Relationship between Pythagoren Triangles & Woodall Primes

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Abstract - In this article, we focus on generating the Pythogoren triangles using Woodall prime numbers by equating the ratio Area/Perimeter to different woodall prime numbers. Of these, a few interesting patterns are displayed.

***Key Words*:** Pythagorean Triangle, Woodall prime number.

1.INTRODUCTION

Numbers play an significant role in Mathematics which is the universal language. Number theory is the area of pure mathematics which concentrates on integer solutions. In number theory, Pythagoren triangle is a most interesting topic under study. Many authors have worked on generating these Pythagorean triangle with the help of different number patterns which can be found in [1–15]. In this paper, the ratio Area/ Perimeter of a Pythogorean triangle is expressed as a Woodall prime number and we look into the exciting results.

2. BASIC DEFINITIONS

Definition 1:- Let (p, q, r) be a Pythagorean triple if it satisfies the Pythagorean equation $p^2 + q^2 = r^2$ where p, q, r are positive integers. A triangle which contains the sides as Pythagoren triple is known as Pythagoren Triangle and it is denoted by $T(p,q,r): p^2 + q^2 = r^2$, where p,q are legs and r is hypotenuse

Definition 2:- A Pythagoren equation has the suitable solution of the form $p = a_1^2 - a_2^2, q = 2a_1a_2$ and $r = a_1^2 + a_2^2$, where $a_1, a_2 > 0$ such that $a_1 > a_2$. If a_1, a_2 are of opposite parity and $\gcd(a_1, a_2) = 1$, then the solution is said to be primitive.

Woodall Prime Numbers :-Any natural number of the form $n.2^n - 1$ is known as Woodall number.T Some of thefew woodall numbers are 1,7,63,159, 383, 895,... A woodall prime number is a woodall number which is prime. Few examples of woodall primes are 7, 23,383, 32212254719, 2833419889721787128217599,...

3. METHOD OF ANALYSIS

Let A_1 and P_1 be the area and perimeter of the triangle respectively.

Assume

$$\frac{A_1}{P_1} = \text{Woodall Prime number}$$

This relationship results in the equation

$$\frac{a_2(a_1 - a_2)}{2} = \text{Woodall Prime number}$$

Case 1:

When

$$\frac{a_2(a_1 - a_2)}{2} = 7 \text{ (one digit Woodall Prime number)}$$

a_2	$a_1 - a_2$	a_1	$p = a_1^2 - a_2^2$	$q = 2a_1a_2$	$r = a_1^2 + a_2^2$	A_1	P_1	A_1/P_1
1	14	15	224	30	226	3360	480	7
2	7	9	77	36	85	1386	198	7
7	2	9	32	126	130	2016	288	7
14	1	15	29	420	421	6090	870	7

Table 1

Case 2:

When

$$\frac{a_2(a_1 - a_2)}{2} = 23 \text{ (one digit Woodall Prime number)}$$

a_2	$a_1 - a_2$	a_1	$p = a_1^2 - a_2^2$	$q = 2a_1a_2$	$r = a_1^2 + a_2^2$	A_1	P_1	A_1/P_1
1	46	47	2208	94	2210	103776	4512	23
2	23	25	621	100	629	31050	1350	23
23	2	25	96	1150	1154	55200	2400	23
46	1	47	93	4324	4325	201066	8742	23

Table 2

Case 3:

When

$$\frac{a_2(a_1 - a_2)}{2} = 383 \text{ (three digit Woodall Prime number)}$$

a_2	$a_1 - a_2$	a_1	$p = a_1^2 - a_2^2$	$q = 2a_1a_2$	$r = a_1^2 + a_2^2$	A_1	P_1	A_1/P_1
1	766	767	588288	1534	588290	451216896	1178112	383
2	383	385	148221	1540	148229	114130170	297990	383
383	2	385	1536	294910	294914	226490880	591360	383
766	1	767	1533	1175044	1175045	900671226	2351622	383

Table 3

4. OBSERVATIONS

- Every woodall prime number generates four Pythagoren triangle of which two are primitive and two are non-primitive triangles.
- There is exactly one Pythagoren triangle with two consecutive sides for all woodall primes.
- For all cases, $\frac{p+q-r}{4}$ = Woodall prime number.

5. CONCLUSIONS

In this work, generation of Pythagoren Triangles with Area/Perimeter as woodall prime number is shown. Further, One may find the Pythagoren Triangles for any other number pattern.

REFERENCES

1. Bert Miller. Nasty numbers. The mathematics teacher, 17, 1997.
2. Charles Bown. K. Nasties are primitives. The mathematics teacher, 74(9):502–504, 1981.
3. Mita Darbari and Prashans Darbari. Pythagoren triangles with sum of its two legs as dodecic. GSC Advanced Engineering and Technology, 3(1):011–015, Feb 2022.
4. M. A. Gopalan and A. Gnanam. Pythagorean triangles and polygonal numbers.International Journal of Mathematical Sciences,, 9(1-2):211–215, 2010.
5. M. A. Gopalan and G. Janaki. Pythagorean triangle with area/perimeter as a special polygonal number. Bulletin of Pure & Applied sciences, 27(2):393–405, 2008.
6. M. A. Gopalan and G. Janaki. Pythagorean triangle with nasty number as a leg. Journal of applied Mathematical Analysis and Applications, 4(1-2):13–17, 2008.
7. M. A. Gopalan and A. Vijayasankar. Observations on a pythagorean problem. Acta Ciencia Indica, Vol.XXXVI M(No 4):517–520, 2010.
8. G. Janaki and Saranya .C. Special rectangles and jarasandha numbers. Bulletin of Mathematics and Statistics Research, 4(2):63–67, April - June 2016.
9. G. Janaki and Saranya .C. Special pairs of pythagorean triangles and jarasandha numbers. American International Journal of Research in Science, Technology, Engineering & Mathematics, (13):118–120, Dec 2015- Feb 2016.
10. G. Janaki and Saranya .C. Connection between special pythogoren triangle and jarasandha numbers. International Journal of Multidisciplinary Research & Development, 3(3):236–239, March 2016.
11. G. Janaki and Saranya .C. Special pairs of rectangles and jarasandha numbers. Asian Journal of Science & Technology, 7(5):3015–3017, May 2016.
12. J. N. Kapur. Dhuruva numbers. Fascinating world of Mathematics and Mathematical sciences, Trust society, 73(No.9):649, 1980.
13. A. Gnanam M. A. Gopalan and G. Janaki. A remarkable pythagorean problem. Acta Ciencia Indica, Vol.XXXIII M(No 4):1429–1434, 2007.
14. P. S. N. Sastry. Jarasandha numbers. The mathematics teacher, No.9 Vol.37(3,4):502– 504, 2001.
15. W.Sierpinski. Pythagorean triangles. Doverpublications, INc, New York, 2003.