Relationship between Pythagorean Triangles & Woodall Primes

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Abstract - In this article, we focus on generating the Pythagorean triangles using Woodall prime numbers by equating the ratio Area/Perimeter to different woodall prime numbers. Of these, a few interesting patterns are displayed.

Key Words: Pythagorean Triangle, Woodall prime number.

1.INTRODUCTION

Numbers play an important role in Mathematics which is the universal language. Number theory is the area of pure mathematics which concentrates on integer solutions. In number theory, Pythagorean triangles is a most interesting topic under study. Many authors have worked on generating these Pythagorean triangles with the help of different number patterns which can be found in [1–15]. In this paper, the ratio Area/Perimeter of a Pythagorean triangle is expressed as a Woodall prime number and we look into the exciting results.

2. BASIC DEFINITIONS

Definition 1:- Let (p, q, r) be a Pythagorean triple if it satisfies the Pythagorean equation \( p^2 + q^2 = r^2 \) where \( p, q, r \) are positive integers. A triangle which contains the sides as Pythagorean triple is known as Pythagorean Triangle and it is denoted by \( \text{T}(p, q, r): p^2 + q^2 = r^2 \), where \( p, q \) are legs and \( r \) is hypotenuse.

Definition 2:- A Pythagorean equation has the suitable solution of the form \( p = a_1^2 - a_2^2, q = 2a_1a_2 \) and \( r = a_1^2 + a_2^2 \), where \( a_1, a_2 > 0 \) such that \( a_1 > a_2 \). If \( a_1, a_2 \) are of opposite parity and gcd\( (a_1, a_2) = 1 \), then the solution is said to be primitive.

Woodall Prime Numbers:- Any natural number of the form \( n.2^n - 1 \) is known as Woodall number. Some of the few woodall numbers are 1, 7, 63, 159, 383, 895, ... A woodall prime number is a woodall number which is prime. Few examples of woodall primes are 7, 23, 383, 3221254719, 283341988972178712821759, ...

3. METHOD OF ANALYSIS

Let \( A_1 \) and \( P_1 \) be the area and perimeter of the triangle respectively.

Assume \( \frac{A_1}{P_1} = \text{Woodall Prime number} \)

This relationship results in the equation

\[
\frac{a_2(a_1 - a_2)}{2} = \text{Woodall Prime number}
\]

Case 1:

When

\[
\frac{a_2(a_1 - a_2)}{2} = 7 \quad \text{(one digit Woodall Prime number)}
\]

<table>
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<tr>
<th>( a_2)</th>
<th>( a_1 - a_2)</th>
<th>( a_1)</th>
<th>( p = a_1^2 - a_2^2)</th>
<th>( q = 2a_1a_2)</th>
<th>( r = a_1^2 + a_2^2)</th>
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Table 1

Case 2:

When

\[
\frac{a_2(a_1 - a_2)}{2} = 23 \quad \text{(one digit Woodall Prime number)}
\]

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Table 2

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Case 3:
When \( a_2(a_1 - a_2) = 383 \) (three digit Woodall Prime number)

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Table 3

4. OBSERVATIONS
- Every woodall prime number generates four Pythagorean triangle of which two are primitive and two are non-primitive triangles.
- There is exactly one Pythagorean triangle with two consecutive sides for all woodall primes.
- For all cases, \( \frac{p+q-r}{4} \) = Woodall prime number.

5. CONCLUSIONS
In this work, generation of Pythagorean Triangles with Area/Perimeter as woodall prime number is shown. Further, one may find the Pythagorean Triangles for any other number pattern.

REFERENCES