PARAMETER ESTIMATION FOR NON-LINEAR MUSKINGUM FLOOD ROUTING MODEL: NATURAL INSPIRED ALGORITHM

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Abstract - In any watershed the surface runoff is routed through the main course of the river to estimate the flow hydrograph at the outlet of the basin. One of the most widely used flood routing model in a river course is Muskingum model. The Muskingum model has three parameters, namely; k, x, and m, which are estimated based on the known input and outflow hydrograph of the channel. The accuracy of routing depends on the preciousness in estimating the model parameters. Many research works had been reported on improvising the parameter estimation by employing various advanced computational techniques. In this paper, we introduce a bat optimization algorithm for estimating the three parameters of the Muskingum model. Bat algorithm finds the global optimum for the parameters with random fly within the domain space. The results of the proposed model have been compared with various parameter estimation techniques reported for the Muskingum flood routing model. The comparative evaluation has been done based on four performance indicators, namely; a) Root Mean Square Error (RMSE), b) Mean absolute error (MAE), c) Coefficient of Correlation (R) , and d) Nash-Sutcliffe efficiency (E). From performance indicators it is observed that the proposed parameter estimation using a bat optimization algorithm outperforms in capturing the observed flow compared to other reported techniques.

Key Words: *Non-linear Muskingum model, Flood routing, parameter estimation, Bat optimization*

1.INTRODUCTION

The most common phenomenon in water resource modelling is to route the sub-basin flood hydrograph through the main river course to estimate the flood hydrograph at the outlet of the watershed basin.In flood routing, there are two basic approaches, namely; a) Hydrological routing, and b) Hydraulic routing. Hydrological routing is based on the storage-continuity equation, and hydraulic routing is based on solving Saint-Venant equation. One of the most frequent and famous hydrological flood routing model used in water resource modelling is the Muskingum model developed by Mc Carthy (1938). The Muskingum model has been used frequently by the researchers for routing the flood discharge because of its simplicity. The Muskingum model has been developed based on storage-continuity equation (1).

$$\frac{ds}{dt} = I_t - O_t \tag{1}$$

where (ds/dt), represents the change in storage in the channel, It is the inflow at time 't' into the channel, and Ot is the outflow at time 't' at the end of the channel. The Muskingum model is given by (2)

$$S_t = k[xI_t + (1-x)O_t] \tag{2}$$

Where St, storage at time 't', K and x are the storage and weighting parameter of the model. The parameters K and x are estimated based on the known inflow and its corresponding outflow response at time 't'. The simplest method for estimating parameters is through graphical solution. In graphical solution, the values of $[xI_t + (1-x)O_t]$ has been plotted versus the storage. The value of 'x' is iterated until the width of the loop reduces almost to zero. The slope of the line represents the parameter 'K'. The iteration process involved in graphical solution is a time-consuming process and the approximation / uncertainty of the model varies based on the researcher. Thus, to improve the reliability on the estimated parameters, Yoon and Padmanaban (1993), considered the Muskingum model as a linear equation and routed the flow using three methods / approaches, namely; a) Trial and error, b) orthogonal least square regression, and c) Iterative filtering outlying data points for regression analysis. However, Muskingum model is a non-linear expression, considering non-linear as linear or linear-piecewise (Yoon and Padmanaban, 1993) might result in the approximate solution. Tung (1985), considered the non-linearity that exists in Muskingum model and proposed a three-parameter model (3).

$$S_t = k[xI_t + (1-x)O_t]^m \tag{3}$$

where m, is an exponent parameter that considers the non-linearity in the expression (3). The above non-linear has been solved (Tung, 1985) using three approaches, namely; i) Hooke-Jeeves (HJ) pattern search in conjunction with linear regression, ii) HJ in conjunction with conjugate gradient, and iii) HJ in conjunction with Davidon-Fletcher-Powell method. Further, to reduce the complexity in estimation approach, least square based parameter estimation approach has been developed by Aldama (1995). The limitation with the least square based approach is arbitrarily selecting points to solve the simultaneous nonlinear equation (Tung, 1985). Yoon and Padmanabhan (1998), proposed an optimization based non-linear parameter estimation model 'NONLR' for routing the flood flow through the channel. All the above reported models require an initial solution for initiating the parameter estimation process. The time taken for routing the flow and accuracy of the model solution depends on the initial solution. To overcome the above limitation, evolution based (Genetic algorithm) parameter estimation Muskingum model was developed by Mohan (1997). Evolution based optimization model starts its search for different initial solutions to achieve the global optimum. Father the accuracy of parameter estimation through integrating optimization models has emerged as a main focus in the research domain. This has lead the way to application of a) iterative process (Das, 2004), b) Broyden Fletcher Goldfarb Shanno (BFGS) Technique (Geem,2006), c) Chance-constrained (Das, 2007), d) Immune clonal selection (Luo and Xie, 2010), e) Hybrid Chaotic genetic algorithm(Wang et al., 2009), f) Harmony search (Geem,2011), g) Particle swarm optimization (PSO) (Chu and Chang, 2009), h) Gray encoded genetic algorithm (Chen and Yang, 2007), i) Nelden-Mead simplex algorithm (NMS) (Barati, 2011), j) Differential evolution (DE) (Xu et al., 2012), and k) hybrid harmony search algorithm (Karahan, 2013). The above highlighted algorithms used the strength of optimal search within the solution space with an objective of reducing the sum of squared error (SSE).

$$z = \min \sum_{t=1}^{n} (ob_t - sim_t)^2 \tag{4}$$

where, ob_t and $sim_t$is the observed and simulated outflow in the channel at time 't'. Except the evolution based genetic algorithm (GA), all the other optimization algorithms reported requires an initial solution to begin the search. Even though GA has been well proven optimization technique, and also does not require an initial solution to begin the search within the solution space. However, the accuracy or dependability depends on the type of crossover function, crossover rate, mutation rate, number of strings / chromosomes used to represent the decision variables, and population size. If the highlighted are estimated based on sensitivity analysis and then the dependability of the solution is ensured. Different combinations of parameters may result in different solution. In such a case a random search optimization algorithm with minimum number of parameters might ensure the dependability on the end / optimal solution.

Bat optimization algorithm developed by Yang (2010) based on swarm intelligence and from observing the bat movement / flight in search of prey. Bat searches the optimal solution / prey by making numerous random flights from the current / present location utilizing echolocation (Yang, 2010). The echolocation has two parameters, namely; a) wavelength (λ), and b) loudness (A_o). In this paper, we have applied the bat algorithm for estimating the parameters of the nonlinear Muskingum flood routing model. This paper has been organized in the following manner; the paper begins with the introduction of a bat optimization algorithm and its application to the Muskingum model. The application will discuss on the stepwise procedure of parameter estimation. A comparative analysis of the computed results with reported methods has been made to understand the pro's and cons of the proposed model. The final remarks with further scope of the study has been highlighted as conclusions.

2. Bat optimization algorithm

There are about 996 types of bats that exists in this world, Gonzajlez et al., (2010) developed a meta-heuristic bat algorithm (BA) by observing one species of such called "Micro bats". Micro bats use a type of sound wave, called echolation to identify the path of travel without any obstacles and also to locate the prey. These ultrasonic sound waves has been emitted with a frequency ranging from 25 kHz to 100 kHz. The emitted ultrasonic waves have a wavelength and loudness. Microbats observes the time delay between emitted to received wavelength to track the obstacles. Gonzajlez et al., (2010) observed that wavelength increases and loudness decreases as the bat approaches closer to the prey. For simplicity, Yang and Gandomi (2012) used the following approximations or rules.

1. All bats use echolocation to sense distance, and they also 'know' the difference between food/ prey and background barriers in some magical way.
2. Bats fly randomly with velocity V_i at position X_i with a fixed frequency f_{min}, varying wavelength λ and loudness A_o to search for prey. They can automatically adjust the wavelength of their emitted pulses and adjust the rate of pulse emission r ε [0,1], depending on the proximity of their target.
3. Although the loudness can vary in many ways, we assume that the loudness varies from a large (positive) A_o to a minimum constant value A_{min}.

Figure 1, shows the stepwise flow of the bat optimization algorithm in search of optimal within the solution domain space.

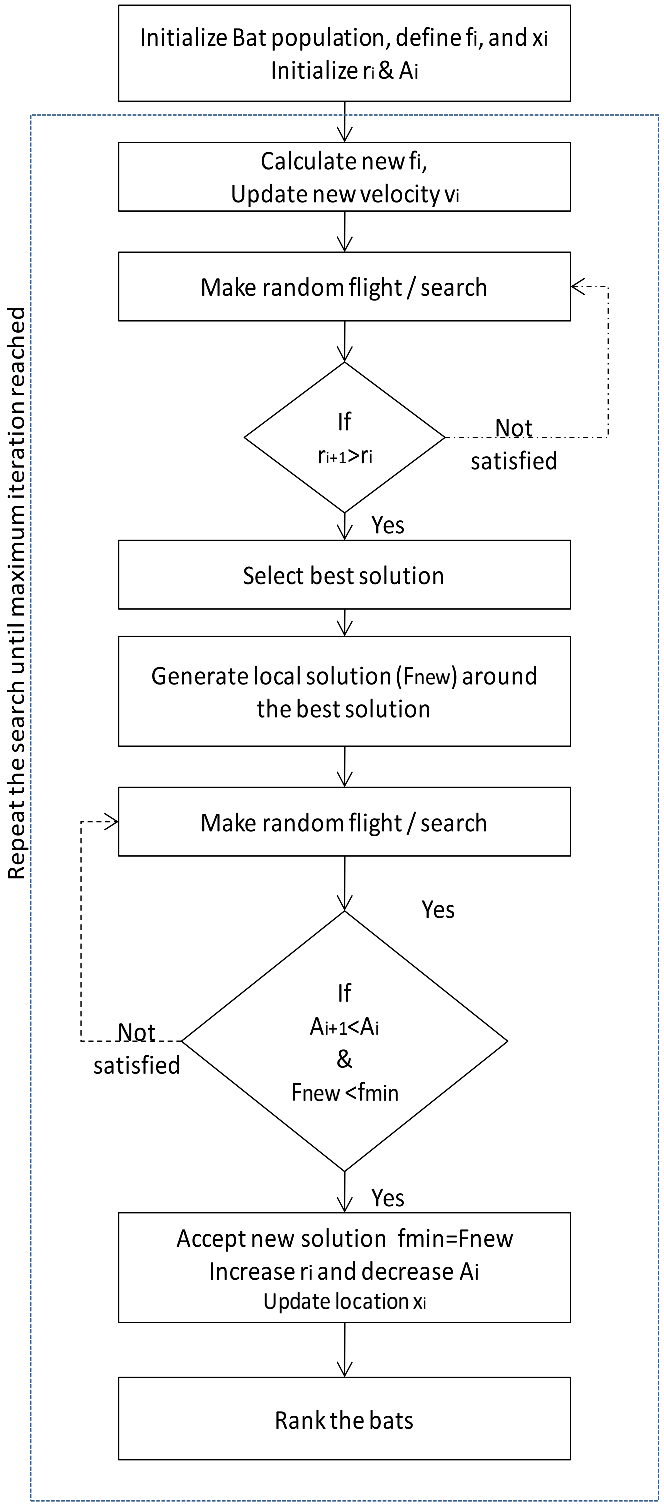

Figure 1: Bat inspired optimization algorithm

From Figure 1, it can be observed that to achieve the solution, the bat intiates its flight from a known location (x_i) with a well defined pulse frequency (f_i), pulse rate (r_i), and initial loudness (A_i). The bat during its flight adjust its pulse frequency using the equation (5) and generates a new solution.

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{5}$$

where, β, is a random value resembling the random flight made with the bat varying from 0 to 1, f_{max} and f_{min} are the maximum and minimum value of the pulse frequency. The generated solution (equation 5) has been used to update the travel velocity and the solution using the equation (6).

$$v_i^t = v_i^{t-1} + (x_i^t - x_x) f_i \tag{6}$$

where, v_i is the travel velocity of the bat at location x_i at time t. xx indicates the location of the current best solution. To select the best solution, bat makes a random flight such that the pulse is greater than the initial pulse rate. Based on pulse rate (r_i) the best solution is chosen from a pool solutions in the current location (x_i). New local solutions are generated in and around the current best solution, to select the best among available local solution, the bat makes a random flight and looks for minimum loudness (A_i) compared to pervious loudness used for flight. The selected local solution to be evaluated by the objective function. If the objective function value is minimum at position of the selected local solution compared to previous position, then bat takes the selected local solution as the new position. The bat before the next flight for new solution, it will increase the emission pulse rate and decrease the loudness wavelength compared to previous flight. The bat iterates the entire process until it finds the best solution / prey among the entire solution domain.

The bat algorithm application for solving engineering problems has been well exhibited by Yang and Gandomi (2012). In particular, Tsai et al., (2012), applied bat optimization algorithm for solving numerical optimization problems. Bat inspired optimization algorithm has found its first applicability in the research domain of deriving optimal releases for a multi reservoir system (Haddad et al., 2014). The random flight by bat through entire solution space, ensures better exploration of the search space without tapping into local optima. This characteristic of bat inspired optimization algorithm encourages the research community to utilize its capability for achieving a global optimal solution.

3. Parameter estimation

To investigate the applicability of the bat optimization algorithm for estimating the parameters of nonlinear Muskingum flood routing model (equation 3) a typical problem with the Wilson data set has been considered (Mohan, 1997). By reorganizing the equation (3), the rate of outflow can be expressed as;

$$O_t = \left(\frac{1}{1-x}\right)\left(\frac{S_t}{k}\right)^{\frac{1}{m}} - \left(\frac{x}{1-x}\right) I_t \tag{7}$$

By combining equation (7) and (1), change in storage with respect to time can be expressed as

$$\frac{\Delta s}{\Delta t} = -\left(\frac{1}{1-x}\right)\left(\frac{S_t}{k}\right)^{\frac{1}{m}} + \left(\frac{x}{1-x}\right) I_t \tag{8}$$

The accumulated storage may be expressed as

$$S_{t+1} = S_t + \Delta s_t \tag{9}$$

The simulation-optimization, routing procedure involves the following steps;

Step 1: the bat algorithm with an objective of minimizing the sum of squared error (SSE), as highlighted in equation 4 will generate values for k, x, and m

Step 2: calculate the initial storage (St) volume by using equation (3), where the initial outflow is same as initial inflow.

Step 3: calculate the change in storage during a time interval using equation (8).

Step 4: calculate the accumulated storage for the next time period using equation (9)

Step 5: calculate the outflow quantity at the next time period using equation (7)

Step 6: repeat the procedure from Step 2 to Step 5 for the entire time period of simulation

Step 7: check for the objective function, repeat the process by initiating Step 1 until the minimum function value is achieved.

The above simulation-optimization, routing procedure has been applied to route the flow for the considered inflow dataset, and the outflow hydrograph ordinates are computed. The routed flood hydrograph has been compared with that of observed hydrograph. The performance evaluation of the developed model has been carried out based on five performance measurements, namely; i) mean absolute error (MAE), ii) coefficient of correlation (R), iii) sum of squared error (SSE), iv) root mean square error (RMSE), and v) Nash Sutcliffe efficiency (E)

i) Mean absolute error (MAE) $=\frac{1}{n}\sum_{i=1}^{t}(y_i^c - y_i^o)$ (10)

ii) Coefficient of Correlation (R) $=\frac{\sum_{i=1}^{t}(y_i^o-\overline{y^o})(y_i^c-\overline{y^c})}{\sqrt{\sum_{i=1}^{t}(y_i^o-\overline{y^o})^2}\sqrt{\sum_{i=1}^{t}(y_i^c-\overline{y^c})^2}}$ (11)

iii) Sum of Squared Error (SSE) $= \sum_{i=1}^{t}(y_i^o - y_i^c)^2$ (12)

iv) Root Mean Square Error (RMSE) $= \sqrt{\frac{\sum_{i=1}^{t}(y_i^o-y_i^c)^2}{n-1}}$ (13)

v) Nash Sutcliffe Efficiency (E) $= 1 - \frac{\sum_{i=1}^{t}(y_i^o-y_i^c)^2}{\sum_{i=1}^{t}(y_i^o-\overline{y^o})^2}$ (14)

where y^o& y^c , are the observed and predicted outflow hydrograph ordinates at time 't'. Thus, the developed bat algorithm based parameter estimation model is evaluated for its accuracy in capturing the actual outflow hydrograph. These performance indicators will indicate the dependability of the developed model. For example, RMSE and Mean absolute error estimated towards zero, indicates that the predicted outflow ordinates match the actual discharge with minimum deviation. Nash-Sutcliffe efficiency, if works out to be one, then the model predicts the actual outflow hydrograph ordinates exactly. Nash-Sutcliffe efficiency explains how exactly the peak flows are captured or predicted by the model.

5.RESULT AND DISCUSSION

Table 1, summarizes the outflow hydrograph ordinates predicted by various parameter estimation techniques and also the outflow ordinates predicted by the developed bat algorithm based parameter estimation model. The various models used for comparative analysis with the developed model are; i) Genetic algorithm (GA) (Mohan, 1997), ii) BFGS (Geem, 2006), iii) PSO (Chu and Chang, 2009), iv) Immune clonal selection algorithm (ICSA) (Luo and Xie, 2010), v) parameter setting free harmony search (psHS) (Geem, 2011), vi) NMS (Barati, 2011), vii) DE) (Xu et al., 2012), and viii) Explicit numerical solving (ENS) using the rungakutta 4th order (Vatankhah, 2014). From Table 1 and Figure 2, it can be observed that all the parameter estimation techniques are able to route the inflow to the channel outlet with an accuracy more than are equal to 90%.

Table 1: Routed outflow hydrograph by various routing techniques

Time	Inflow (m^3/s)	Observed outflow (m^3/s)	Predicted outflow (m^3/s)								
			GA	BFGS	PSO	ICSA	psHS	NMS	ENS	DE	BA
0	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00
6	23.00	21.00	22.00	22.00	22.00	22.00	22.00	22.00	22.10	22.00	22.24
12	35.00	21.00	22.40	22.42	22.60	22.40	22.40	22.42	23.10	22.40	22.45
18	71.00	26.00	26.30	26.61	28.10	26.60	26.60	26.61	25.30	26.60	26.41
24	103.00	34.00	34.20	34.46	32.20	34.40	34.50	34.46	32.70	34.50	33.43
30	111.00	44.00	44.20	44.18	45.00	44.20	44.20	44.17	46.00	44.20	43.83
36	109.00	55.00	56.90	56.86	57.00	56.90	56.90	56.85	58.10	56.90	55.13

42	100.00	66.00	68.20	68.07	67.50	68.10	68.10	68.06	68.70	68.10	67.20
48	86.00	75.00	77.10	77.08	75.90	77.10	77.10	77.07	77.50	77.10	75.18
54	71.00	82.00	83.20	83.33	81.20	83.30	83.30	83.32	83.10	83.30	83.64
60	59.00	85.00	85.70	85.91	85.60	85.90	85.90	85.90	84.70	85.90	84.74
66	47.00	84.00	84.20	84.54	84.20	84.50	84.50	84.54	84.00	84.50	83.24
72	39.00	80.00	80.20	80.58	79.60	80.50	80.60	80.58	79.80	80.60	79.21
78	32.00	73.00	73.30	73.71	73.30	73.60	73.70	73.71	73.80	73.70	72.51
84	28.00	64.00	65.00	65.40	65.00	65.30	65.40	65.41	65.70	65.40	64.08
90	24.00	54.00	55.80	55.99	56.20	55.90	56.00	56.00	56.90	56.00	54.48
96	22.00	44.00	46.70	46.66	46.50	46.60	46.70	46.67	47.10	46.70	45.56
102	21.00	36.00	38.00	37.75	37.30	37.70	37.70	37.76	37.80	37.80	37.50
108	20.00	30.00	30.90	30.46	29.70	30.50	30.50	30.47	30.20	30.50	30.94
114	19.00	25.00	25.70	25.22	24.30	25.30	25.20	25.23	24.60	25.20	25.88
120	19.00	22.00	22.10	21.74	20.60	21.80	21.70	21.74	21.20	21.70	22.44
126	18.00	19.00	20.20	19.99	19.60	20.00	20.00	20.00	19.60	20.00	20.28

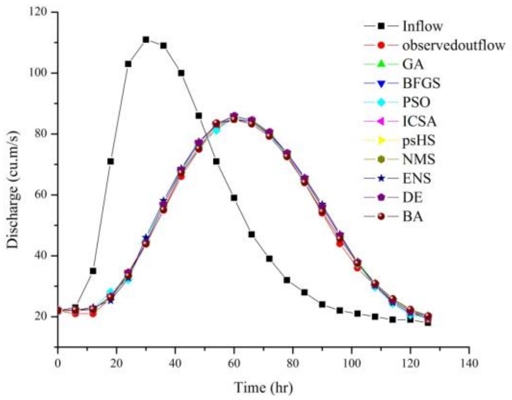

Figure 2: Comparison of routed outflow hydrograph of various models

Table 2, shows the performance of each parameter estimation technique in routing the flow. From Table 2, it can be inferred that the R varies from 0.998 to 0.999, i.e., all the 9 models (including bat algorithm based parameter estimation) capture the mean that exists in the actual outflow hydrograph. Nash Sutcliffee efficiency (E) varies from 0.997 to 0.998, i.e., that the models are able to predict the peak flow with an accuracy of 99 %. RMSE measures the deviation between the actual and predicted, from Table 2 it can be visualized that RMSE varies from 35 to 62 cu.m/s (previously reported models), in case of bat algorithm based parameter estimation model, RMSE is 18.25 cu.m/s. This gives a clear picture about the performance of the bat algorithm in estimating the parameters of Muskingum Model. To have a clear picture of the performance of the developed model, the maximum residual in capturingdischarge is measured. The maximum resuidal is about 1.64 cu.m/sec, in case of GA and NMS, the residuals are 2.70 cu.m/s and 2.67 cu.m/s. From the above, it can be inferred that BA based parameter estimation model routes the flow hydrograph with minimum discharge error compared to other existing models.

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Table 2: comparsion of estimated parameter values for various techniques

Parameter estimation technique	Reported by	k	x	m	R2				
GA	Mohan, 1997	0.1033	0.2813	1.8282	[illegible]				
BFGS	Geem, 2006	0.0966	0.2851	1.8434	[illegible]				
PSO	Chu and Chang, 2009	0.1824	0.3330	2.1458	[illegible]				
ICSA	Luo et al., 2010	0.0884	0.2862	1.8624	[illegible]				
psHS	Geem, 2011	0.0863	0.2869	1.8679	[illegible]				
NMS	Bharat, 2011	0.0862	0.2869	1.8681	0.9995	1.0660	36.7650	0.9970	0.8192
ENS	Vatankhan, 2014	0.0542	0.2830	2.3720	0.9984	1.3364	62.2800	0.9942	1.3530
DE	Xu et al., 2012	0.5175	0.2869	1.8680	0.9995	1.0773	37.5100	0.9969	0.8396
BA	proposed	0.6663	0.3469	1.8669	0.9995	0.7478	18.2550	0.9985	0.4596

Figure 3, shows the developed model outflow predictions with that of the actual outflow hydrograph. From Figure 3, it can be observed that the predicted outflow hydrograph ordinates traces the actual hydrograph with minimum errors (Table 2). Figure 4, displays the 95% prediction and confidence band, narrower the width of the band is a good indication towards dependability of model results. From Figure 4, it can be inferred actual and the predicted scatter plot almost falls in 45 degree line (perfect model fit), highlighting the performance of the developed model. In order to have further insight into the model performance, the variation of ‘k’, ‘x’ and ‘m’ are analyzed (Figure 5).

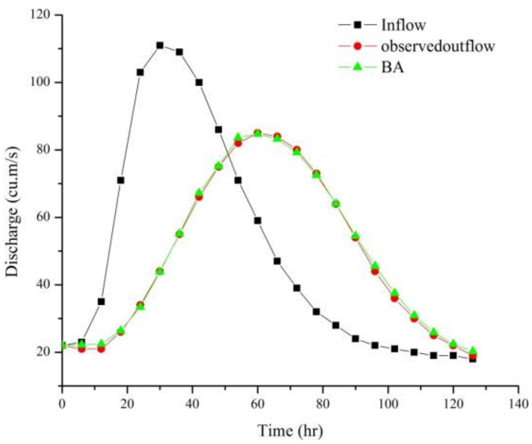


Figure 3: Comparison of routed hydrograph by Bat algorithm with that of observed outflow

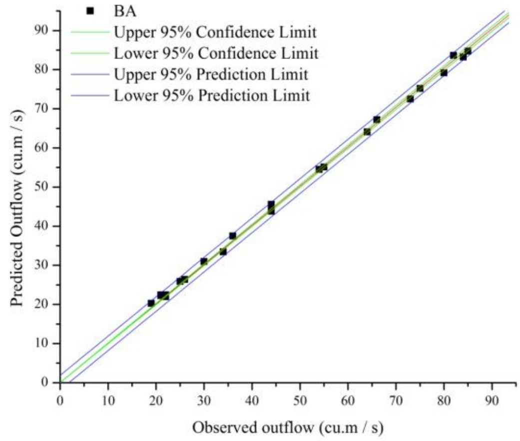


Figure 4: Prediction range outflows of developed bat algorithm

From Figure 5, it can be observed that visually that variation in storage parameter ‘k’ controls or depicts a variation in SSE compared to weighting parameter (x) and exponent parameter (m). From the correlation analysis between the three parameters, it is observed that storage parameter has a strong negative correlation of 0.96 with exponent parameter ‘m’ and a positive correlation 0.66 with that SSE (objective function). Whereas, weighting parameter ‘x’ has minimum positive correlation of 0.15 with SSE. From the above, it can be stated that accuracy in the estimation of ‘k’ and ‘m’ governs the performance of nonlinear three parameter Muskingum model.

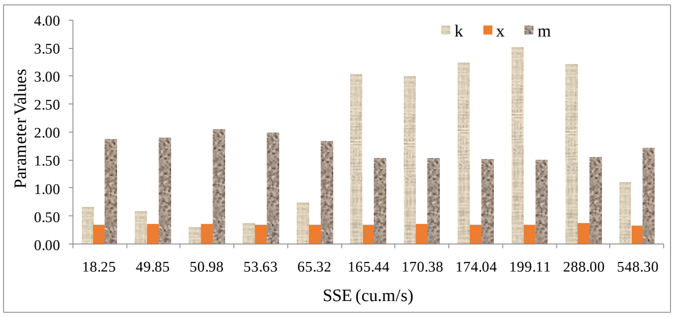
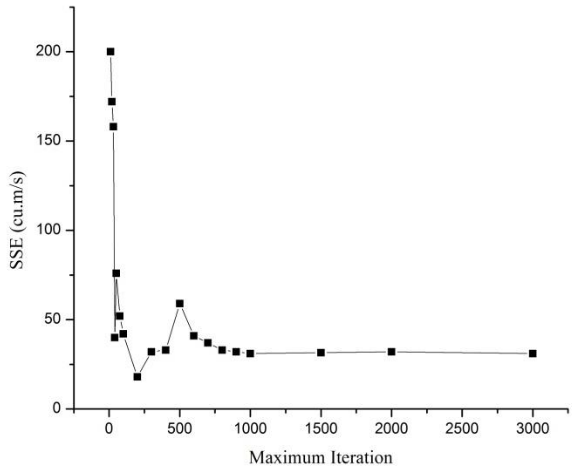
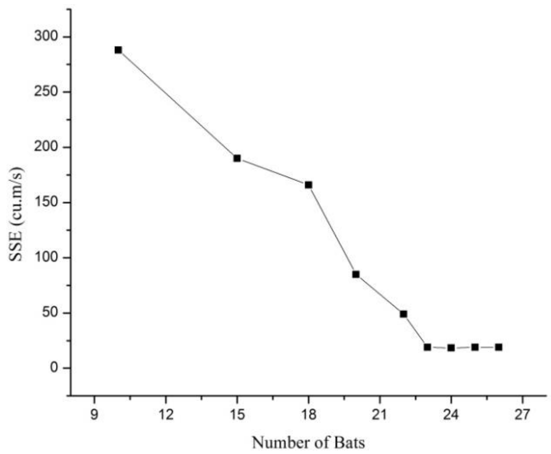


Figure 5: Comparison of developed Muskingum Model parameters with that of SSE

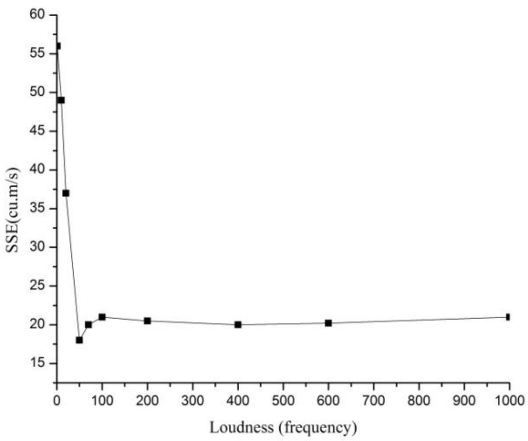
To understand the role of parameters of the bat algorithm in influencing the development model predictions, sensitivity analysis has been performed. The sensitivity analysis of the three parameters are performed, namely; a) maximum number of iteration, b) number of bats, and c) initial loudness (λ). The reason to select the above three parameters are; a) From Figure1, it can be inferred that specified maximum iteration controls the search of bats towards the solution in the domain space, b) sufficient number of bats are required to make random fly around a generated local solution, and c) loudness or sound wavelength, specified initial value will govern the convergence time towards a solution. Figure 6, shows the sensitivity analysis performed on the parameters of the developed bat algorithm based Muskingum parameter estimation model. From Figure 6 (a), it can be observed that minimum SSE has been achieved for maximum iteration in 200. Simiarlly minimum objective function value (SSE) has been achieved for 23 numbers of bats and initial loudness of 50 (Figure 6 (b & c)).



(a)



(b)



(C)

Figure 6: Senstivity anlaysis for the parameters of bat optimization algorithm

The Muskingum model parameters ‘k’,’x’, and ‘m’ resulted through a bat optimization algorithm are listed in Table 2. The characteristic of bat to explore the best solution among the generated local solutions in all directions by making a random flight or a search, enhances the capability to explore the solution space in a higher dimension compared to reported parameter estimation algorithms. These above characteristics have influenced the developed bat algorithm-based Muskingum parameter estimation model to outperform in routing the flood wave compared to all other existing models.

6.CONCLUSION

The simplicity of Muskingum model encouraged the water resource community to utilize the same more frequently for routing the flood wave. The challenge lies in calibrating the parameters of the Muskingum model. Many research works are reported towards achieving accuracy in estimating the model parameters. Optimization models such as GA, NMS, DE etc., found its applicability in estimating the parameters. In the process of increasing the parameter estimation accuracy, in this research a bat optimization algorithm is used. BA excels with minimum number of algorithm parameters compared to GA and NMS, and also explores an entire solution space by conducting ‘n’ number of random flights in search of the best solution. From application to Muskingum model parameter estimation, the developed model has outperformed all other existing parameter estimation models with minimum RMSE of 18.25 cu.m/s, with a maximum residual of 1.69 cu.m/s. This reveals that the developed bat algorithm-based Muskingum parameter estimation model works better for the consider dataset. In this study, a simple bat algorithm with an assumption on a positive variation of loudness has been used, but loudness can vary based on the distance from the solution. Further research has to be carried out to overcome the above assumption that may improve the optimal solution and thus might encourage the applicability of the bat optimization algorithm in various areas of water resources domain.

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