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Resonance In the Motion of a Geocentric Satellite Due To the Earth's Oblateness

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Abstract - This paper deals with the effects of the oblateness of the Earth on the resonant motion of a geocentric satellite. Three resonances, 1:1, 2:1, and 3:1, occur due to the perturbation of the oblateness of the Earth.

Key Words: Inertial frame, Rotating frame, Oblateness, Geocentric Satellite and Resonance.

1. Introduction

Problems of resonance play an important role in solar dynamics while solving the equations of motion. Resonance may be manifested as the appearance of small divisors in the solution of the equation of motion of the satellite. Orbital resonance of Earth satellite concerning lunisolar gravity and direct solar radiation pressure, with particular reference to those resonances, the occurrence of which is dependent only on the satellite's orbital inclination, was studied by Hughes in (1980).

Bhatnagar *et al.* (1986) examined the motion of a satellite by taking gravitational forces of the Moon, Earth and the Sun (with radiation pressure). Quarles *et al.* (2012) have studied the resonances for the coplanar Circular Restricted Three-body Problem for the mass ratio between 0.10 and 0.15 and used the method of maximum Lyapunov exponent to locate the resonances. They showed that for a high value of resonance, orbital stability is ensured where single resonance is present.

Sushil *et al.* (2013) worked on resonance in a geocentric satellite due to the Earth's equatorial ellipticity. Kaur *et al.* (2018) studied resonance in the motion of a geocentric satellite due to PR-drag. Furthermore, they have discussed the resonance in the motion of a geocentric satellite due to the combined effects of PR drag and equatorial ellipticity of the Earth. Presently, we have proposed to study the effect of the oblateness of the Earth on the resonant motion of a geocentric satellite.

2. Equations of Motion of a Geocentric Satellite

Considering the inertial frame (E, X_0Y_0Z) with the oblate Earth E at the origin and a rotating frame (E, XYZ) relative to the inertial one, where $\overrightarrow{EX_0}$ passes through the vernal equinox γ. The oblateness of the Earth is given by $A = I_a - I_e$, where I_a is the moment of inertia of the oblate Earth about its polar axis and I_e is the moment of inertia of the oblate Earth about its equatorial axis.

Let $\hat{i}_0, \hat{j}_0$ and $\hat{i}, \hat{j}$ be the unit vectors along the axes of the inertial frame and the rotating frame, with a common unit vector $\hat{k}$ along the common vertical z-axis EZ (not seen in the figure). Let $\overrightarrow{EP} = \vec{r}$ be the position vector of the satellite P, $\overrightarrow{SE} = \vec{\rho}$ the displacement of the Sun S relative to the Earth E and $\overrightarrow{SP} = \vec{R}$. If M, m and μ be the masses of the Sun, Earth and the satellite, respectively, then their mutual gravitational forces are given by

$$\left.\begin{aligned} \vec{F}_{SP} &= -\frac{GM\mu}{R^3}\vec{R}, \\ \vec{F}_{EP} &= -\frac{Gm\mu}{r^3}\vec{r} - \frac{3GA}{2r^5}\vec{r}, \\ \vec{F}_{SE} &= -\frac{GMm}{\rho^3}\vec{\rho} - \frac{3GA}{2\rho^5}\vec{\rho}. \end{aligned}\right\} \quad \text{(McCuskey 1967)} \qquad (1)$$

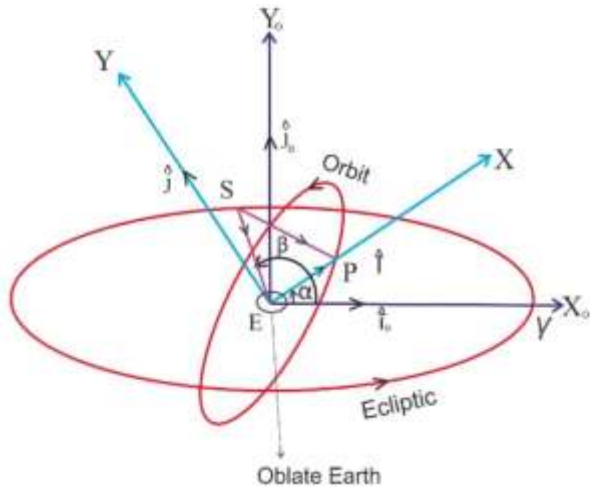

Fig 1: *Ecliptic is the orbit of the Sun 'S' relative to the inertial frame* (EX_0Y_0Z) *and rotating frame* (E, XYZ) *with oblate Earth 'E' at the origin.*

Let at any time $t, \vec{\omega}$ be angular velocity of the rotating frame relative to the inertial frame with the Earth at the origin, then the velocity of the satellite relative to the inertial frame is given by

$$\Rightarrow \frac{d\vec{r}}{dt} = \frac{\partial \vec{r}}{\partial t} + \vec{\omega} \times \vec{r} \qquad (2)$$

The acceleration of the satellite in the rotating frame can be written as

$$\Rightarrow \frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}\left(\frac{\partial \vec{r}}{\partial t} + \vec{\omega} \times \vec{r}\right)$$

$$= \left(\frac{\partial}{\partial t} + \vec{\omega} \times\right)\left(\frac{\partial \vec{r}}{\partial t} + \vec{\omega} \times \vec{r}\right)$$

$$=\frac{\partial^2\vec{r}}{\partial t^2}+2\vec{\omega}\times\frac{\partial\vec{r}}{\partial t}+\frac{\partial\vec{\omega}}{\partial t}\times\vec{r}+\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right) \quad (3)$$

If $\hat{i}$ be the unit vector along the direction of the satellite, then $\vec{r}=r\hat{i}$ and the equation of motion of the satellite in a rotating frame is reduced to

$$\Rightarrow\frac{d^2\vec{r}}{dt^2}=\frac{\partial^2 r}{\partial t^2}\hat{i}+2\frac{\partial r}{\partial t}\left(\omega\times\hat{i}\right)+r\left(\frac{\partial\vec{\omega}}{\partial t}\times\hat{i}\right)+r\vec{\omega}\times(\vec{\omega}\times\hat{i}) \quad (4)$$

Let α be the angle of direction of the satellite with the direction of the vernal equinox, then $\vec{\omega}=\dot{\alpha}\vec{k}$ where, $\dot{\alpha}$ is the angular velocity of the satellite relative to the Earth. Thus, Equation (4) is reduced to

$$\frac{d^2\vec{r}}{dt^2}=\hat{i}\left(\frac{\partial^2 r}{\partial t^2}\right)+2\frac{\partial r}{\partial t}\left(\dot{\alpha}\hat{k}\times\hat{i}\right)+r\left(\frac{\partial\dot{\alpha}}{\partial t}\hat{k}\times\hat{i}\right)+r\dot{\alpha}\hat{k}\times(\dot{\alpha}\hat{k}\times\hat{i})$$

$$=\left(\frac{\partial^2 r}{\partial t^2}\right)\hat{i}+2\frac{\partial r}{\partial t}\left(\dot{\alpha}\hat{j}\right)+r\left(\ddot{\alpha}\hat{j}\right)+r\dot{\alpha}^2\left[\hat{k}\times(\hat{k}\times\hat{i})\right]$$

$$\Rightarrow\frac{d^2\vec{r}}{dt^2}=\left(\frac{\partial^2 r}{\partial t^2}-r\dot{\alpha}^2\right)\hat{i}+\left(2\dot{\alpha}\frac{\partial r}{\partial t}+r\ddot{\alpha}\right)\hat{j} \quad (5)$$

In the triangle EPS, $\overrightarrow{EP}+\overrightarrow{SE}=\overrightarrow{SP}\Rightarrow\vec{r}+\vec{\rho}=\vec{R}$

$$\Rightarrow\vec{r}=\vec{R}-\vec{\rho}$$

$$\Rightarrow\ddot{\vec{r}}=\ddot{\vec{R}}-\ddot{\vec{\rho}}$$

$$\Rightarrow\ddot{\vec{r}}=\frac{\vec{F}_{SP}+\vec{F}_{EP}}{\mu}-\frac{\vec{F}_{SE}}{m}$$

$$\Rightarrow\ddot{\vec{r}}=-\frac{GM\mu}{\mu R^3}\vec{R}-\frac{Gm\mu}{\mu r^3}\vec{r}-\frac{3GA}{2\mu r^5}\vec{r}+\frac{GMm}{\rho^3 m}\vec{\rho}+\frac{3GA}{2\rho^5 m}\vec{\rho}$$

$$\Rightarrow\ddot{\vec{r}}=-\left(\frac{GM}{R^3}\right)\vec{R}-\frac{Gm}{r^3}\vec{r}-\frac{3GA}{2\mu r^5}\vec{r}+\frac{GMm}{\rho^3}\vec{\rho}+\frac{3GA}{2\rho^5 m}\vec{\rho} \quad (6)$$

If $\hat{\rho}$ be the unit vector along $\overrightarrow{SE}=\vec{\rho}$ and β be the angle of the direction of the sun with the direction of the vernal equinox γ then $\hat{\rho}=\cos\beta\hat{i}_0+\sin\beta\vec{j}_0$.

Thus

$$-\frac{GMm}{\rho^3 m}\vec{\rho}-\frac{3GA}{2\rho^5 m}\vec{\rho}=-\dot{\beta},$$

$$\Rightarrow\frac{GM}{\rho^3}\vec{\rho}+\frac{3GA}{2\rho^5 m}\vec{\rho}=\dot{\beta}^2\vec{\rho},$$

$$\Rightarrow\dot{\beta}^2=\frac{GM}{\rho^3}+\frac{3GA}{2\rho^5 m}$$

(7)

Introducing Equation (7) in Equation (6), one has

$$\ddot{\vec{r}}=-\left(\frac{GM}{R^3}\right)\vec{R}-\frac{Gm}{r^3}\vec{r}-\frac{3GA}{2\mu r^5}\vec{r}+\dot{\beta}^2\vec{\rho}$$

$$\Rightarrow\ddot{\vec{r}}=-\left(\frac{Gm}{r^3}+\frac{3GA}{2\mu r^5}\right)\vec{r}+\dot{\beta}^2\vec{\rho}-\frac{GM}{R^3}\vec{R}$$

$$\ddot{\vec{r}}=-\left(\frac{Gm}{r^3}+\frac{3GA}{2\mu r^5}\right)r\hat{i}+\dot{\beta}^2\rho(\cos\beta\hat{i}_0+\sin\beta\hat{j}_0)$$
$$-\frac{GM}{R^3}(r\hat{i}+\rho\cos\beta\hat{i}_0+\rho\sin\beta\hat{j}_0) \quad (8)$$

Taking the scalar product of $\hat{i}$ with Equations (4) and (8), and comparing the results, one can find the first equation of motion of the geocentric satellite in polar form as

$$\frac{\partial^2 r}{\partial t^2}-r\dot{\alpha}^2=-\left(\frac{Gm}{r^3}+\frac{3GA}{2\mu r^5}\right)r+\dot{\beta}^2\rho\cos\left(\alpha-\beta\right)$$
$$-\frac{GM}{R^3}(r+\rho\cos(\alpha-\beta))$$

$$\frac{d^2 r}{dt^2}-\dot{\alpha}^2 r+\left(\frac{Gm}{r^2}+\frac{3GA}{2\mu r^4}\right)$$
$$=\left(\dot{\beta}^2-\frac{GM}{R^3}\right)\rho\cos\left(\alpha-\beta\right)-\frac{GMr}{R^3} \quad (9)$$

Again, taking a scalar product of $\hat{j}$ with Equations (4) and (8), and comparing the results, one can find the second equation of motion of the geocentric satellite in polar form as

$$2\dot{\alpha}\frac{\partial r}{\partial t}(\dot{\alpha})+r\ddot{\alpha}=\dot{\beta}^2\rho\left(-\cos\beta\sin\alpha+\sin\beta\cos\alpha\right)$$
$$-\frac{GM}{R^3}\rho\left[-\cos\beta\sin\alpha+\sin\beta\cos\alpha\right],$$

$$2\frac{\partial r}{\partial t}(\dot{\alpha})+r\left(\frac{\partial\dot{\alpha}}{\partial t}\right)=\dot{\beta}^2\rho\left[-\sin\left(\alpha-\beta\right)\right]-\frac{GM\rho}{R^3}\sin\left(\alpha-\beta\right),$$

$$2r\frac{\partial r}{\partial t}\left(\dot{\alpha}\right)+r^2\frac{\partial\dot{\alpha}}{\partial t}=\dot{\beta}^2\rho r\sin\left(\alpha-\beta\right)-\frac{GMr\rho}{R^3}\sin\left(\alpha-\beta\right),$$

$$\Rightarrow\frac{\partial}{\partial t}\left(r^2\dot{\alpha}\right)=\dot{\beta}^2\rho r\sin(\alpha-\beta)-\frac{GMr\rho}{R^3}\sin(\alpha-\beta). \quad (10)$$

Equations (9) and (10) are not integrable, so we replace r and $\dot{\alpha}$ by their steady state value r_0 and $\dot{\alpha}_0$ by the perturbation technique, which can be introduced in Equations (9) and (10).

$$\frac{d^2 r}{dt^2}-\dot{\alpha}^2 r+\frac{Gm}{r^2}+\frac{3GA}{2\mu r^4}=\rho\left(\dot{\beta}^2-\frac{GM}{R^3}\right)\cos\left(\dot{\alpha}_0-\dot{\beta}\right)t$$
$$-\frac{GMr_0}{R^3} \quad (11)$$

$$\frac{d}{dt}\left(\dot{r}^2\alpha\right)=\dot{\beta}^2\rho r_0\sin\left(\dot{\alpha}_0-\dot{\beta}\right)t-\frac{GMr_0\rho}{R}\sin\left(\dot{\alpha}_0-\dot{\beta}\right)t \quad (12)$$

Since in the central orbit $r^2\dot{\alpha}=$ constant = h (say) and $r=1/u$

$$\Rightarrow\frac{dr}{dt}=-r^2\frac{du}{d\alpha}\cdot\frac{h}{r^2}=-h\frac{du}{d\alpha}\Rightarrow\frac{d^2 r}{dt^2}=-h\frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right)\cdot\frac{d\alpha}{dt}$$
$$\Rightarrow\frac{d^2 r}{dt^2}=-h\dot{\alpha}\frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right)$$

$$\Rightarrow\frac{d^2 r}{dt^2}=-r^2\dot{\alpha}^2\frac{d^2 u}{d\alpha^2}\Rightarrow\frac{d^2 r}{dt^2}=-\frac{\dot{\alpha}^2}{u^2}\frac{d^2 u}{d\alpha^2}$$

Now, putting the value of $\frac{d^2 r}{dt^2}$ in Equation (11), we get

$$-\frac{\dot{\alpha}^2}{u^2}\frac{d^2 u}{d\alpha^2}-\frac{\dot{\alpha}^2}{u}+Gmu^2+\frac{3GAu^4}{2\mu}=$$
$$\left(\dot{\beta}^2-\frac{GM}{R^3}\right)\rho\cos(\dot{\alpha}_0-\dot{\beta})t-\frac{GMr_0}{R^3}$$

$$\Rightarrow -\frac{\dot{\alpha}^2}{u^2}\left[\frac{d^2u}{d\alpha^2}+u\right] = -Gmu^2 - \frac{3GAu^4}{2\mu} + \rho\left(\dot{\beta}^2 - \frac{GM}{R^3}\right)\cos(\dot{\alpha}_0-\dot{\beta})t - \frac{GMr_0}{R^3}$$

$$\Rightarrow \frac{d^2u}{d\alpha^2}+u = \frac{Gm}{r_0^4\dot{\alpha}_0^2} + \frac{3GA}{2\mu\dot{\alpha}_0^2}u^6 - \frac{\rho}{\dot{\alpha}_0^2}u^2\left(\dot{\beta}^2 - \frac{GM}{R^3}\right)\cos(\dot{\alpha}_0-\dot{\beta})t + \frac{GMr_0u^2}{R^3\dot{\alpha}_0^2}$$

$$\Rightarrow \frac{d^2u}{d\alpha^2}+u = \frac{Gm}{r_0^4\dot{\alpha}_0^2} + \frac{3GAu^6}{2\mu\dot{\alpha}_0^2} - \frac{\rho}{\dot{\alpha}_0^2}\left(\dot{\beta}^2 - \frac{GM}{R^3}\right)u^2\cos(\dot{\alpha}_0-\dot{\beta})t + \frac{GMr_0u^2}{\dot{\alpha}_0^2R^3} \quad (13)$$

This is the perturbed equation of motion due to the oblateness of the Earth.

Now, we take $\vec{R} = \vec{\rho} + \vec{r}$

$$-\rho\cos(\beta-\alpha)\hat{i} - \rho\sin(\beta-\alpha)\hat{j} + r\hat{i}$$

$$= [r-\rho\cos(\beta-\alpha)]\hat{i} - \rho\sin(\beta-\alpha)\hat{i}$$

$$R^2 = \{r-\rho\cos(\beta-\alpha)\}^2 + \{\rho\sin(\beta-\alpha)\}^2,$$

$$R = \rho\left[1-\frac{2r}{\rho}\cos(\beta-\alpha)+\frac{r^2}{\rho^2}\right]^{1/2},$$

Neglecting higher-order terms $\left(\frac{r}{\rho}\right)^3, \left(\frac{r}{\rho}\right)^4, \ldots$ above the second, we get

$$\frac{GM}{R} = \frac{GM}{4\rho}\left[\left\{1+3\cos 2(\beta-\alpha)\left(\frac{r}{\rho}\right)^2\right\} + 4\cos(\beta-\alpha)\left(\frac{r}{\rho}\right)+4\right] \quad (14)$$

From Equation (12),

$$\dot{\beta}^2\rho r_0\sin(\dot{\alpha}_0-\dot{\beta})t - \frac{GMr_0\rho}{R}\sin(\dot{\alpha}_0-\dot{\beta})t = 0,$$

Using Equation (14), we get

$$\dot{\beta}^2\rho r_0\sin(\dot{\alpha}_0-\dot{\beta})t - \frac{GM}{4\rho}\left[\{1+3\cos 2(\beta-\alpha)\}\left(\frac{r}{\rho}\right)^2 + 4\cos(\beta-\alpha)\left(\frac{r}{\rho}\right)+4\right]r_0\rho\sin(\dot{\alpha}_0-\dot{\beta})t = 0,$$

$$r_0\sin(\dot{\alpha}_0-\dot{\beta})t\left\{\dot{\beta}^2\rho - \frac{GM}{4}\left[\{1+3\cos 2(\beta-\alpha)\}\left(\frac{r}{\rho}\right)^2 + 4\cos(\beta-\alpha)\left(\frac{r}{\rho}\right)+4\right]\right\} = 0, \quad (15)$$

This is a quadratic equation in r/ρ so here r can be derived from Equation (15), but resonance can't be found from this equation.

3. Resonance in the Motion of a Geocentric Satellite of the Oblate Earth

The resonances can be obtained from the perturbed Equation (13) only. The unperturbed equation of motion is $\frac{d^2u}{d\alpha^2}+u = \frac{Gm}{r_0^4\dot{\alpha}_0^2}$ whose complete solution is given by $\frac{l}{r} = 1+e\cos(\alpha-\psi)$. where $l = a(1-e^2)$ and e, ψ are constants.

$$\Rightarrow u = \frac{1+e\cos(\alpha-\psi)}{a(1-e^2)}. \quad (16)$$

Let us consider $\alpha-\psi = \dot{\alpha}_0 t = nt(say)$ where n is the frequency of the satellite. Since eccentricity $e<1$ hence, $(1+e\cos nt)^p \approx 1+pe\cos nt$, where p is a positive integer.

Also,

$$\frac{du}{dt} = \frac{du}{d\alpha}\cdot\frac{d\alpha}{dt} = \frac{du}{d\alpha}\cdot\dot{\alpha}_0$$

$$\frac{d^2u}{dt^2} = \frac{d}{dt}\left(\frac{du}{dt}\right) = \frac{d}{dt}\left(\frac{du}{d\alpha}\cdot\dot{\alpha}_0\right) = \dot{\alpha}_0\frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right)\cdot\frac{d\alpha}{dt} = \dot{\alpha}_0^2\frac{d^2u}{d\alpha^2}$$

$$\frac{d^2u}{dt^2} = \dot{\alpha}_0^2\frac{d^2u}{d\alpha^2} \quad (17)$$

Collaborating Equations (13), (16) and (17), we get

$$\Rightarrow \frac{d^2u}{dt^2}+n^2u = \left(\frac{Gm}{r_0^4} + \frac{3GA}{2\mu a^6(1-e^2)^6} + \frac{GMr_0}{R^3a^2(1-e^2)^2}\right) + \frac{18GAe}{\mu a^6(1-e^2)^6}\cos nt + \frac{\rho\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2}\cos(n-\dot{\beta})t + \frac{\rho e\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2}\{\cos(2n-\dot{\beta})+\cos\dot{\beta}t\} + \frac{2GMr_0e}{R^3a^2(1-e^2)^2}\cos nt.$$

$$\Rightarrow \frac{d^2u}{dt^2}+n^2u = \left[\frac{Gm}{r_0^4} + \frac{3GA}{2\mu a^6(1-e^2)^6} + \frac{GMr_0}{R^3a^2(1-e^2)^2}\right] + \left[\frac{18GAe}{\mu a^6(1-e^2)^6} + \frac{2GMr_0e}{R^3a^2(1-e^2)^2}\right]\cos nt + \frac{\rho e\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2}\cos\dot{\beta}t + \frac{\rho\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2}\cos(n-\dot{\beta})t + \frac{\rho e\left(-\dot{\beta}^2 - \frac{3GA}{2\rho^5m} + \frac{GM}{R^3}\right)}{a^2(1-e^2)^2}\cos(2n-\dot{\beta})t$$

$$\Rightarrow \frac{d^2u}{dt^2}+n^2u = \gamma_1 + \gamma_2\cos nt + \gamma_3\cos\dot{\beta}t + \gamma_4\cos(n-\dot{\beta})t + \gamma_5\cos(2n-\dot{\beta})t \quad (18)$$

where,

$$\gamma_1=\left[\frac{Gm}{r_0^{\,4}}+\frac{3GA}{2\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0}{R^3a^2\left(1-e^2\right)^2}\right],$$

$$\gamma_2=\left[\frac{18GAe}{\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0e}{R^3a^2\left(1-e^2\right)^2}\right],$$

$$\gamma_3=\frac{\rho e\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2\left(1-e^2\right)^2}=\frac{\rho e\left(\frac{GM}{R^3}-\frac{GM}{\rho^3}-\frac{3GA}{2\rho^5 m}\right)}{a^2\left(1-e^2\right)^2},$$

$$\gamma_4=\frac{\rho\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2\left(1-e^2\right)^2}=\frac{\rho\left(\frac{GM}{R^3}-\frac{GM}{\rho^3}-\frac{3GA}{2\rho^5 m}\right)}{a^2\left(1-e^2\right)^2},$$

$$\gamma_5=\frac{\rho e\left(\frac{GM}{R^3}-\dot{\beta}^2\right)}{a^2\left(1-e^2\right)^2}=\frac{\rho e\left(\frac{GM}{R^3}-\frac{GM}{\rho^3}-\frac{3GA}{2\rho^5 m}\right)}{a^2\left(1-e^2\right)^2}=\gamma_3.$$

Thus, the complementary function of Equation (18) is given by $C.F=K\cos(nt-\epsilon_1)$ where K,ϵ_1 are constants.

$$P.I=\frac{1}{D^2+n^2}\left[\gamma_1+\gamma_2\cos nt+\gamma_3\cos\dot{\beta}t+\gamma_4\cos(n-\dot{\beta})t+\gamma_5\cos(2n-\dot{\beta})t\right]$$

$$=\frac{\gamma_1}{n^2}+\frac{\gamma_2 t\sin t}{2n}+\frac{\gamma_3\cos\dot{\beta}t}{n^2-\dot{\beta}^2}+\frac{\gamma_4\cos(n-\dot{\beta})t}{n^2-\left(n-\dot{\beta}\right)^2}+\frac{\gamma_5\cos(2n-\dot{\beta})}{n^2-(2n-\dot{\beta})^2}$$

$$\frac{\gamma_1}{n^2}+\frac{\gamma_2 t\sin nt}{2n}+\frac{\gamma_3\cos\dot{\beta}t}{(n-\dot{\beta})(n+\dot{\beta})}+\frac{\gamma_4\cos(n-\dot{\beta})t}{\dot{\beta}(2n-\dot{\beta})}-\frac{\gamma_5\cos(2n-\dot{\beta})}{(n-\dot{\beta})(3n-\dot{\beta})}$$

The solution of Equation (18) is given by u = C.F. + P.I.

$$u=k\cos(nt-\epsilon_1)+\frac{\gamma_1}{n^2}+\frac{\gamma_2 t\sin nt}{2n}+\frac{\gamma_3\cos\dot{\beta}t}{(n-\dot{\beta})(n+\dot{\beta})}+\frac{\gamma_4\cos(n-\dot{\beta})t}{\dot{\beta}(2n-\dot{\beta})}-\frac{\gamma_5\cos(2n-\dot{\beta})}{(n-\dot{\beta})(3n-\dot{\beta})}\qquad(19)$$

On vanishing the denominator of any term of Equation (19), we get some points at which motion becomes indeterminate, and hence, resonances occur at these points. Thus, the resonances occur at the points $n=\dot{\beta}, 2n=\dot{\beta}$ and $3n=\dot{\beta}$. Thus, the resonances $1:1$, $2:1$ and $3:1$ occur due to the oblateness of the Earth.

4. Conclusion

This manuscript is a collection of three sections, excluding references. The first section encompasses all important previous research works on geocentric satellites and their applications in various branches of space science. In the second, the equations of motion of a geocentric satellite have been derived in polar form under the gravitational field of the Sun and the oblate Earth. In the third section, resonant points have been investigated by making zero the denominator of each term of the solution of the equation of motion of the geocentric satellite. It is to be noted that three resonances 1:1,2:1,3:1, occur due to the oblateness of the Earth.

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