, 1	84
	3	**1, 2, 3, 5, 6, 7, 4, 1**	**83**
2	1	2, 3, 4, 5, 6, 7, 1, 2	89
	2	2, 3, 4, 5, 7, 6, 1, 2	84
	3	2, 3, 5, 6, 7, 4, 1, 2	83
3	1	3, 4, 5, 6, 7, 2, 1, 3	84
	2	3, 5, 6, 7, 4, 2, 1, 3	83
4	1	4, 5, 6, 7, 3, 2, 1, 4	84
	2	4, 5, 6, 7, 2, 3, 1, 4	83
5	1	5, 6, 7, 4, 3, 2, 1, 5	84
	2	5, 6, 7, 4, 2, 3, 1, 5	83
6	1	6, 7, 5, 4, 3, 2, 1, 6	84
	2	6, 7, 5, 4, 2, 3, 1, 6	83

7	1	7, 6, 5, 4, 3, 2, 1, 7	89
	2	7, 6, 5, 4, 3, 1, 2, 7	84
	3	7, 6, 5, 4, 1, 3, 2, 7	83

From the results derived from the table 3.1.2 by simulation, optimal sequence of each place is obtained with the cycle – path weight 83 and represented in the table with different colour.

3.2. Design of Travelling Salesman Model as Non - Complete Graph:

A Travelling Salesman model consist of six place with the corresponding work should be completed is designed as the weighted Non - complete graph , since each and every place are not connected with the edges and here the weight represents the average expanse of the salesman in the corresponding place one unit represents thousand rupee.

The Mathematical model of Travelling Salesman Problem, constructed with the following weights is represented in the table 3.2.1.

Table 3.2.1: Representation of the Average Expanse

Place	1	2	3	4	5	6
1	0	5	6	-	-	3
2	5	0	-	4	8	6
3	6	-	0	5	8	-
4	-	4	5	0	-	6
5	-	8	8	-	0	-
6	3	3	6	-6	-	0

The designed Weighted graph as Non - Complete graph is represented in the Fig. 3.2.1.

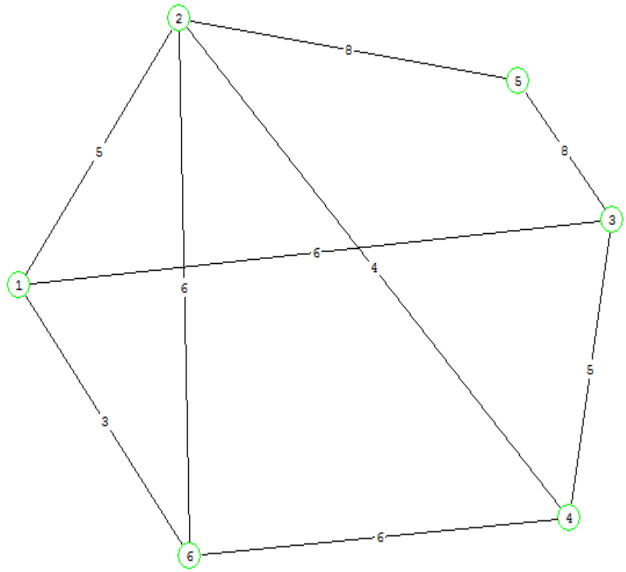

Fig. 3.2.1: Weighted Non - Complete Graph Model

Compiling the Weighted graph in the open source software and feeding the dates according to the designed model. Simulations are done to get the all possible travelling sequence from all the vertex of the graph is tabulated as the table 3.2.2.

Table 3.2.2: All Possible Cycle Path

Place	S.No	Cycle – Path	Weight
1	1	1, 6, 4, 3, 5, 2, 1	35
2	1	2, 4, 6, 1, 3, 5, 2	35
3	1	3, 4, 6, 1, 2, 5, 3	35
4	1	4, 6, 1, 3, 5, 2, 4	35
5	1	5, 3, 4, 6, 1, 2, 5	35
6	1	6, 4, 3, 5, 2, 1, 6	35

From the results derived from the table 3.2.2 by simulation, optimal sequence of each place is obtained with the cycle – path weight 35.

Conclusion:

The main objective of the design is to get an optimal schedule sequence for the salesman to achieve the target. A design of the Travelling salesman model of mathematics from the general formation is stated with its parameters and varying constraints equations. The application of graph theory is applied and the design of the Travelling salesman is modeled as a Weighted edge simple graphs and according to the constraint present in the models the structure of the graphs are divided in to two categories as Complete graph and Non-Complete graph. A novel model for travelling salesman is designed as a graph structure and by applying the methodology of cycle path to the modeled graph we have many sequences for salesman. To get optimal sequence, Salesman paths from each node were evaluated using open software and solutions are presented as output.

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