SOLVE THE TRANSPORTATION PROBLEM OF TRAPEZOIDAL FUZZY NUMBERSUSING RUSSELL'S APPROXIMATION METHOD

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Abstract: Fuzzy set theory has been applied in many fields such as management, engineering, theory of matrices and so on. In this paper, some elementary operations on proposed trapezoidal fuzzy numbers (TrFNs) are defined and also have been defined some operations on trapezoidal fuzzy matrices(TrFMs).Using Russell's approximation method to solve the Fuzzy transportation problem of Trapezoidal Fuzzy numbers.

INTRODUCTION

Fuzzy sets have been introduced by Lofti.A. Zadeh. Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Hisdal discussed the interval-valued fuzzy sets if higher type. Interval arithmetic was first suggested by Dwyer in 1951, by means of Zadeh's extension principle, the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubosis and Prade have defined any of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers. Jhon studied an appraisal of theory and applications on type-2 fuzzy sets.

Trapezoidal fuzzy number's (TrFNs) are frequently used in application, due to the presence of uncertainty in many mathematical formulations in different branches of science and technology. Presenting a new ranking function and arithmetic operations on type-2 generalized Trapezoidal fuzzy numbers by Stephen Dinagar and Anbalagan.

Fuzzy matrices were introduced for the first time by Thomason who discussed the convergence of power of fuzzy matrix. Several authors had presented a number of results on the convergence of power sequences of fuzzy matrices. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in Shymal.A.K. and Pal.M. .

Ragab et.al presented some properties of the min-max composition of fuzzy matrices. Kim presented some important results on determinant of square fuzzy matrices and contributed with many research works. The adjoint of square fuzzy fuzzy matrix was defined by Thomson and Kim Jaisankar and Mani proposed the Hessenberg of Trapezoidal fuzzy number matrices.

In this paper a new method is presented for solving the fuzzy transportation problem using Russell's approximation method for the representative values of the fuzzy number. Using the proposed ranking method is discussed with illustration. It is very simple and easy to understand the fuzzy optimal solution of fuzzy transportation problems occurring in the real life situations.

BASIC PRELIMINARIES

Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

(i) $\tilde{A}$ is normal (ii) $\tilde{A}$ is convex

(iii) The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Trapezoidal fuzzy number

A fuzzy number $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{A^{TzL}}(x) = \begin{cases} 0 & ; \quad x \le a_1 \\ \frac{x - a_1}{a_2 - a_1} & ; \quad a_1 \le x \le a_2 \\ 1 & ; \quad a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3} & ; \quad a_3 \le x \le a_4 \\ 0 & ; \quad x > a_4 \end{cases}$$

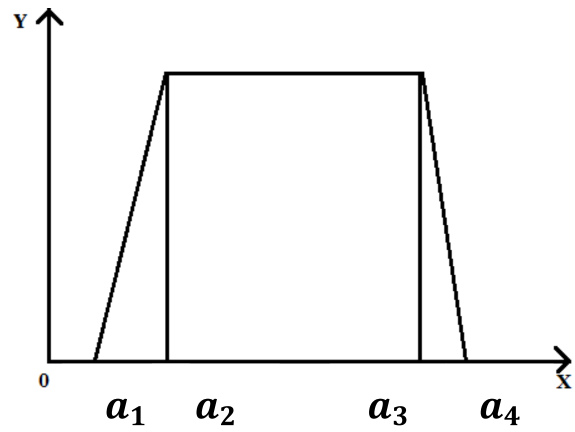

Fig:1 Trapezoidal fuzzy number

Definition

Let $\tilde{A}$= (a_1, b_1, α, β) and $\tilde{B}$= (a_2, b_2, α, β) be two Trapezoidal fuzzy numbers, then

$\tilde{A} \oplus \tilde{B}$= ($a_1$, b_1, α_1, β_1) ⊕ (a_2, b_2, α_2, β_2)

= (a_1+a_2, b_1+b_2, $\alpha_1+\alpha_2$, $\beta_1+\beta_2$)

$\tilde{A} \ominus \tilde{B}$= ($a_1$, b_1, α_1, β_1) ⊖ (a_2, b_2, α_2, β_2)

= (a_1-a_2, b_1-b_2, $\alpha_1+\alpha_2$, $\beta_1+\beta_2$)

Ranking functions: A convenient method for comparing of fuzzy number is by use of ranking function (Zimmermann, 1991 ; Maleki, 2002). A ranking function $\mathfrak{R}$: F(R)→R, where, F(R) (a set of all fuzzy numbers defined on set of real numbers), maps each fuzzy number into a real number of F(R).

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers in F(R), then:

- $\tilde{A}^{zL} \ge_{\mathfrak{R}} \tilde{B}^{zL}$ if and only if $\mathfrak{R}(\tilde{A}^{zL}) \ge \mathfrak{R}(\tilde{B}^{zL})$
- $\tilde{A}^{zL} \le_{\mathfrak{R}} \tilde{B}^{zL}$ if and only if $\mathfrak{R}(\tilde{A}^{zL}) \le \mathfrak{R}(\tilde{B}^{zL})$
- $\tilde{A}^{zL} =_{\mathfrak{R}} \tilde{B}^{zL}$ if and only if $\mathfrak{R}(\tilde{A}^{zL}) = \mathfrak{R}(\tilde{B}^{zL})$

International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 08 Issue: 02 | February - 2024 SJIF Rating: 8.176 ISSN: 2582-3930

Let $\mathfrak{R}$ be any linear ranking function, then $\tilde{A} \geq_{\mathfrak{R}} \tilde{B}$ if and only if $\tilde{A} - \tilde{B} \geq_{\mathfrak{R}} \tilde{0}$ if and only if $-\tilde{B} \geq_{\mathfrak{R}} -\tilde{A}$. If $\tilde{A} \geq_{\mathfrak{R}} \tilde{B}$ and $\tilde{C} \geq_{\mathfrak{R}} \tilde{D}$ then $\tilde{A}+\tilde{C} \geq_{\mathfrak{R}} \tilde{B}+\tilde{D}$

Ranking functions for Trapezoidal Fuzzy Number

For Trapezoidal fuzzy number $\tilde{A}^{zL}$ = (a, b, α, β), the ranking function is defined by

$$\mathfrak{R}(\tilde{A}) = 1/2 \int_0^1 [\tilde{A}_\lambda^L, \tilde{A}_\lambda^U] d\lambda$$

$$= 1/2 \int_0^1 [(l' + (a - l')\lambda) + (r' - (r' - b)\lambda)] d\lambda.$$

This reduces to

$$\mathfrak{R}(\tilde{A}) = \frac{a+b}{2} + \frac{\beta - \alpha}{4}$$

The Trapezoidal fuzzy number $\tilde{A}^{zL}$ = (a, b, α, β) and $\tilde{B}^{zL}$ = (c, d, γ, δ), we have $\tilde{A} \geq_{\mathfrak{R}} \tilde{B}$ if and only if

$$\mathfrak{R}(\tilde{A}) = \left(\frac{a+b}{2} + \frac{\beta - \alpha}{4}\right) \geq \left(\frac{c+d}{2} + \frac{\delta - \gamma}{4}\right) = \mathfrak{R}(\tilde{B})$$

Fuzzy transportation problem

The mathematical formulation of the FTP is of the following form (Table 1):

$minimize\ \tilde{\Psi} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} X_{ij}$

Subject to $\sum_{j=1}^{n} X_{ij} \leq \alpha_i,\ i = 1,2,\ldots,m$

$\sum_{i=1}^{m} X_{ij} \geq \beta_i \quad j = 1,2,\ldots,n \ X_{ij} \geq 0$

for all i and j,

Table 1: The fuzzy transportation table

	1	**2**	$\cdots$	**N**	α_i
1	c_{11}	c_{12}	$\cdots$	$\tilde{c}_{1n}$	α_1
2	c_{21}	c_{22}	$\cdots$	$\tilde{c}_{2n}$	α_2
⋮	⋮	⋮	⋮	⋮	⋮
β_j	β_1	β_2	$\cdots$	β_n	$\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j$

Where $\tilde{c}_{ij}$ is the fuzzy cost of transportation one unit of the goods ith source to the jth destination.

X_{ij} is the quantity transportation from i th source to the jth destination.

α_i is the total availability of the goods at ith source. β_j is the total demand of the goods at jth destination.

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \tilde{X}_{ij}$, is total fuzzy transportation cost.

If $\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j$, then FTP is said to be balanced.

If $\sum_{i=1}^{m} \alpha_i \neq \sum_{j=1}^{n} \beta_j$, then FTP is said to be unbalanced. .

Algorithm of fuzzy russe1l's approximation method (FRAM): FRAM is proposed to obtain the IFBFS of a particular type of the FTP. Russell's approximation method provides another excellent criterion that is still quick to implement on a computer (but not manually) (Russell, 1969). Although is unclear as to which is more effective on average, this criterion frequently does obtain a better solution than Vogel's. For a large problem, it may be worthwhile to apply both criteria and then use the better solution to start the iterations of the transportation simplex method. One distinct advantage of Russell's approximation method is that it is patterned directly after stepl for the transportation simplex method which somewhat simplifies the overall computer code (Hillier and Liberian, 2001).

In FRAM the $\tilde{u}$ and $\tilde{v}$ values have been defined as the largest fuzzy unit transportation cost for each row and column respectively. In this method, each allocation is made on the basis of the maximum

$$\tilde{\Delta}_{ij} = \tilde{u}_i \oplus \tilde{v}_j \ominus \tilde{c}_{ij}$$

The algorithm of this method for the FTP when all the cost coefficients are fuzzy numbers and all demands and supply are crisp numbers are follows:

Step 1: Let $\tilde{u}_i$, be the largest fuzzy unit transportation cost c_{ij} (i e., select the unit cost whose the fuzzy unite cost has the largest rank) for each source row i remaining consideration.

Step 2: Let $\tilde{v}_j$, be the largest fuzzy unit transportation cost c_{ij} (i e., select the unit cost whose the fuzzy unite cost has the largest rank) for each destination column j remaining under consideration.

Step 3: For each variable x_{ij} not previously selected in these rows and columns.
Compute

$$\tilde{\Delta}_{ij} = \tilde{u}_i \oplus \tilde{v}_j \ominus \tilde{c}_{ij}$$

or

$$\tilde{\Delta}_{ij} = \Re(\tilde{u}_i) \oplus \Re(\tilde{v}_j) \ominus \Re(\tilde{c}_{ij})$$

for each row and each column under consideration.

Step 4: Allocate as much as possible for the row and column with the maximum $\tilde{\Delta}_{ij}$ (i e., select the $\tilde{\Delta}_{ij}$ whose the fuzzy $\tilde{\Delta}_{ij}$ has the largest rank).

In case the maximum $\tilde{\Delta}_{ij}$ is not unique (i.e., the maximum $\tilde{\Delta}_{ij}$ of more than one is same), then select the variable x_{ij} where maximum allocation can be made.

Step 5: Adjust the supply and demand requirements to reflect the allocations already made. Eliminate any rows and columns in which supply and demand have been exhausted.

Step 6: If all supply and demand requirements have not been satisfied, go to the first step and recalculate new $\tilde{\Delta}_{ij}$. If all row and column values have been satisfied the initial fuzzy solution has been obtained.

Algorithm of fuzzy modified distribution method (FMDM): In this study, FMDM has been illustrated to find optimal solution of the FTP. The steps for finding optimal solution are as follows:

Step 1: Construct the IFBFS of the FTP by any of the initial methods.

Step 2: Derive the values of $\tilde{u}_i$ and $\tilde{v}_j$ corresponding to each ith row and jth column, respectively. Write $\tilde{u}_i$ in front of each ith row and $\tilde{v}_j$ in bottom of each jth column.

Step 3: For a basic variable x_{ij}, , let $\tilde{u}_i$ and $\tilde{v}_j$ satisfy the set of equations $\tilde{c}_{ij} = \tilde{u}_i \oplus \tilde{v}_j$ for each (i, j) such that x_{ij} is basic there are m+n-1 basic variables and so there are m+n-1 of these equations. Since the number of unknown is m*n, none of these variables can be assigned a value arbitrarily without violating the equations. The choice of this one variable and its value does not affect the value of any $\tilde{u}_i \oplus \tilde{v}_j \ominus c_{ij}$ even when x_{ij} is non-basic.

Step 4: Take any one of the $\tilde{u}_i$ or $\tilde{v}_j$ to be zero ranked fuzzy number.

Step: 5 Calculate the rank of $\tilde{d}_j = \tilde{u}_i \oplus \tilde{v}_j \ominus \tilde{c}_{ij}$ for every (i, j) such that x_{ij} is non-basic. The IFBFS is fuzzy optimal if and only if $\tilde{u}_i \oplus \tilde{v}_j \ominus \tilde{c}_{ij} \leq_{\Re} 0$ for each (i, j) such that x_{ij} is non-basic. If there exist at least one $\tilde{d}_j$, such that $\tilde{d}_j >_{\Re} 0$ then this IFBFS is not fuzzy optimal solution. Go to the step 6.

Step 6: Determine the entering basic variable: Because the FTP seeks to minimize cost, the entering variable is the one having the most positive coefficient in the z-row. In the FTP chose that $\tilde{d}_j$ whose rank is most positive.

Step 7: Determine the leaving basic variable: The leaving variable in the following manner. First, construct a closed loop that starts and ends at the entering variable cell. The loop consists of connected horizontal and vertical segments only (no diagonals are allowed). Except for the entering variable cell, each corner of the closed loop must coincide with a basic variable. Identify the chain reaction require to retain feasibility when the entering basic variable is increased. From the donor cells, select the basic variable having the smallest value.

Step 8: Determine the new fuzzy basic feasible solution: Add the value of the leaving basic variable to the allocation for each recipient cell. Subtract this value from the allocation for each donor cell.

Step 9: Again use the latest IFBFS and repeat steps 1-8 until $\tilde{d}_{ij} \leq_{\Re} 0$ for all i and j.

FUZZY TRANSPORTATION PROBLEM USING RUSSELL'S TECHNIQUE

NUMERICAL EXAMPLE

Consider fuzzy transportation problem with three sources that is S_1, S_2, S_3 and three destinations D_1, D_2, D_3 . The cost of transporting one unit of the goods from ith source to the jth destination given whose elements are trapezoidal fuzzy numbers, and is shown in Table 2. Find out the minimum cost total fuzzy transportation.

Since $\sum_{i=1}^{3} \alpha_i = \sum_{j=1}^{3} \beta_j = 125$, the FTP is balanced.

From Table 2, the cost of transporting one unit of the goods from i th source to the j th destination are TrFNs. In Table 2, the cost of the TrFN of the form (a-α, a, b, β-b). Convert all the costs into the form (a, b, α, β) and find the ranks of the costs using formula (a+b)/2+(β-α)/4 (Table 3).

Table 2: The fuzzy transportation table

SOURCE	D_1	D_2	**D_3**	**SUPPLY (a_i)**
S_1	(3, 5, 7, 14)	(2, 4, 8, 13)	(3, 5, 9, 15)	35
S_2	(2, 5, 8, 10)	(3, 6, 9, 12)	(4, 7, 10, 16)	40
S_3	(3, 6, 8, 13)	(4, 8, 10, 15)	(5, 9, 13, 15)	50
DEMAND (β_j)	45	55	25	

Table 3: Ranks of the Fuzzy transportation costs using Russell's method

SOURCE	D_1	D_2	D_3	SUPPLY
S_1	(5, 7, 2, 7) RANK=10.75	(4, 8, 2, 5) RANK=12	(5, 9, 2, 6) RANK=12.5	35
S_2	(5, 8, 3, 2) RANK=10	(6, 9, 3, 3) RANK=11	(7, 10, 3, 6) RANK=12	40

S_3	(6, 8, 3, 5) RANK=11.5	(8, 10, 4, 5) RANK=11	(9, 13, 4, 12) RANK=9.5	50
DEMAND	45	55	25	125

RESULTS AND DISCUSSION

Using the criterion for FRAM, the results, including the sequence of basic variables (allocations), are shown in Table 3. At iteration 1, the largest fuzzy unit transportation cost in row 1 is $\tilde{u}_1$ = (5, 9, 2, 6) the largest in column 1 is $\tilde{v}_1$ = (6, 8, 3, 5) and so forth. Thus:

$$\Re(\tilde{\Delta}_{11}) = \tilde{u}_1 \oplus \tilde{v}_1 \ominus c_{11}$$

$\Re(\tilde{\Delta}_{11})$ = (5, 9, 2, 6) ⊕ (6, 8, 3, 5) ⊖ (5, 7, 2, 7)

= (6, 10, 7, 18)

$\Re(\tilde{\Delta}_{11})$ = 10.75

Calculating all the $\Re(\tilde{\Delta}_{ij})$ values for i = 1, 2, 3 and j shows that $\Re(\tilde{\Delta}_{13})$ = 12.5 has the largest positive value, so, x_{13} = 25 is selected as the first basic variable (allocation). This allocation uses up 25 unit from the supply in row 1 and fully meet the demand in column 3, so this column is eliminated from further consideration.

The second iteration requires recalculating the $\Re(\tilde{\Delta}_{ij})$. The largest positive value now is:

$\Re(\tilde{\Delta}_{12}) = \tilde{u}_1 \oplus \tilde{v}_2 \ominus c_{12}$

$\Re(\tilde{\Delta}_{12})$ = (5, 7, 2, 7) ⊕(8, 10, 4, 5) ⊖(4, 8, 2, 5)

= (9, 9, 8, 17)

$\Re(\tilde{\Delta}_{12})$ = 11.25

and $\Re(\tilde{\Delta}_{11})$ = 10, $\Re(\tilde{\Delta}_{21})$ = 8.25

$\Re(\tilde{\Delta}_{22})$ = 9.25, $\Re(\tilde{\Delta}_{31})$ = 10.25, $\Re(\tilde{\Delta}_{32})$ = 9.75

So, x_{12} = 10 becomes the second basic variable (allocation), eliminating row 1, so this row is eliminated from further consideration.

The subsequent iterations proceed similarly. The final table after applying the algorithm of the FRAM (Table 5).

The objective function value at the IFBFS of the FTP:

Table 4: Results including the sequence of basic variables (allocations) from fuzzy Russell's approximation method.

Iteration	$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{u}_3$	$\tilde{v}_1$	$\tilde{v}_2$	$\tilde{v}_3$	Largest positive $\Re(\tilde{\Delta}_{ij})$	Allocation
1	(5, 9, 2, 6)	(7, 10, 3, 6)	(9, 13, 4, 2)	(6, 8, 3,5)	(8, 10, 4, 5)	(9, 13, 4, 2)	$\Re(\tilde{\Delta}_{13})$= 12.5	X_{13} = 25
2	(5, 7, 2, 7)	(6, 9, 3, 3)	(8, 10 4, 5)	(6, 8, 3,5)	(8, 10, 4, 5)	-	$\Re(\tilde{\Delta}_{12})$= 11.25	X_{12} = 10
3	-	(6, 9, 3, 3)	(8, 10 4, 5)	(6, 8, 3,5)	(8, 10, 4, 5)	-	$\Re(\tilde{\Delta}_{31})$= 10.25	X_{22} = 40
4	-	(6, 9, 3, 3)	(8, 10 4, 5)	-	-	-	Irrelevant	X_{32} = 5
5								X_{31} = 45

Table 5: Initial fuzzy basic feasible solution from Russell's approximation method

SOURCE	D_1	D_2	D_3	SUPPLY
S_1	(3, 5, 7, 14) RANK=10.75	(2, 4, 8, 13) RANK=12 **10**	(3, 5, 9, 15) RANK=12.5 **25**	35
S_2	(2, 5, 8, 10) RANK=10	(3, 6, 9, 12) RANK=11 **40**	(4, 7, 10, 16) RANK=12	40

S_3	(3, 6, 8, 13) RANK=11.5 **45**	(4, 8, 10, 15) RANK=11 **5**	(5, 9, 13, 15) RANK=9.5	50
DEMAND	45	55	25	125

Table 6: Optimal fuzzy basic feasible solution from MODI method

SOURCE	D_1	D_2	D_3	SUPPLY	
S_1	(5, 7, 2, 7)[1] RANK=10.75 [-3.25]	(4, 8, 2, 5) RANK=12 **10**	(5, 9, 2, 6) RANK=12.5 **25**	35	u_1=(4, 8, 2, 5)
S_2	(5, 8, 3, 2) RANK=10 [-2]	(6, 9, 3, 3) RANK=11 **40**	(7, 10, 3, 6) RANK=12 [-2.5]	40	u_2=(6, 9, 3, 3)
S_3	(6, 8, 3, 5) RANK=11.5 **45**	(8, 10, 4, 5) RANK=11 **5**	(9, 13, 4, 12) RANK=9.5 [-2.5]	50	u_3=(8, 10, 4, 5)
DEMAND	45	55	25	125	
	v_1=(-2, -2, 10, 7)	v_2=(0, 0, 0, 0)	v_3=(1, 1, 4, 11)		

Transportation cost = (2, 4, 8, 13)(10)+ (3, 5, 9, 15)(25)+ (3, 6, 9, 12)(40)+ (3, 6, 8, 13)(45)+ (4, 8, 10, 15)(5) = (370, 715, 1075, 1645) = 951.25

Step 4: Using MODI Method

$C_{12} = U_1 \oplus V_2 = (4, 8, 2, 5)$

$\Rightarrow U_1 = (4, 8, 2, 5)$

$C_{22} = U_2 \oplus V_2 = (6, 9, 3, 3)$

$\Rightarrow U_2 = (6, 9, 3, 3)$

$C_{32} = U_3 \oplus V_2 = (8, 10, 4, 5)$

$\Rightarrow U_3 = (8, 10, 4, 5)$

$C_{13} = U_1 \oplus V_3 = (5, 9, 2, 6)$

$\Rightarrow V_3 = (5, 9, 2, 6) - (4, 8, 2, 5)$

$\Rightarrow V_3 = (1, 1, 4, 11))$

$C_{31} = U_3 \oplus V_1 = (6, 8, 3, 5)$

$\Rightarrow V_1 = (6, 8, 3, 5) - (8, 10, 4, 5)$

$\Rightarrow V_1 = (-2, -2, 10, 7)$

Evaluate the net evaluations of non-basic variable using the formula $\tilde{u}_i \oplus \tilde{v}_j \ominus \tilde{c}_{ij}$

C_{11}= [-3.25], C_{21} = [-2], C_{23} = [-2.5], C_{33} = [-2.5]

All net evaluations are less than zero. Hence the table is optimal.

Transportation cost = (2, 4, 8, 13)(10)+ (3, 5, 9, 15)(25)+ (3, 6, 9, 12)(40)+ (3, 6, 8, 13)(45)+ (4, 8, 10, 15)(5) = (370, 715, 1075, 1645) = 951.25

CONCLUSION

In this paper, concentrated the notion of the Trapezoidal fuzzy matrices and operations of trapezoidal fuzzy matrices are defined. Few illustrations based on operations of trapezoidal fuzzy matrices have also been justified. To solve the Transportation Problem using the Russell's approximation method and ATM method and also the Ranking of Trapezoidal fuzzy numbers with illustration. To solve the Assignment Problem, Sequencing Problem and so on is using the

Russell's approximation method of Trapezoidal fuzzy numbers will discuss in future.

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