International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 08 Issue: 12 | Dec - 2024 SJIF Rating: 8.448 ISSN: 2582-3930

Special Pair of Rectangles and Wagstaff Primes

G. Janaki[1], S. Shanmuga Priya*[2]

[1]*Department of Mathematics, Cauvery college for Women (Autonomous),Affiliated to Bharathidasan University , Trichy, Tamil Nadu, India.*
[2]*Department of Mathematics, Cauvery college for Women (Autonomous), Affiliated to Bharathidasan University Trichy, Tamil Nadu, India.*

---***---

Abstract - The objective of this work is to identify the pairs of rectangles such that a Wagstaff prime number represents the sum of their areas in each pair. Each Wagstaff prime number is also accompanied by the number of primitive and non-primitive rectangles.

***Key Words*:** Pair of rectangles, Wagstaff prime number, Primitive, Non-primitive, Area.

1. INTRODUCTION

In mathematics, which is the universal language, numbers are important. The branch of pure mathematics that focusses on integer solutions is called number theory. Finding solutions to Diophantine problems involves using integers and involves fewer equations than unknown variables. A universal approach for determining if a Diophantine equation has a solution or identifying all of its solutions is not yet available. In this regard, [1-4] may be cited. Similarly, one can have knowledge about Dhurva numbers and some other special numbers from [5-7].

Any numerical sequence that can be represented by a mathematical function can be regarded as a pattern. Actually, it is possible to think of mathematics as a description of patterns. Any regularity that may be demonstrated by a scientific theory is a pattern for the purposes of clarity. Stated otherwise, a pattern is a collection of shapes, numbers, or things that adhere to a set of rules. The number of sides of a polygon and its polygonal numbers correlate one to one, as any pattern-conscious observer might perceive. Apart from these, many authors dealt with special pairs of rectangles and Pythagorean triangles related with many special polygonal numbers like Jarasandha numbers, sphenic numbers etc., and they found the number of primitive and non-primitive rectangles and triangles based on the dimensions of rectangles and sides of triangles which can be found from [8-24] .

In this study, sum of the areas of two rectangles is taken as Wagstaff prime number of 1,2,3 and 4 digits. The number of primitive rectangles and non-primitive rectangles are also evaluated and tabulated based on the dimensions of the two rectamgles. In the section 2, Some basic definitions are given and the methodology of finding the rectangles are given in section 3. Section 4 presents some remarkable observations based on the table 1.

2. BASIC DEFINITIONS

In a talk at the Eurocrypt 1990 conference, François Morain is credited by the prime sites with naming Wagstaff primes after the mathematician Samuel S. Wagstaff Jr. Wagstaff primes are used in cryptography and are mentioned in the New Mersenne conjecture.

Definition 1:- Wagstaff Prime number is a prime number of the form $\frac{2^p+1}{3}$, where p is an odd prime.

The first few Wagstaff primes are:

3, 11, 43, 683, 2731, 43691, 174763, 2796203, 715827883, 2932031007403, 768614336404564651,…..

Definition 2:- A rectangles with dimensions (m,n) is said to be primitive if $gcd(m,n) = 1$.

Definition 3:- A rectangles with dimensions (m,n) is said to be non-primitive if $gcd(m,n) \neq 1$

3. METHOD OF ANALYSIS

Let $R_1(m,n)$ and $R_2(p,q)$ be the two different rectangles whose areas are represented as A_1', A_2' and (m,n), (p,q) are the dimensions of two distinct rectangles R_1, R_2 respectively.

Let

$$A_1' + A_2' = W_p \quad (1)$$

where 'W_p is a Wagstaff number.

That is,

$$mn + pq = W_p \quad (2)$$

Let $u, v, w \in \mathbb{Z}_+$, where all three are distinct and non-zero with $v > w$.

Take the linear transformations

$$m = w, \ n = 2u + w, \ p = v - w, \ q = v + w$$

On using this transformation in (2), one may have

$$w(2u + w) + (v - w)(v + w) = W_p$$

$$v^2 = W_p - 2uw \quad (3)$$

On solving (3), one can obtain the values for u, v, w for the 1,2,3 and 4 digits Wagstaff number W_p. Also, the dimensions of the rectangles R_1, R_2 are evaluated. Based on the value of the dimensions, it is easy to categorize the primitive and non-primitive rectangles.

Few numerical examples of different Wagstaff primes are listed in the Table 1.

Table 1

$A'_1 + A'_2 = W_p$	R_1	R_2	Observation	
			Primitive	Non-Primitive
11	(1,3)	(2,4)	R_1	R_2
43	(1,35)	(2,4)	R_1	R_2
43	(1,19)	(4,6)	R_1	R_2
43	(3,9)	(2,8)		R_1, R_2
683	(1,675)	(2,4)	R_1	R_2
683	(1,659)	(4,6)	R_1	R_2
683	(1,635)	(6,8)	R_1	R_2
683	(1,603)	(8,10)	R_1	R_2
683	(7,93)	(2,16)	R_1	R_2
683	(1,563)	(10,12)	R_1	R_2
683	(1,515)	(12,14)	R_1	R_2
683	(1,459)	(14,16)	R_1	R_2
683	(1,395)	(16,18)	R_1	R_2
683	(1,323)	(18,20)	R_1	R_2
683	(7,53)	(12,26)	R_1	R_2
683	(1,243)	(20,22)	R_1	R_2
683	(11,33)	(10,32)		R_1, R_2
683	(1,155)	(22,24)	R_1	R_2
683	(7,29)	(16,30)	R_1	R_2
683	(11,25)	(12,34)	R_1	R_2
683	(1,59)	(24,26)	R_1	R_2
2731	(13,23)	(38,64)	R_1	R_2
2731	(5,31)	(46,56)	R_1	R_2
2731	(1,131)	(50,52)	R_1	R_2
2731	(1,331)	(48,50)	R_1	R_2
2731	(5,71)	(44,54)	R_1	R_2
2731	(33,43)	(16,82)	R_1	R_2
2731	(3,113)	(46,52)	R_1	R_2
2731	(11,41)	(38,60)	R_1	R_2
2731	(15,37)	(34,64)	R_1	R_2
2731	(3, 177)	(44,50)		R_1, R_2
2731	(29,47)	(18,76)	R_1	R_2
2731	(9,67)	(38,56)	R_1	R_2
2731	(1, 523)	(46,48)	R_1	R_2
2731	(1, 707)	(44,46)	R_1	R_2
2731	(1, 883)	(42,44)	R_1	R_2
2731	(3, 297)	(40,46)		R_1, R_2
2731	(7, 133)	(36,50)		R_1, R_2
2731	(9, 107)	(34,52)	R_1	R_2
2731	(21,63)	(22,64)		R_1, R_2
2731	(1, 1051)	(40,42)	R_1	R_2
2731	(3, 353)	(38,44)	R_1	R_2
2731	(5, 215)	(36,46)		R_1, R_2

$A'_1 + A'_2 = W_p$	R_1	R_2	Observation	
			Primitive	Non-Primitive
2731	(7, 157)	(34,48)	R_1	R_2
2731	(15,85)	(26,56)		R_1, R_2
2731	(35,65)	(6,76)		R_1, R_2
2731	(21,71)	(20,62)	R_1	R_2
2731	(25,67)	(16,66)	R_1	R_2
2731	(1, 1211)	(38,40)	R_1	R_2
2731	(5, 247)	(34,44)	R_1	R_2
2731	(11, 121)	(28,50)		R_1, R_2
2731	(1, 1363)	(36,38)	R_1	R_2
2731	(3, 457)	(34,40)	R_1	R_2
2731	(1, 1507)	(34,36)	R_1	R_2
2731	(3, 505)	(32,38)	R_1	R_2
2731	(1, 1643)	(32,34)	R_1	R_2
2731	(1,1771)	(30,32)	R_1	R_2
2731	(3,593)	(28,34)	R_1	R_2
2731	(5,359)	(26,36)	R_1	R_2
2731	(15,133)	(16,46)	R_1	R_2
2731	(1,1891)	(28,30)	R_1	R_2
2731	(3,633)	(26,32)		R_1, R_2
2731	(5,383)	(24,34)	R_1	R_2
2731	(7,277)	(22,36)	R_1	R_2
2731	(9,219)	(20,38)		R_1, R_2
2731	(15,141)	(14,44)		R_1, R_2
2731	(21,111)	(8,50)		R_1, R_2
2731	(27,97)	(2,56)	R_1	R_2
2731	(1,2003)	(26,28)	R_1	R_2
2731	(7,293)	(20,34)	R_1	R_2
2731	(11,193)	(16,38)	R_1	R_2
2731	(13,167)	(14,40)	R_1	R_2
2731	(1,2107)	(24,26)	R_1	R_2
2731	(3,705)	(22,28)		R_1, R_2
2731	(9,243)	(16,34)		R_1, R_2
2731	(13,175)	(12,38)	R_1	R_2
2731	(1,2203)	(22,24)	R_1	R_2
2731	(3,737)	(20,26)	R_1	R_2
2731	(1,2291)	(20,22)	R_1	R_2
2731	(5,463)	(16,26)	R_1	R_2
2731	(1,2371)	(18,20)	R_1	R_2
2731	(3,793)	(16,22)	R_1	R_2
2731	(5,479)	(14,24)	R_1	R_2
2731	(15,173)	(4,34)	R_1	R_2
2731	(1,2443)	(16,18)	R_1	R_2
2731	(3,817)	(14,20)	R_1	R_2
2731	(11,233)	(6,28)	R_1	R_2

$A'_1 + A'_2 = W_p$	R_1	R_2	Observation	
			Primitive	Non-Primitive
2731	(1,2507)	(14,16)	R_1	R_2
2731	(7,365)	(8,22)	R_1	R_2
2731	(1,2563)	(12,14)	R_1	R_2
2731	(3,857)	(10,16)	R_1	R_2
2731	(7,373)	(6,20)	R_1	R_2
2731	(1,2611)	(10,12)	R_1	R_2
2731	(3,873)	(8,14)		R_1, R_2
2731	(5,527)	(6,16)	R_1	R_2
2731	(9,299)	(2,20)	R_1	R_2
2731	(1,2651)	(8,10)	R_1	R_2
2731	(5,535)	(4,14)		R_1, R_2
2731	(1,2683)	(6,8)	R_1	R_2
2731	(3,901)	(4,10)	R_1	R_2
2731	(1,2707)	(4,6)	R_1	R_2
2731	(3,905)	(2,8)	R_1	R_2
2731	(1,2723)	(2,4)	R_1	R_2

REMARKABLE OBSERVATIONS

- It is found that no pair of rectangles exists for the one digit Wagstaff prime number.
- It is observed that all the dimensions of the rectangle R_1 for 2,3,4 digit Wagstaff prime number are odd.
- It should be noted that all the dimensions of the rectangle R_2 are even for 2,3,4 digit W_p.
- For each value of 2,3,4 digit W_p, R_2 is non- primitive.
- For each value of W_p, there exists atleast one non- primitive rectangle R_1.

5. CONCLUSION

This study attempts to find pairs of rectangles where the sum of their areas is represented by a Wagstaff prime number for each pair. For each 1,2,3,4 digits Wagstaff prime number W_p, readers of this paper may look for pairings of rectangles other than the ones shown above.

REFERENCES

1. Carmichael.R.D, The Theory of Numbers and Diophantine Analysis, Dover Publication, New York (1959).
2. Dickson.L.E, History of the theory of Numbers, Vol II, Diophantine Analysis, New York, Dover 2005.
3. John.H, Conway and Richard K.Guy, the Book of Numbers, Springer Verlag, New York, 1995.
4. Mordell.L.J, Diophantine Equations, Academic press, London (1969).
5. Kapoor.J.N, Dhuruva numbers, Fascinating world of Mathematics and mathematical Sciences, Trust Society, Vol.17, 1997.
6. Bert Miller, Nasty numbers, The Mathematics Teacher, No.9, Vol.73,649, 1980.
7. Charles Boum.K, Nasties are primitives, The Mathematics Teacher, No.9, Vol.74,502-504, 1981.
8. Sastry,P,S,N, Jarasandha numbers, The Mathematics Teacher, N0.9, Vol.37, issue 3 and 4200.
9. Gopalan.M.A, Sangeetha. V and Manju Somanath, "Pythagorean triangle and Polygonal numbers", Cayley J. Math, Vol.2 (2), 151-156, 2013
10. Gopalan.M.A, Janaki.G, "Pythagorean triangle with Nasty number as a leg", Journal of Applied Mathematical Analysis and Applications, Vol.4, No. 1-2, 13- 17, 2008.
11. Dr. Mita Darbari, A connection between Hardy-Ramanujan number and Special Pythagorean triangle", Bulletin of Society for Mathematical Services and Standards, Vol.3, No.2, 71-73, 2014.
12. Gopalan.M.A, Vidhyalakshmi.S, Premalatha.E and Presenna.R, "Special Pythagorean triangles and 5-digit Dhuruva numbers, IRJMEIT, Vol.1 (4), 29-33, Aug 2004.
13. Gopalan.M.A, Vidhyalakshmi.S, Premalatha.E and Presenna.R, "Special Pairs Pythagorean triangles and Dhuruva number, Global Journal of Science Frontier Research, Vol.15, p.33-38, 2015.
14. Janaki. G, Vidhya S, " Special pairs of rectangles and sphenic numbers", International Journal for Research in Applied

Science & Engineering Technology (IJRASET), Vol. 4(2), 376-378, 2016.

15. Charles Bown. K. Nasties are primitives. The mathematics teacher, 74(9):502–504, 1981.
16. Mita Darbari and Prashans Darbari. Pythagoren triangles with sum of its two legs as dodecic. GSC Advanced Engineering and Technology, 3(1):011–015, Feb 2022.
17. M. A. Gopalan and A. Gnanam. Pythagorean triangles and polygonal numbers.International Journal of Mathematical Sciences,, 9(1-2):211–215, 2010.
18. M. A. Gopalan and G. Janaki. Pythagorean triangle with area/perimeter as a special polygonal number. Bulletin of Pure & Applied sciences, 27(2):393–405, 2008.
19. M. A. Gopalan and G. Janaki. Pythagorean triangle with nasty number as a leg. Journal of applied Mathematical Analysis and Applications, 4(1-2):13–17, 2008.
20. M. A. Gopalan and A. Vijayasankar. Observations on a pythagorean problem. Acta Ciencia Indica, Vol.XXXVI M(No 4):517–520, 2010.
21. G. Janaki and Saranya .C. Special rectangles and jarasandha numbers. Bulletin of Mathematics and Statistics Research, 4(2):63–67, April - June 2016.
22. G. Janaki and Saranya .C. Special pairs of pythagorean triangles and jarasandha numbers. American International Journal of Research in Science, Technology, Engineering & Mathematics, (13):118–120, Dec 2015- Feb 2016.
23. G. Janaki and Saranya .C. Connection between special pythogoren triangle and jarasandha numbers. International Journal of Multidisciplinary Research & Development, 3(3):236–239, March 2016.
24. G. Janaki and Saranya .C. Special pairs of rectangles and jarasandha numbers. Asian Journal of Science & Technology, 7(5):3015–3017, May 2016.