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Review Paper on Application of Laplace Transform in Engineering

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Abstract - The Laplace Transform is one of the most powerful mathematical tools used for solving various engineering problems. In this paper, we will discuss the properties and applications of Laplace Transform in engineering problems. We will examine its applications in Nuclear Physics, Control Engineering, and Signal Processing which will helps to solve differential equations.

Key Words: Laplace Transform, Inverse Laplace Transform, Differential Equation, Properties of Laplace Transform

1.INTRODUCTION

The Laplace Transform is a mathematical tool that transforms variables into parameters under certain conditions, known as Dirichlet's conditions. For any function of time f(t) to be Laplace Transformable, it must satisfy the following Dirichlet's conditions-
1.f(t) must be piecewise continuous which means that it must be single valued but can have a finite number of finite isolated discontinuities for t >0.
2.f(t) must be exponential order which means that f(t) must remain less than s $e^{a_0 t}$ as t approaches ∞ where s is a positive constant and a_0 is real positive number.
The Laplace Transform has wide applications in various areas of engineering.

Definition:

Let f(t) be a function of t (t >0) then the integral $\int_0^\infty e^{-st} f(t)dt$ is called Laplace Transform of f(t) which is denoted as L[f(t)] or F(s).

i.e. L[f(t)] = $\int_0^\infty e^{-st} f(t)dt$ = F(s)

Important Properties of Laplace Transform:

Linearity Property: If f(t) & g(t) are any two functions of t and α, β are any two constants then

L $[\alpha\, f(t) + \beta g(t)] = \alpha\, L[f(t)] + \beta\, L[g(t)]$

Shifting Property:

If L [f(t)]= F(s) then L $[e^{at} f(t)]$ = F (s −a)

And If L [f(t)]= F(s) then L $[e^{-at} f(t)]$ = F (s +a)

Inverse Laplace Transform: If L [f(t)]= F(s) then L^{-1}[F(s)] = f(t) is called Inverse Laplace Transform of F(s).

Inverse Laplace Transform by Convolution Theorem:

If $L^{-1}[\emptyset_1(s)] = f_1(t)$ and $L^{-1}[\emptyset_2(s)] = f_2(t)$ then $L^{-1}[\emptyset_1(s)\emptyset_2(s)] = \int_0^t f_1(u) f_2(t-u)\, du$

Application of Laplace Transform in Various Engineering Streams:

Laplace Transform is very powerful mathematical tool applied in various areas of Engineering & Science. This section defines the application of Laplace Transform in engineering problem.

Nuclear Physics: Laplace Transform is used in the study of radioactive disintegration that is radioactive decay.

Control Engineering: Laplace Transform can simplify the analysis and design of control systems. It can convert complex differential equation that describes the dynamic behavior of the system into simpler algebraic equation that describes the frequency response of a system.

Signal Processing: Laplace Transforms are employed in signal processing applications, such as filtering, modulation and noise reduction. By analyzing signals in the Laplace domain, engineers can manipulate and process signals to extract useful information or remove unwanted components.

Examples to solve the problems using Laplace Transform:

Nuclear Physics:

Consider the following first order linear differential equation

$$\frac{dN}{dt} = -\lambda N$$

This equation is a fundamental relationship describing radioactive decay, where N = N(t) Represents the number of undecayed atoms remaining in a radioactive isotope at time t(in seconds) and λ is decay constant. We can use Laplace Transform to solve this equation

$$\frac{dN}{dt} + \lambda N = 0$$

Taking Laplace Transform on both sides,

$$(\text{s } \bar{N}\text{ (s)} - N_0) + \lambda \bar{N}\text{ (s)} = 0$$

where $\bar{N} = L[N(t)]$ and $N_0 = N(0)$

Solving we get

$$\bar{N}\text{ (s)} = \frac{N_0}{s+\lambda}$$

Taking inverse Laplace Transform on both sides,

$$N(t) = L^{-1}[\bar{N}\text{ (s)}]$$

$$= L^{-1}[\frac{N_0}{s+\lambda}]$$

$$= N_0\, e^{-\lambda t}$$

which indeed the correct form for radioactive decay.

Control Engineering: Consider the system described by following transfer function

$$\text{G(s)} = \frac{s^2+3s+3}{s^3+6s^2+11s+6}$$

Find an analytical expression for an impulse response of the system.

To find analytical expression taking Laplace Transform on both sides,

$$\text{L[G(s)]} = \text{L}[\frac{s^2+3s+3}{s^3+6s^2+11s+6}]$$

$= \text{L}[\frac{s^2+3s+3}{(S+1)(S+2)(S+3)}]$ { By factorization method

For finding Laplace Transform use method of Partial Fraction to resolve denominator as

$$\text{L}[\frac{s^2+3s+3}{(S+1)(S+2)(S+3)}] = \frac{\frac{1}{2}}{(s+1)} + \frac{(-1)}{(s+2)} + \frac{\frac{3}{2}}{(s+3)}$$

Taking Inverse Laplace Transform we get an expression of impulse response of the system.

$$\frac{s^2+3s+3}{s^3+6s^2+11s+6} = \frac{1}{2}e^{-t} - e^{-2t} + \frac{3}{2}e^{-3t}$$

Signal Processing:

Suppose we have continuous time signal $x(t)$ defined as

$$\text{X(t)} = e^{-at}u(t)$$

Where $u(t)$ is the unit step function and a is a positive constant.

We want to find the Laplace Transform X(s) of this signal. The Laplace Transform of a function $x(t)$ is defined as:

$$\text{X(s)} = \int_0^{\infty} e^{-st}x(t)dt$$

Substituting the expression for $x(t)$ into the Laplace Transform definition, we get

$$X(s) = \int_0^{\infty} e^{-st}\, e^{-at}u(t)dt$$

$$= \int_0^{\infty} e^{-(s+a)t}u(t)dt$$

The Unit Step Function $u(t)$ is equal to 1 for $t \geq 0$ and 0 for $t < 0$.Therefore the integral becomes

$$X(s) = \int_0^{\infty} e^{-(s+a)t}\, u(t)dt.$$

This integral can be solved to obtain the Laplace Transform $X(s)$.The result is

$$X(s) = \frac{1}{s+a}$$

So, the Laplace Transform of the signal

$x(t) = e^{-at}u(t)$ is $X(s) = \frac{1}{s+a}$

In signal processing, the Laplace Transform is used to analyze signals in that Frequency domain which can provide insights into their behavior, frequency Content and system responses.

3. CONCLUSIONS

Throughout the paper we have discussed about applications of Laplace Transform in various Engineering problems like Control Engineering, Nuclear Physics, Signal Processing etc. Laplace Transform gives solution of any problem in simple way.

REFERENCES

1. H.K.Dass and Er. Rajnish Verma, Higher Engineering Mathematics, S. Chand Publication, New Delhi.
2. Sarina Adhikari, Laplace Transform and its applications, University of Tennessee. (Department of Electrical Engineering).
3. Sunil Shrivastava , Department of Mathematics AISECT University India , Introduction of Laplace Transform and Elzaki.
4. International Research Journal of Engineering and Technology (IRJET)e-ISSN: 2395-0056 Volume: 05 Issue: 05 | May-2018 www.irjet.net p-ISSN: 2395-0072
5. International Journal of Creative Research Thoughts (IJCRT) Volume 8, Issue 5 May 2020 | ISSN: 2320-28820
6. Higher Engineering Mathematics by B. S. Grewal, Khanna Publishers, New Delhi