

## Role of PERT and CPM based on operation Research method in management in making Decision under uncertainty to improve the prospects of Industrial organization

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Abstract - As the global environment becomes fiercely competitive, Operations Research has gained significance in applications like world-class Manufacturing systems(WCM), Lean production, Six-sigma quality management, Benchmarking, Just-intime (JIT) inventory techniques. The growth of global markets and the resulting increase in competition have highlighted the need for Operation Research. To survive and lead the today's highly competitive and demand driven market, pressure is on management to make economical decisions. One of the essential managerial skills is ability to allocate and utilize resources appropriately in the efforts of achieving the optimal performance efficiently. In some cases such as small-scale low complexity environment, decision based on intuition with minimal quantitative basis may be reasonably acceptable and practical in achieving the goal of the organization. However, for a largescale system, both quantitative and qualitative (i.e. intuition, experience, common sense) analyses are required to make the most economical decisions. Using Operations Research techniques including Linear Programming, Discrete Event Simulation and Queuing Theory, organization leaders can make high quality decisions. Project Management with PERT (Project Evaluation and Review Techniques) and CPM (Critical Path method) CPM/PERT is based on the basis that a small set of activities, which make up the longest path through the activity network control the entire project. If these "critical" activities could be identified and assigned to responsible persons, management resources could be optimally used by concentrating on the few activities which determine the fate of the entire project. On-critical activities can be rescheduled and resources for them can be reallocated flexibly, without affecting the whole project. Both are project management techniques, which have been created out of the need of Western industrial and military establishments to plan, schedule and control complex projects. Operations managers are not expected to be experts in any decision science tools; however, he or she must have fundamental knowledge of such tools to acquire right resources and to make the most economically sounding decisions for the company as a whole. Present paper is an attempt to study the importance of Operation research and various techniques used to improve the operational efficiency of the organization.

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Key Words: OR, PERT, CPM, WCM, JIT, and AHP

### **1.INTRODUCTION**

OR in which firstly the beginning and progress of operations research has been explained. Then classification of problems in operation research has the identified then mathematical modeling has been made. Mathematical models have been constructed for each of these categories, and methods for solving the models are available in many cases.

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Sequencing problems involve placing items in a certain sequence or order for service. For example, in a job shop, N jobs requiring different amounts of time on different machines must each be processed on M machines in the same order with no passing between machines. How should the jobs be ordered for processing to minimize the total time to process all of the jobs on all of the machines? The solution is quite simple for the two-machine problem and for a special case of the three machine problem but is several magnitudes more difficult for the general Mmachine problem.



Routing problems involve finding the optimal route from an origin to a destination when a number of possible routes are available. The classical traveling salesman problem is a example. A salesman wishes to visit each of N cities once and only once before returning to his home office. In what order should he visit the cities to minimize the overall distance traveled? This problem arises as a sub problem of the vehicle dispatch or delivery problem. Once a set of distinct locations have been assigned to a certain truck route in the delivery problem, in what order should the locations be visited to minimize the total distance traveled?

Replacement problems occur when one must decide the optimal time to replace equipment that deteriorates or fails immediately. When, if ever, should an automobile be replaced with a new one? This is a problem faced by most of us today. Of course, we each have our own measure of effectiveness, so there would not be a single optimal answer for everyone even if each automobile gave exactly the same service. Much depends on the purpose of the car, the role prestige plays in our lives, how fast we drive, etc. Another type of replacement problem involves equipment that works perfectly until it fails, such as a light bulb or an intricate computer component. What is the optimal replacement policy for this type of equipment?

The problem of deciding how much of a certain product to hold in inventory one of real concern IS is. If a customer requests a certain quantity of the product but it is not available, this could mean a lost sale. On the other hand, if an excess of the product is carried in inventory, the many costs associated with inventory may be unacceptable. Hence, the inventory problem is to determine the level of inventory that will optimize some measure of effectiveness.

Queuing problems plague us from the time we rise in the morning until we retire at night. Wait

for the bathroom, wait for breakfast, wait at stoplights, wait for the computer, wait, wait, wait; that's the story of our lives. Any problem that involves waiting for service is classified as a queuing or waiting-line problem. The OR literature is bulging with solutions for many types of queuing models; however, most realistic queuing problems are so complex and the components so interrelated that simulation is a vital technique in this general area.

Competitive problems arise when two or more people are competing for a precious resource. The resource may range from an opponent's king in chess to a larger share of the market in business. Quite often a competitive problem involves bidding for a contract to perform a service or to obtain some type of privilege. A number of different types of bidding procedures are used, but in each case competition is involved. Formal models of realistic problems in this area are scarce; however, the underlying concepts of the decisionmaking process are worthy of some study.

Search problems differ from the other types of problems we have discussed in that they all involve searching for information that is necessary to make a decision. Some examples are:

Searching the ocean for enemy ships

Auditing books for errors

Exploring for valuable natural resources, such as oil, copper, or coal

Retrieving information from computer storage

Shopping for a new suit

In each case, the objective is to minimize the costs associated with collecting and analyzing data to reduce decision errors and to minimize the costs associated with the decision errors themselves. We will see later that statistical decision theory provides a basis for solving many search problems.

### 2. METHODOLOGIES

This is organized into two groups. Group I covers deterministic models and methods of solution, while group II covers probabilistic models and methods of solution.

Each group is independent of the other. The deterministic methods are presented first so that the reader who is unfamiliar with probability can become acquainted with the general computer-oriented algorithmic approach first. The deterministic methods on probability theory is not intended as a comprehensive coverage of the subject. Rather, it presents only the essential element of probability that are required throughout the remaining probabilistic models in group II.



Fig.1 Diagram of different methodologies of operation research

### 2.1 DETERMINISTIC OPERATION RESEARCH MODEL :-

The deterministic models include linear programming, networks, goal programming, integer

programming, deterministic dynamic programming, and deterministic inventory models. The organization of the linear programming allows the beginning student to gain an immediate appreciation of the linear of the practical use of the technique, including the powerful concept of dual prices and sensitivity analysis. The dynamics programming has been streamlined to include more applications and more generalized algorithms, including investment problem. The dynamics programming approach to the solution of the problem of allocation resource to activities in order to optimize the total return from all activities.

To this point we have dealt with mathematical models and techniques that were completely deterministic in nature. There was no randomness or variation due to chance in the assumed models or in the methods that were used to solve the models. The models were exact representations of the real systems under study and, in the absence of round-off errors; the methods would find the optimal solution of the models in a finite number of steps.

The general approach throughout is;The development of most of the methods of solution is preceded by one or more illustrative examples, many with the computer solutions. In dynamic programming using parts 1-3 learn about the types of problems for solution . Part 4 which is written to cover the method in general , is a detailed step-by- step procedure that can be easily programmed for the computer. Then finally, Part 5 gives opportunity to solve a large variety of problems and to emphasize the analysis of the result , thus enhancing the reader's problem –solving ability.

### 2.2 PROBABILISTIC OPERATION RESEARCH MODEL :-

Probabilistic model start with review of basic probability and statistics, followed by a representation of forecasting models. Since there is random variation or

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some degree of uncertainty in the components of many realistic systems, it is important that we study some basic probability and statistical concepts that can be incorporated into mathematical models of such systems. These same concepts can be further extended to the techniques needed to solve probabilistic models. The elements of probability and statistics that have direct application to the probabilistic models and techniques in this thesis will be presented in an intuitive manner for ease of understanding. The material on decision analysis now includes the analytic hierarchy approach (AHP)

1.FORMULATION OF PROBLEM TO BE SOLVED	)
2.CONSTRUCTION OF A MODEL OF THE PROBLEM	)
3.DEVELOPMENT OF A METHOD TO SOLVE THE MODEL	)
4, PRESENTATION OF A CONCISE COMPUTER - ORIENTED ALGORITHM TO SOLVE THE	)
5. PRESENTATION OF A FORTRAN COMPUTER PROGRAM FOR THE ALGORITHM	]

and the role of utility function in decision analysis. The method of solving Regression analysis, Queuing model allows the study both the application and theory, or to concentrates solely on the application aspect of the topic.

Stochastic OR methods generally are classified as tools that can be used to assist management in making decisions under uncertainty. For example, predicting a future response or outcome may involve a great deal of uncertainty. Consequently, a statistical tool, such as regression analysis, can be used to determine the relationship between the important factors and the response to provide management with an equation that will adequately predict future responses. Likewise, queuing theory can be used to determine the number of service facilities to provide for customer service, the arrangement of queues to feed the service facilities, etc. In general, each stochastic OR method is used to solve problems that can be formulated as a certain type of model.

Despite the impressive advance in mathematical modeling, many real situations still are well beyond the capabilities of representation systems mathematically. for one thing , the "rigidity " of the mathematical representation may make it impossible to describe the decision problem by a mathematical model adequately, alternatively, even when it is plausible to the formulate a proper mathematical model, the resulting optimization problem may prove too complex for available solution algorithms. An alternative approach to modeling complex system is simulation. Simulation modeling is the next best thing to observing a real system. It differ from mathematical modeling in that the relationship between the input and output need not be stated explicitly. Instead, it breaks down the real system into (small) modules and then imitates the actual behavior of the system by using logical relationship to link the module together. Starting with the input modules, the simulation computations move among the appropriate modules until the output result is realized.

Simulation computation through usually simple, are voluminous. It is unthinkable to execute a simulation model without the use of the computer.

Simulation models are much more flexible in representating systems than their mathematical counterparts. The main reason for this flexibility is that simulation views the system at elemental level, where as mathematical counterparts. The main reason for this flexibility is that simulation view the system at elemental level, where as the mathematical model tend to represent the system from global stand point.

1. The flexibility of simulation is not without drawbacks. The development of a simulation



model is usually costly in both time and resource. Moreover, the execution of some simulation models, even on the fastest computers, may be slow.

2. COMPARISION AND RESULT ANALYSIS BETWEEN DETERMINISTIC MODEL AND PROBABLISTIC MODEL :

.CPM (Critical Path Method) and PERT (Program Evalution and Review Technique) are network-based methods designed to assist in the planning. Scheduling and control of projects. A project is defined as a collection of interrelated activities with each activity consuming time and resources. The objective of CPM and PERT is to provides analytic means for scheduling the activities.Fig.3 summarizes the step of techniques. First , we define the activities of the project, their precedence relationships, and their requirements. Next, the project is translated into a network that shows the precedence relationship among the activities. The third step involves carring out specific network computation the facilitate the development of the time schedule for the project.



### Fig. 2

The two techniques, CPM and PERT, which were developed independently, differ in that CPM assumes deterministic activity durations. PERT, conversely, assumes probabilistic durations. This presentation concentrates on CPM only.

# 3.1.1 MATHEMATICAL CALCULATION USING DETERMINISTIC MODEL

There is following step to determine critical path

- (a) Earliest expected completion time for each event
- (b) Latest allowable event completion time for each event
- (C) Slack time for each event
- (d) Critical path

### (e) STEP 1

Forward path - Earliest expected completion time of events (TE)

 $TE(J^*) = max [TE(I)+ET(I,J^*)]$ 

- ET(I,J) = expected completion time of activity (I,J)
- TE(J) = earliest expected completion time of event J



TE(I) = 0

TE(2) = TE(I) + ET(1,2) = 0 + 2 = 2

TE(3) = TE(I) + ET(1,3) = 0 + 7.2 = 7.2

$$TE(4) = Max \{ [TE(I) + ET(1,4) [TE(3) + ET(3,4)] \}$$
$$= Max [(0 + 4.7), (7.2 + 2)] = 9.2$$

TE(5) = TE(2) + ET(2,5) = 2 + 4 = 6

$$TE(6) = TE(4) + ET(4,6) = 9.2 + 4 = 13.2$$

 $TE(7) = Max \{ [TE(5) + ET(5,7)], [TE(4) + ET(4,7) \},\$ 

$$[TE(6) + ET(6,7)]$$

$$= \max [(6 + 4), (9.2 + 6.2), (13.2 + 4.3)] = 17.5$$

### **STEP 2**

Backward path- Latest allowable event completion time (TL)

 $TL(I *) = \min[TL(J) - ET(I^*, J)]$ 



TL(7) = TE(7) = 17.5

TL(6) = TL(7) - ET(6,7) = 17.5 - 4.3 = 13.2

TL(5) = TL(7) - ET(5,7) = 17.5 - 4.0 = 13.5

$$TL(4) = \min \{ [TL(7) - ET(4,7)], [TL(6) - ET(4,6)] \}$$

 $= \min [(17.5 - 6.2), (13.2 - 4.0)] = 9.2$ 

TL(3) = TL(4) - ET(3,4) = 9.2 - 2.0 = 7.2

TL(2) = TL(3) - ET(2,5) = 13.5 - 4.0 = 9.5

 $TL(1) = min \{ [TL(2) - ET(1,2)], [TL(3) - ET(1,3)],$ 

[TL(4) - ET(1,4)]

= min [(9.2 – 2.0),(7.2 – 7.2), (9.2 – 4.7)] = 0 STEP 3 EVENT SLACK TIMES (SE)

SE(J) = TL(J) - TE(J)

STEP 4

CRITICAL PATH SE(M)

SE(M) = TL(M) - TE(M) = 0

To be an activity on the critical path, an activity (I,J) must satisfy three conditions:

1. TL(I) - TE(I) = SE(M)2. TL(J) - TE(J) = SE(M)3. TE(J) - TE(I) = TL(J) - TL(I) =ET(I,J)

Where SE(M) is the slack time for the final event. All activities on the path 1 - 3 - 4 - 6 - 7 in

Figure 4 satisfy these conditions, so the path is a critical path.

### **3.1.2 USE OF DETERMINICTIC RESULT**

The actual work begins on a project, the network should be updated either weekly, monthly, or as needed, with all of the latest available information. This information might include scheduled dates, available resources, activity time estimates, actual activity completion times, etc.

# 3.1.3 PROBABILISTIC MODEL OF COMPLETING ON SCHEDULE

There is following step to determine probablity

Activities

(a)Mean completion time for each activity

### (b)Variance of the completion time for each activity

(c)Value x(k) such that the integral of the N(0,1) distribution from  $-\infty$  to x(k) is the probability of completing event K on or before the scheduled completion time

Note in (g) that x(k) is the value such that

$$P[T(k) \le SD(k)] = \int_{-\infty}^{x(k)} N(0,1) dt$$

where T(k) is the completion time of event k. It is calculated using the formula

$$x(k) = \frac{SD(k) - TE(k)}{\sigma TE(k)}$$

Once x(k) is given, the user must find the area under the N(0,1) curve from  $-\infty$  to x(k) using the cumulative N(0,1) distribution in Table A.1. The value obtained is the probability of completing event k on or before the scheduled completion time.

These scheduled completion times can be used with the mean and variance of the completion time of each activity to determine the probability of completing event on schedule.

### 1 TABULAR FORM OF INPUT DATA FOR NETWORK

Estimates 
$$1-2$$
  $1-3$   $1-4$   $2-5$   $3-4$   $3-6$   $4-5$   $4-7$   $5-7$   $6-7$   
a 2.20 1.51 5.00 6.00 2.00 5.51  
1.76 4.00 5.00 2.51  
m 2.45 6.12 9.90 7.50 5.00  
10.21 4.06 8.16 10.38 5.62  
b 6.00 10.00 15.39 12.00 8.00  
14.00 6.00 11.35 13.49 11.00



Fig 5 Bar chart representation of given input data for network

#### We assume

a =optimistic time – the expected time if everything goes better then expected without delays

m =most likely time – the most realistic time available b = pessimistic time – the expected time if just about everything goes wrong Table which probability of completing events before scheduled completion time

TE = expected value of T

 $\sigma^2 TE$  = Variance of T

SD = scheduled completion time of

an event

### **3.1.2.1 USE OF PROBABILISTIC RESULTS**

After calculating the probability of completing each event on schedule ,events with low probabilities are noted and measure are taken to be "speed up" the completion.

# 4. CONCLUSION, SUGGESTION AND SCOPE FOR FUTURE WORK:

### 4.1 CONCLUSIONS:-

Project management with PERT (project Evalution and Review Techniques) and CPM (critical path method) is based on the basis that a small set of activities , which make up the longest path through the activity network control the entire project. If these "critical " activities could be identified and assigned to responsible person , management resource could be optimally used by concentrating on the few activities which determine the fate of the entire project . Non –critical activities can be reallocated flexibly, without affecting the whole project. Both are project management techniques, which have been created out of the need of western industrial and military establishments to plan , schedule and control complex project.

When we solve any problem by using the mathematical tools of operation research in that situation. We have to understand and accordingly value are calculated. In this case it take lots of time to reach the results. But by using the computer oriented operation research based analysis we can easily understand the situation and can solve

Event	TE	SD	$\sigma 2TE$	x	p(T
				$-\frac{SD-T}{T}$	SD)
				$\sigma TE$	
					$\int_{-\infty}^{x}$
2	3	4	0.4	1.58	0.94
3	6	6	2.0	0.0	0.5
4	11	10	3.0	-0.58	0.28
5	15	16	3.5	0.53	0.70
6	16	18	4.0	1.0	0.84
7	25	25	5.5	0.0	0.50



Figure 6 Bar chart representation of probabilistic model of network

where

T = Completion time of an event



problem very soon without any difficulties and in a very small time period. Thus we conclude that by the use of operation research and computer oriented analysis all the situation are very clear in no time and the problem is solved very soon. Hence we can have ability to solve the number of problem is a very small time.

### 4.2 SUGGESTION :-

Decision analysis provides tool for quantitative analyzing decision with uncertainty and multiple conflicting objectives, and these tools can be especially useful when there is limited directly relevant data so that expert judgment plays a significant role in the decision making process. It provides a systematic quantitative approach for making better decisions, rather than a description.

Now a days it has become very popular to solve the such critical situation by the operation research tools for deterministic and probabilistic problem. In the above two cases it has become very much useful tool to analysis the situation and these selection of the required values and correspondingly to feed in the computer mathematical model. We suggest that in any industrial organization or construction organization such type of implementation of this method will be very much helpful and thus the nation will lead to its development in the industrial revolution.

#### 4.3 SCOPE FOR FUTURE WORK:-

The implementation of such type of tools can be used in the other sector such as medical, agriculture, railway etc. It will be much helpful in the marketing. Construction of building, road and dams also. It will be much helpful to complete the work in very short time. In power plant whether in thermal or hydel where the power generation has to be done in very short time with greater output, this can be implemented.

#### REFERENCES

- 1. Afifi, A. A., and S.P. Azen: "Statistical Analysis, a computer oriented approach, " Acadmic Press,Inc., New York, 1972.
- Ackoff,R.(ed...) Progress in operation Researchvol. III. New York Wiley & Sons, Inc. 1968.
- Alford, L.D.(ed...), Cost and Production Handbook (2rd ed.). New York: The Ronald Press Company,1955.
- Bellman , R., and S.E. Dreyfus: "Applied Dynamic programming ," Princeton University press, Princeton , N.J.,1962.
- Birnbaum, Z. W.: Numerical Tabulation of the Distribution of Kolmogorov's Statistic for Finite Sample Size, J. Am. Statistical Assoc., Vol.47, pp.425-441, 1952.
- Chernoff, H., and L. E. Moses;" Elementray Decision Theory," John Wiley & Sons, Inc., New York, 1959.
- Edwards, A. L.:" Probablity and Statistics," Holt, Rinehart and Winston, Inc., New work, 1971.
   Taha, H.A., Operation Research: An Introduction, Prentice Hall of India Sixth Edition, 1997.
- Williams ,E.J.: "Regression Analysis," John Wiley & Sons, Inc., New York, 1959
- 9. Winston, W.L, Operation Research: Applications and Algorithms, Fourth Edition, 2007.
- W.T.George, "Operation Research and Evolution", Journal of Operation Research Society, Vol.37, No.8, 1986, pp.725-733.
- Wilson J.M., "Classification of Models in Operation Research", Journal of Operation Research Society, Vol.36, No.3, 1985, pp.253-256

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