International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 10 Issue: 10 | Oct - 2026 SJIF Rating: 8.659 ISSN: 2582-3930

Preliminary Structural Design and Criterion Prediction of Steel Beams Using the Deflection Governance Ratio

Ansh H. Vadgama[1], Utsav M. Dholakiya[2]

[1]Student, Department of Mechanical Engineering, Dr. Subhash University, Junagadh, Gujarat, India
[2]Assistant Professor, Department of Mechanical Engineering, Dr. Subhash University, Junagadh, Gujarat, India

---

Abstract - A steel beam designed for ultimate limit state (bending strength) may exceed the serviceability limit state for deflection, so section selection must consider both strength and stiffness. This paper reviews beam-deflection theory, support conditions, and calculation methods. For an ISMB 400 beam spanning 8 m under combined loading, Macaulay's method, the unit load method, and finite element analysis yielded an identical mid-span deflection of 22.57 mm. The paper introduces the Deflection Governance Ratio (DGR), a closed-form measure that compares the load permitted by the deflection limit with the load permitted by bending strength, and shows that it equals the ratio of strength utilization to deflection utilization. The governing criterion can then be identified from a critical span-to-depth ratio that depends on the loading pattern, steel properties, deflection limit, and share of load considered, not on the selected section size. Critical ratios are presented for ten support and loading cases and three steel grades. A preliminary design procedure based on a balanced depth selects ISMB 400 directly for a 7 m floor beam. Support restraint, cracking in reinforced-concrete beams, and excessive deflection in service are also examined. The criterion helps engineers identify the governing check early in preliminary design.

Key Words: Beam deflection, Deflection Governance Ratio, stiffness criterion, span-to-depth ratio, serviceability limit state, high-strength steel.

1. INTRODUCTION

When a beam carries a load, its neutral axis moves away from its original position. This transverse displacement is called deflection. Some deflection is unavoidable; the engineering question is whether it remains within an acceptable limit. Structural design must satisfy both ultimate limit states, which concern the member's ability to carry loads safely, and serviceability limit states, which concern its performance during normal use.

Deflection is especially important in long-span floors, roof purlins, timber joists, precast members, machine shafts and beds, aircraft structures, and micro-cantilever sensors. A beam can have adequate bending strength and yet be unsuitable because its deformation causes discomfort, damages finishes, or interferes with equipment.

In practice, designers often check strength first and deflection afterwards. If the section passes the strength check but fails the deflection limit, the design must be revised. This paper addresses the question of whether the governing criterion can be predicted before a particular section is selected.

The main contribution is a closed-form Deflection Governance Ratio (DGR), defined as the ratio of the load allowed by the deflection limit to the load allowed by bending strength. The paper develops the criterion, compares three methods for calculating deflection, tabulates critical span-to-depth ratios for ten support and loading cases and three steel grades, and presents a preliminary design procedure based on balanced depth. The analysis is limited primarily to straight, prismatic, linearly elastic beams under static loading; cracked reinforced-concrete behaviour is discussed separately.

2. LITERATURE REVIEW

The theory of elastic beam deflection developed through contributions from several researchers. Bernoulli related the curvature of a bent elastic beam to its bending moment, and Euler developed the theory of the elastica in 1744. Navier's work on the neutral axis helped establish the familiar beam equation, $EI \cdot y'' = M$ [1]. Clebsch introduced singularity functions in 1862 [2], and Macaulay later popularised the bracket notation used to represent changing loads and moments along a beam [3].

Mohr's work introduced the moment-area and conjugate-beam approaches [4], while Castigliano's theorems connected deflection with strain energy [5]. Timoshenko's work extended classical beam theory to account for shear deformation and rotary inertia [6]. Later, the finite element method made it practical to analyse structures with variable sections and more complex material behaviour [7].

For reinforced concrete, Branson proposed an effective moment of inertia that interpolates between gross and cracked section stiffness; this approach was adopted in the American Concrete Institute (ACI) code ACI 318 [8]. Bischoff and Scanlon reported that Branson's method can overestimate stiffness at reinforcement ratios below approximately 1% [9]. Marino examined ponding stability in roof systems [10], while the American Institute of Steel Construction (AISC) Design Guide 11 discusses the relationship between floor stiffness and vibration comfort [11].

Standard textbooks explain the established deflection methods separately [12]–[14]. Rattan distinguishes the strength criterion, which limits stress, from the stiffness criterion, which limits deflection, and notes that a strong beam can still deflect excessively [14]. Indian Standards (IS) specify deflection requirements for steel and reinforced-concrete structures through IS 800 and IS 456, respectively [15], [16]. The present study builds on this established background by proposing a closed-form ratio intended to indicate which criterion is likely to govern a steel beam before a section is selected.

3. THEORY OF BEAM DEFLECTION

The deflected shape of a beam's axis is known as its elastic curve, represented here by y(x) [14]. The slope at any point is tan θ = dy/dx, and the flexural rigidity EI describes the section's resistance to bending. In this notation, x is measured from the left support and y is positive upward (Fig -1); downward deflection is therefore negative [14].

The curvature of the elastic curve is related to the bending moment by:

$$\frac{1}{\rho} = \frac{M(x)}{EI} \tag{1}$$

For small beam slopes, the square of the slope is negligible compared with 1, so the curvature can be approximated by d^2y/dx^2. The familiar elastic curve equation is therefore:

$$EI\,\frac{d^2y}{dx^2} = M(x) \tag{2}$$

Differentiating the equation gives the corresponding relationships with shear force and distributed load: $EI \cdot d^3y/dx^3 = V(x)$ and $EI \cdot d^4y/dx^4 = -w(x)$, subject to the sign convention adopted.

For steel, E is approximately 200 GPa; typical values are 25–35 GPa for concrete and 8–14 GPa for timber. For a rectangular section, $I = bd^3/12$. Because depth appears to the third power, doubling the depth increases the second moment of area by a factor of eight, whereas doubling the width only doubles it. For a standard case, the maximum deflection can be written as $\delta_{max} = k \cdot WL^3/EI$, where W is the total load and k depends on the support arrangement and load distribution. Span therefore has a strong influence on deflection.

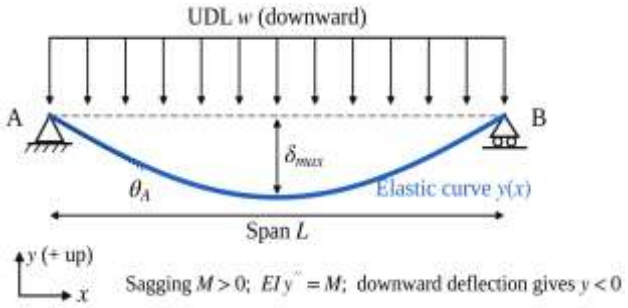


Fig -1: Elastic curve of a simply supported beam under a uniformly distributed load (UDL) and the sign convention used

The classical theory assumes that the beam is initially straight and prismatic; the material is homogeneous, isotropic, and linearly elastic; plane sections remain plane; slopes are small, so $(dy/dx)^2 \ll 1$; shear deformation is negligible for sufficiently slender beams (approximately span/depth above 10); and superposition is valid.

4. TYPES OF BEAM SUPPORTS

Support conditions determine the boundary conditions of the elastic curve. A support that restrains more rotation or translation generally makes the beam stiffer (Table 1, Fig -2). Real connections may fall between ideal pinned and fully fixed conditions, so assuming full fixity where a connection can rotate may underestimate deflection.

Table -1: Support types and boundary conditions

Support	Reactions	Boundary condition
Roller	V	y = 0, M = 0
Pinned (hinged)	H, V	y = 0, M = 0
Fixed (built-in)	H, V, M	y = 0, θ = 0
Free end	None	M = 0, V = 0
Guided (sliding)	H, M	θ = 0, V = 0

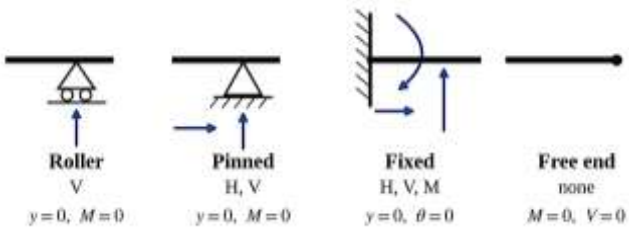


Fig -2: Support symbols, the reactions they provide and the conditions they impose

Common beam forms include simply supported beams, cantilevers, fixed beams, propped cantilevers, continuous beams, and overhanging beams [14]. Simply supported beams, cantilevers, and overhanging beams can generally be solved using equilibrium alone. Fixed, propped, and continuous beams are statically indeterminate because they have more unknown reactions than independent equilibrium equations. Compatibility conditions based on deflection are needed to determine those reactions.

5. METHODS FOR CALCULATING DEFLECTION

The principal methods include double integration, Macaulay's method, the moment-area method, the strain-energy method, and the conjugate-beam method [14] (Table 2). The finite element method provides a numerical alternative for more complex cases.

5.1 Double integration

The moment equation is integrated twice, and the integration constants are determined from the boundary conditions [14]. This method gives the complete elastic curve, but the algebra can become lengthy when the bending-moment expression changes along the beam.

5.2 Macaulay's method

Macaulay's method uses one bending-moment expression for the full span, with bracket terms such as $\langle x - a \rangle$ that are zero when $x < a$ [2], [3]. The bracket terms can be integrated without expanding every loading interval, allowing one pair of integration constants to be used for the beam [14].

5.3 Moment-area method

Mohr's first theorem states that the change in slope between two points equals the area under the M/EI diagram between them. The second theorem relates the vertical offset from a tangent to the first moment of that area [4], [14]. This method is particularly useful when the deflection at a specific point is required.

5.4 Conjugate-beam method

In the conjugate-beam method, a fictitious beam is loaded with the real beam's M/EI diagram. The shear force in the conjugate beam corresponds to the slope of the real beam, while the conjugate-beam moment corresponds to the real beam's

deflection. The method is useful when flexural rigidity varies along the span.

5.5 Energy methods and numerical analysis

Castigliano's theorem gives deflection by differentiating bending strain energy, $U = \int M^2/2EI\ dx$, with respect to the relevant load: $\delta = \partial U/\partial P$ [5]. The unit load method gives $\delta = \int M \cdot m/EI\ dx$, where m is the bending moment produced by a unit load applied at the point and in the direction of the required displacement. These methods are useful for frames, trusses, and curved members.

For linear behaviour, the deflections caused by separate loads can be added using superposition. The finite element method assembles element stiffness matrices and can accommodate variable sections, cracking, and dynamic loading [7]. Numerical results should still be checked against analytical calculations or other appropriate validation methods.

Table -2: Comparison of methods

Method	Best suited to	Full elastic curve?
Double integration	Simple loading; deriving formulas	Yes
Macaulay's method	Several point loads or partial loads	Yes
Moment-area method	Deflection at a selected point	No
Conjugate-beam method	Varying flexural rigidity	Point by point
Energy methods	Frames and curved members	Depends on application
Finite element method	Complex geometries or material behaviour	Yes, within model assumptions

6. FORMULAS, DERIVATION AND WORKED EXAMPLE

6.1 Standard formulas

Table 3 summarises commonly used maximum-slope and maximum-deflection expressions. The determinate-beam expressions follow Rattan [14, Eqs. 7.4–7.26], using W for a point load, w for a uniformly distributed or peak load intensity, and l for span. Expressions for indeterminate cases are drawn from [12], [13].

Table -3: Maximum slope and deflection

No.	Beam and loading	End slope θ (magnitude)	Maximum deflection δ_{max}
1	Cantilever, point load W at free end	$Wl^2/2EI$	$Wl^3/3EI$
2	Cantilever, UDL w	$wl^3/6EI$	$wl^4/8EI$
3	Cantilever, end couple M	Ml/EI	$Ml^2/2EI$
4	Cantilever, triangular load rising from 0 to w	$wl^3/24EI$	$wl^4/30EI$
5	Simply supported, central point load W	$Wl^2/16EI$	$Wl^3/48EI$
6	Simply supported, off-center point load W at distances a and b from the supports (a > b)	$Wb(l^2-b^2)/6EIl$	$Wb(l^2-b^2)^{1.5}/9\sqrt{3}\cdot EIl$
7	Simply supported, UDL w	$wl^3/24EI$	$5wl^4/384EI$
8	Simply supported, triangular load rising from 0 to w	$7wl^3/360EI$	$0.00652\ wl^4/EI$
9	Propped cantilever, UDL w	$wl^3/48EI$	$\approx wl^4/185EI$
10	Fixed–fixed beam, UDL w	0 (at ends)	$wl^4/384EI$

For a uniformly distributed load, fixing both ends reduces the maximum deflection by 80% compared with a simply supported beam of the same span. A cantilever under the same UDL has 9.6 times the maximum deflection of the simply supported beam.

6.2 Derivation for a simply supported beam under uniformly distributed load (Case 7)

For a simply supported beam carrying a uniformly distributed load w over span L, the bending moment at a distance x from the left support is $M(x) = wLx/2 - wx^2/2$.

Integrating the elastic-curve equation twice and applying $y(0) = y(L) = 0$ gives $C_2 = 0$ and $C_1 = -wL^3/24$, and hence the elastic curve:

$$y(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3) \qquad (3)$$

At mid-span, x = L/2, this gives $y = -5wL^4/384EI$, so the maximum deflection magnitude is $\delta_{max} = 5wL^4/384EI$. The slope at the left support has magnitude $\theta_A = wL^3/24EI$, consistent with the cited expressions in Rattan [14, Eqs. 7.23–7.24].

6.3 Worked example: one beam, three methods

Consider a simply supported ISMB 400 beam (ISMB: Indian Standard Medium Weight Beam) with $I = 20{,}458.4 \times 10^4$ mm⁴ and E = 200 GPa, giving EI = 40,917 kN·m². The span is 8 m and the beam carries a 10 kN/m UDL plus a 40 kN point load located 3 m from support A. The support reactions are $R_A = 65$ kN and $R_B = 55$ kN.

$$EI\,y = \frac{65}{6}x^3 - \frac{5}{12}x^4 - \frac{20}{3}\langle x-3\rangle^3 + C_1 x \qquad (4)$$

Using Macaulay's method, the moment expression is integrated and the boundary condition y(8) = 0 is applied, which gives $C_1 = -375.83$ kN·m²; at x = 4 m, EI·y = −923.33 kN·m³. The resulting mid-span deflection is 22.57 mm. The unit load method, with m = x/2 to mid-span, gives the same value, and the finite element model returns 22.57 mm for meshes containing 8 to 32 cubic beam elements. The calculated maximum, 22.59 mm at x = 3.88 m, is approximately 0.1% greater than the mid-span value.

The beam meets a span/300 limit of 26.7 mm but narrowly exceeds a span/360 limit of 22.2 mm, which may apply where brittle finishes are present. This example illustrates why the appropriate serviceability limit must be selected for the actual application.

7. PROPOSED DEFLECTION-GOVERNANCE CRITERION

7.1 Formulation

Consider a laterally restrained, doubly symmetric beam with span L, depth d, and total load Q. For a distributed load, Q = wL; for a point load, Q = W. For a given support and loading case, the peak moment and maximum deflection can be written as $M = k_M \cdot QL$ and $\delta = k_\delta \cdot QL^3/EI$, where k_M and k_δ are coefficients determined by the support arrangement and load shape.

The bending-strength criterion limits the load to $Q_\sigma = 2\sigma_s \cdot I/(k_M \cdot L \cdot d)$, where σ_s is the permissible service stress and $Z = 2I/d$ is used for a doubly symmetric section. The deflection limit $\delta \le L/n$ gives a corresponding load limit $Q_\delta = EI/(\alpha \cdot k_\delta \cdot n \cdot L^2)$, where α represents the share of the total load included in the deflection check; α = 1 when the entire load is considered.

The Deflection Governance Ratio is defined as the ratio of these two load capacities:

$$DGR = \frac{Q_\delta}{Q_\sigma} = \Phi \cdot \frac{E}{2\,\alpha\, n\, \sigma_s} \cdot \frac{d}{L}, \qquad \Phi = \frac{k_M}{k_\delta} \tag{5}$$

The stiffness criterion governs when DGR < 1. Equivalently, this occurs when the span-to-depth ratio exceeds the critical value:

$$\lambda_{cr} = \Phi \cdot \frac{E}{2\,\alpha\, n\, \sigma_s} \tag{6}$$

To bridge Working Stress Method (WSM) and Limit State Design (LSD) per IS 800:2007, the equivalent elastic service stress σ_s is approximated by scaling the yield strength f_y using the product of the material factor γ_{m0}= 1.10 and nominal live-load factor γ_f = 1.50.

The critical ratio does not include a section-size term because Z = 2I/d for the doubly symmetric sections assumed here. The governing criterion depends on the load-shape factor Φ, Young's modulus E, permissible stress σ_s, deflection-limit parameter n, and load share α. In this study, $\sigma_s = f_y/(\gamma_{m0} \cdot \gamma_f) = f_y/1.65$, using γ_{m0} = 1.10 [15] and a mean load factor γ_f = 1.5.

Strength utilisation is $u_\sigma = M/(\sigma_s \cdot Z)$, while deflection utilisation is $u_\delta = \delta/(L/n)$. Because both utilisations vary linearly with the same load under the stated assumptions, their ratio is constant for a given section and span. From the definitions above:

$$DGR = \frac{u_\sigma}{u_\delta} \tag{7}$$

For plastic or compact rolled sections where design strength is dictated by $Z_p = \alpha_p Z_e$ (where $\alpha_p \approx 1.12$--1.15 for I-sections), the critical ratio λ_{cr} can be scaled by $1/\alpha_p$.

7.2 Load-shape factor Φ

Table 4 lists the load-shape factor Φ and critical span-to-depth ratios for ten common support and loading cases (Chart - 1). The deflection coefficients for determinate beams are taken from Table 3 and Rattan [14]; values for indeterminate beams are based on [12], [13], while the moment coefficients are obtained from statics. Values below use α = 1 and E = 200 GPa.

Table -4: Load-shape factor Φ and critical span-to-depth ratio λ_{cr} (α = 1, E = 200 GPa)

Support and loading	Φ	n	E250 λ_{cr}	E350 λ_{cr}	E450 λ_{cr}
Simply supported, UDL	9.6	300	21.1	15.1	11.7
Simply supported, central point load	12.0	300	26.4	18.9	14.7
Simply supported, triangular load	9.8	300	21.6	15.5	12.0
Propped cantilever, UDL	23.1	300	50.9	36.3	28.3
Propped cantilever, central point load	20.1	300	44.3	31.6	24.6
Fixed–fixed, UDL	32.0	300	70.4	50.3	39.1
Fixed–fixed, central point load	24.0	300	52.8	37.7	29.3
Cantilever, UDL	4.0	150	17.6	12.6	9.8
Cantilever, end point load	3.0	150	13.2	9.4	7.3
Cantilever, triangular load	5.0	150	22.0	15.7	12.2

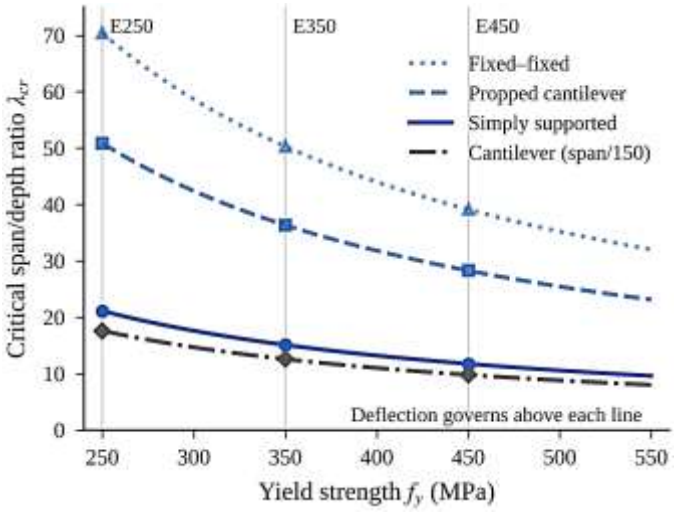


Chart -1: Critical span-to-depth ratio against yield strength for UDL on four support conditions

7.3 Preliminary design procedure based on the Deflection Governance Ratio

The criterion gives a balanced depth at which the strength and deflection limits are reached at the same load:

$$d_b = \frac{L}{\lambda_{cr}} \tag{8}$$

A preliminary design can then follow five steps:

1) Select Φ from Table 4 for the support condition and loading pattern.

2) Calculate λ_{cr} from Eq. (6) using the selected steel grade, deflection limit n, and load share α.

3) Calculate the balanced depth d_b from Eq. (8).

4) For a section deeper than d_b, size the beam using the strength requirement, $Z \geq M/\sigma_s$. For a shallower section, use the stiffness requirement, $I \geq \alpha \cdot k_\delta \cdot n \cdot Q \cdot L^2/E$.

5) Check the other utilisation. Under the stated assumptions, Eq. (7) relates the two utilisations (the other utilisation equals u_σ/DGR for a strength-governed beam or $DGR \cdot u_\delta$ for a stiffness-governed beam), so a second sizing iteration should not be required for the preliminary comparison. All applicable code checks are still necessary.

7.4 Main findings from the criterion

1) Section size: the same critical span-to-depth ratio applies to the rolled doubly symmetric sections considered, so the likely governing check can be identified before selecting a particular section.

2) Steel grade: higher-strength steel becomes deflection-governed at a lower span-to-depth ratio. For a simply supported beam under UDL, λ_{cr} decreases from 21.1 for E250 to 11.7 for E450 because yield strength increases while Young's modulus remains approximately unchanged.

3) Load shape: replacing a UDL with a central point load increases λ_{cr} by 25% for a simply supported beam, but decreases it by 13% for a propped cantilever and by 25% for fixed–fixed beams and cantilevers.

4) End restraint: under UDL, full fixity increases λ_{cr} from 21.1 to 70.4 for E250, shifting the balance towards strength control.

5) Load share: because λ_{cr} varies inversely with α, checking only half the total load ($\alpha = 0.5$) doubles λ_{cr} from 21.1 to 42.2.

8. VALIDATION AND PARAMETRIC STUDY

8.1 Section independence

For five ISMB sections, the span at which $Q_\delta = Q_\sigma$ was found numerically using handbook values of I and Z for a simply supported beam under UDL, with E250 steel and n = 300. Each section reached the crossover at L/d = 21.12, consistent with Eq. (6) (Table 5).

Table -5: Numerical crossover spans for ISMB sections

Section	Depth d (mm)	I (×10⁴ mm⁴)	Crossover span (m)	L/d
ISMB 200	200	2,235.4	4.22	21.1
ISMB 250	250	5,131.6	5.28	21.1
ISMB 300	300	8,603.6	6.34	21.1
ISMB 400	400	20,458.4	8.45	21.1
ISMB 500	500	45,218.3	10.56	21.1

8.2 Design example using the proposed procedure

Consider a simply supported floor beam spanning 7 m and carrying 20 kN/m, with E250 steel, n = 300, and $\alpha = 1$. Table 4 gives $\lambda_{cr} = 21.1$, so the balanced depth is $d_b = 7000/21.1 = 331$ mm. A practical section deeper than 331 mm is therefore treated as strength-governed in this preliminary selection.

The required section modulus is $Z \geq 808.5 \times 10^3$ mm³. ISMB 350, with $Z = 778.9 \times 10^3$ mm³, is insufficient, so ISMB 400, with $Z = 1{,}022.9 \times 10^3$ mm³, is selected. Its strength utilisation is $u_\sigma = 0.79$. From Eq. (7), $u_\delta = 0.79/1.207 = 0.65$, corresponding to a deflection of approximately 15.3 mm against the 23.3 mm span/300 limit.

If the beam were sized for deflection first, ISMB 350 would be selected because its second moment of area is 13,630 × 10⁴ mm⁴, slightly above the calculated requirement of 13,398 × 10⁴ mm⁴. However, that section has a strength utilisation of 1.04 and fails the stated strength check. This comparison illustrates the value of checking strength and stiffness together (Table 6).

Table -6: Utilisation check for the 7 m design example

Section	L/d	u_σ	u_δ	u_σ/u_δ	DGR	Governing check
ISMB 300	23.3	1.410	1.557	0.905	0.905	Deflection
ISMB 350	20.0	1.038	0.983	1.056	1.056	Strength
ISMB 400	17.5	0.790	0.655	1.207	1.207	Strength
ISMB 450	15.6	0.599	0.441	1.358	1.358	Strength

8.3 Effect of span and support condition

The deflection of an ISMB 300 beam (I = 8,603.6 × 10⁴ mm⁴, E = 200 GPa) carrying 20 kN/m was calculated for spans from 3 m to 10 m and compared with the IS 800 span/300 limit [15] (Chart -2). The simply supported beam meets the limit up to approximately 6.04 m, while the bending-strength limit based on $f_y/1.65$ is reached at 5.90 m (L/d = 19.7). The comparison is consistent with the criterion's prediction that strength governs below L/d = 21.1. The propped cantilever meets the limit up to 8 m, and the fixed–fixed beam meets it up to at least 10 m. This demonstrates how end restraint can substantially increase the usable span for a given section.

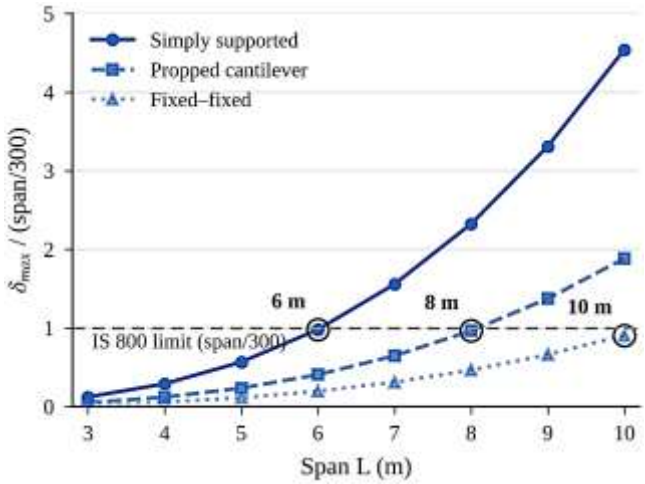


Chart -2: Deflection relative to the span/300 limit, ISMB 300 under 20 kN/m

For a uniformly distributed load, elastic deflection scales with the fourth power of span, whereas the span/300 limit increases linearly with span. Consequently, doubling the span from 5 m to 10 m multiplies the calculated deflection by 16, assuming the other parameters remain unchanged.

9. EXTENSION AND LIMITATIONS REGARDING NON-STEEL AND CRACKED MEMBERS

When the service moment M_a exceeds the cracking moment $M_{cr} = f_{cr} \cdot I_g/y_t$, where $f_{cr} = 0.7\sqrt{f_{ck}}$, concrete cracks and the member's effective stiffness decreases. Branson's equation [8]

estimates effective inertia by interpolating between the gross and cracked section values:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \qquad (9)$$

IS 456 Annex C [16] uses a different expression for effective inertia, $I_{eff} = I_r / [1.2 - (M_r/M)\,(z/d)\,(1 - x/d)\,(b_w/b)]$. To illustrate the difference, consider a 300 × 500 mm M25 reinforced-concrete beam with effective depth d = 450 mm, three 16 mm bars, reinforcement ratio ρ = 0.45%, span 6 m, service moment M_a = 60 kN·m, cracking moment M_{cr} = 43.75 kN·m, and cracked inertia $I_{cr} = 690 \times 10^6$ mm⁴. The short-term deflections are summarised in Table 7.

Table -7: Effect of cracking on deflection

Inertia model	I (×10⁶ mm⁴)	Deflection (mm)
Gross section, I_g	3,125	2.88
Branson (ACI 318 [17])	1,634	5.51
IS 456 Annex C	1,007	8.94
Fully cracked, I_{cr}	690	13.04

For this example, the gross-section estimate is approximately 3.1 times lower than the IS 456 estimate, while Branson's equation gives a deflection about 38% lower than the IS 456 result. This is consistent with reported concerns that Branson's equation may overestimate stiffness at reinforcement ratios below 1% [9]. Creep and shrinkage can also increase long-term deflection, sometimes to two or three times the short-term value; the actual increase depends on material properties, loading history, humidity, and member details.

10. WHY EXCESSIVE DEFLECTION IS DANGEROUS

10.1 Damage to non-structural elements

Excessive beam deflection can crack brittle partitions, plaster, glazing, tiles, and cladding. It can also cause doors to jam and joints to leak. These problems may occur even when the beam has not reached its bending-strength limit, and visible sagging can lead occupants to question the safety of the structure [14].

10.2 Ponding on roofs

On a flat roof, deflection can create a shallow basin that collects water. The added water increases the load and may cause further deflection. In a sufficiently flexible roof, this feedback can lead to progressive collapse. A documented failure involved drains clogged by leaves during a storm that produced about 90 mm of rain in a few hours; trapped water deflected a steel girder, and the growing deformation contributed to a partial roof collapse [18]. Roof drainage and ponding stability should therefore be considered during design [10].

10.3 Vibration and equipment malfunction

Flexible floors can have lower natural frequencies and may respond noticeably to walking or machinery. A commonly used approximate relationship is $f \approx 18/\sqrt{\delta}$ Hz, where δ is the deflection in millimeters [11]. Excessive movement can also cause crane rails to bind, misalign shafts and bearings, and place unwanted stress on attached pipes and services.

10.4 Instability and long-term effects

When an axially loaded member deflects, the additional moment associated with the P–Δ effect can influence stability. In concrete and timber, creep can cause deflection to continue increasing over time. Deflection measurements that differ substantially from predictions should be investigated rather than dismissed.

10.5 Deflection as a warning sign

During construction of the Hartford Civic Center roof, measured deflection was reported to be about twice the predicted value, but concerns were not acted upon. The roof collapsed under snow in January 1978, and investigators found that compression members were severely overloaded [19]. This case illustrates the importance of investigating unexpected structural movement.

11. CODE LIMITS AND DEFLECTION CONTROL

Table 8 summarises typical deflection limits cited in the referenced standards [15]–[17], [20], [21], including the International Building Code (IBC). The applicable limit must be confirmed against the current edition of the relevant code, the structural system, the loading combination, and the finish or occupancy requirements. IS 456 also controls reinforced-concrete deflection through basic span-to-effective-depth ratios, with reference values of 7 for cantilevers, 20 for simply supported beams, and 26 for continuous beams.

Table -8: Typical deflection limits

Code	Condition	Typical limit
IS 456	Final deflection, including creep and shrinkage	L/250
IS 456	Deflection after partitions are built	L/350 or 20 mm
IS 800	Floor, live load, ordinary finishes	L/300
IS 800	Floor, live load, brittle finishes	L/360
ACI 318 / IBC	Floor, live load	L/360
Eurocode 2	Appearance, quasi-permanent load	L/250

Deflection can be controlled by increasing beam depth ($I \propto d^3$), selecting I- or box-shaped sections with greater second moment of area, improving end restraint where structurally justified, reducing span, using composite action, applying pre-camber, or prestressing. In reinforced-concrete beams, additional compression reinforcement and appropriate formwork-removal timing may also help. Deflection should be monitored during construction when the design or construction sequence warrants it.

The classical formulas are most accurate for slender, elastic beams under static loads. They may underestimate deformation in deep beams, where Timoshenko theory or finite element analysis may be more suitable [6]. Large deflections, yielding, cracking, dynamic loading, semi-rigid supports, and axial-force effects may require nonlinear, dynamic, or second-order analysis. The proposed criterion is also limited to laterally restrained, doubly symmetric sections, a single load case, and a uniform service-stress assumption. Lateral-torsional buckling, unsymmetric or composite sections, and combined loading require separate evaluation.

12. CONCLUSIONS

1) A closed-form Deflection Governance Ratio was developed as the ratio of deflection-limited load capacity to strength-limited load capacity. Under the assumptions stated in the paper, it also equals the ratio of strength utilization to deflection utilization. The critical span-to-depth ratio is $\lambda_{cr} = \Phi \cdot E/(2\alpha n \sigma_s)$ and is independent of section size for the doubly symmetric sections considered.

2) The load distribution and support conditions have a substantial influence on the critical ratio. A central point load increases λ_{cr} by 25% for a simply supported beam, while it decreases the ratio for the restrained-beam and cantilever cases examined.

3) Higher-strength steel can become deflection-governed at a lower span-to-depth ratio because the elastic modulus changes little as yield strength increases. For a simply supported beam under UDL, the tabulated ratio falls from 21.1 for E250 to 11.7 for E450.

4) The balanced-depth procedure selected ISMB 400 for the 7 m floor-beam example. Selecting a section based on deflection alone would have led to ISMB 350, which did not satisfy the strength check in the example.

5) Support restraint can significantly reduce deflection. For a UDL, the fixed–fixed case reduces the maximum deflection by 80% compared with the simply supported case. The ISMB 300 study also showed a larger usable span when end restraint was increased.

6) Macaulay's method, the unit load method, and the finite element model produced the same reported mid-span deflection of 22.57 mm for the worked example. The reinforced-concrete example also showed that the estimated deflection depends strongly on the effective stiffness adopted.

Future work could extend the criterion to combined load cases, unsymmetrical and composite sections, and lateral-torsional buckling, and could validate its predictions against laboratory load tests.

DECLARATION

During the preparation of this work, the authors used Claude (Anthropic) to assist with initial text drafting and manuscript formatting. The authors critically reviewed, verified, and edited all generated content, independently performed and validated all mathematical calculations, and take full responsibility for the final integrity of the publication.

NOMENCLATURE

Symbol	Definition	Unit
x	Longitudinal distance along the beam from the left support	m, mm
y(x)	Transverse deflection of the neutral axis (positive upward)	mm
δ, δ_{max}	Magnitude of maximum transverse deflection (positive downward)	mm
θ	Slope of the elastic curve (θ=dy/dx)	rad
L	Clear span length of the beam	m, mm
d,b	Overall depth and flange width of the cross-section	mm
W,P	Concentrated (point) load	kN
w	Uniformly distributed load (UDL) per unit length	kN/m, N/mm
Q	Total applied transverse load (Q=wL or Q=W)	kN
M(x)	Bending moment at distance x	kN·m
V(x)	Shear force at distance x	kN
E	Modulus of elasticity (Young's modulus)	GPa, N/mm^2
I	Second moment of area about the principal bending axis	mm^4
Z, Z_e	Elastic section modulus (Z=2I/d)	mm^3
Z_p	Plastic section modulus	mm^3
EI	Flexural rigidity of the beam	$kN·m^2$, $N·mm^2$
k_M, k_δ	Moment and deflection shape coefficients	–
Φ	Load-shape factor ($\Phi = k_M/k_\delta$)	–
DGR	Deflection Governance Ratio (Q_δ/Q_σ)	–
λ_{cr}	Critical span-to-depth ratio (L/d)	–
d_b	Balanced depth where strength and deflection govern equally	mm
n	Deflection limit denominator parameter (e.g., L/300→n=300)	–
α	Share of total load considered in deflection check	–
σ_s	Equivalent allowable/working service stress	MPa, N/mm^2
f_y	Yield strength of structural steel	MPa
γ_{m0}, γ_f	Partial safety factor for material strength and load factor	–
u_σ, u_δ	Bending strength utilisation and deflection utilisation	–
I_g, I_{cr}, I_{eff}	Gross, cracked, and effective moments of inertia (for concrete)	mm^4

REFERENCES

[1] S. P. Timoshenko, *History of Strength of Materials*. New York, NY, USA: McGraw-Hill, 1953.

[2] A. Clebsch, *Theorie der Elasticität fester Körper*. Leipzig, Germany: B. G. Teubner, 1862.

[3] W. H. Macaulay, "A note on the deflection of beams," *Messenger of Mathematics*, vol. 48, pp. 129–130, 1919.

[4] O. Mohr, "Beitrag zur Theorie der Holz- und Eisenkonstruktionen," *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, vol. 14, 1868.

[5] A. Castigliano, *Théorie de l'équilibre des systèmes élastiques et ses applications*. Turin, Italy: A. F. Negro, 1879.

[6] S. P. Timoshenko, "On the correction for shear of the differential equation for transverse vibrations of prismatic

bars," *Philosophical Magazine*, ser. 6, vol. 41, no. 245, pp. 744–746, 1921.

[7] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt, *Concepts and Applications of Finite Element Analysis*, 4th ed. New York, NY, USA: Wiley, 2002.

[8] D. E. Branson, "Instantaneous and time-dependent deflections of simple and continuous reinforced concrete beams," Alabama State Highway Dept., HPR Rep. No. 7, 1963.

[9] P. H. Bischoff and A. Scanlon, "Effective moment of inertia for calculating deflections of concrete members containing steel reinforcement and fiber-reinforced polymer reinforcement," *ACI Structural Journal*, vol. 104, no. 1, pp. 68–75, 2007.

[10] F. J. Marino, "Ponding of two-way roof systems," *Engineering Journal (AISC)*, vol. 3, no. 3, pp. 93–100, 1966.

[11] T. M. Murray, D. E. Allen, and E. E. Ungar, *Design Guide 11: Vibrations of Steel-Framed Structural Systems Due to Human Activity*, 2nd ed. Chicago, IL, USA: AISC, 2016.

[12] R. C. Hibbeler, *Mechanics of Materials*, 10th ed. Hoboken, NJ, USA: Pearson, 2017.

[13] J. M. Gere and B. J. Goodno, *Mechanics of Materials*, 9th ed. Boston, MA, USA: Cengage Learning, 2018.

[14] S. S. Rattan, *Strength of Materials*, 2nd ed. New Delhi, India: Tata McGraw Hill Education, 2011, ch. 4, "Shear force and bending moment," and ch. 7, "Slope and deflection," pp. 138–139, 290–364.

[15] *IS 800:2007*, General Construction in Steel — *Code of Practice (3rd Revision),* Bureau of Indian Standards, New Delhi, India.

[16] *IS 456:2000, Plain and Reinforced Concrete — Code of Practice (4th Revision),* Bureau of Indian Standards, New Delhi, India.

[17] *ACI 318-19, Building Code Requirements for Structural Concrete*. Farmington Hills, MI, USA: American Concrete Institute, 2019.

[18] T. Martins, "It's just water weight," *The Construction Specifier*, Jul. 31, 2025. [Online]. Available: https://www.constructionspecifier.com/?p=212434

[19] T. Hoke, "Engineers are not infallible," *Civil Engineering* (ASCE), Mar. 2020. [Online]. Available: https://www.asce.org/publications-and-news/civil-engineering-source/civil-engineering-magazine/article/2020/03/engineers-are-not-infallible

[20] *EN 1992-1-1:2004, Eurocode 2: Design of Concrete Structures — Part 1-1*. Brussels, Belgium: CEN, 2004.

[21] *International Building Code*, Table 1604.3, "Deflection limits." Washington, DC, USA: International Code Council, 2021.