STRUCTURAL TOPOLOGY OPTIMIZATION FOR MULTIPLE LOADS

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ABSTRACT—
such that they resist stability of the structure. In modern Engineering, the developer requires light weight, high performance and low-cost structures. So, by Structural optimization, the best possible configuration of structure domain In the last decades the structure design process is based on selecting the materials, their size and configuration, to resist the service load is achieved. Topology optimization is achieving the best with the variable resource while satisfying the constraints. In Topology optimization researchers used simple prismatic member as calculation of FEM is easy. For non-prismatic members or curved structures, Isoperimetric formulation is used.

In paper, they generally explain the simply supported structure domain with only one point load at the centre. But in general the loads can be varied throughout the length (L/3, L/5, L/8 etc.). Which was actually a challenging problem to optimize the structure. In practical, load on the structure may be inclined, vertical and horizontal which again was a major to solve this type of problems. However, after changing some line, all the above problems were sorted and the code gave expected results.

***Keywords*:** Discretization, Iso parametric mapping, SIMP method, Finite element analysis, Sensitivity analysis, Filtering sensitivity,

1.INTRODUCTION

2.1 General: -Topology optimization, first introduced by M. Bendose [2] has matured over the last few decades [3, 4] and has had a significant influence on design optimization research. It involves mathematically redistributing material inside an initial design domain so as to satisfy mechanical and physical performance constraints.

In mechanics, the structure design process is based on selecting the materials, their size and configuration, such that they resist stability of the structure. In modern Engineering, the developer requires light weight, high performance and low-cost structures.

Optimization means to optimize the structure domain on the basis of size, shape or topology. It means reduction of the material from the structure domain conforming the service load on the structure. So, by Structural optimization, the best possible configuration of structure domain to resist the service load is achieved. Out of three Structural optimizations, Topology optimization is the best form of optimization as by doing topology optimization, size and shape optimization is automatically taken care of Topology means surface. It is a computational approach to optimize the structural material within the fixed design domain, given service load, dead load and boundary conditions such that the resulting optimized element meets the given sets of design requirements. The classical topology optimization problem is that of optimizing material distribution in 2D or 3D to minimize the structural compliance, i.e finding the density distribution over a lattice for a chosen volume fraction, so as to minimize the elastic strain energy beneath a given set of external loads and boundary conditions. Density-based methods are broadly used by engineers compared to level-set methods, topological derivative method and phase field method.

In Topology optimization for curved domains while discretizing with rectangular element there may be discretization error, as we will not get the curved shape properly, there was a concept of iso parametric elements which maps rectangular elements with quadrilateral elements for discretizing a curved domain properly.

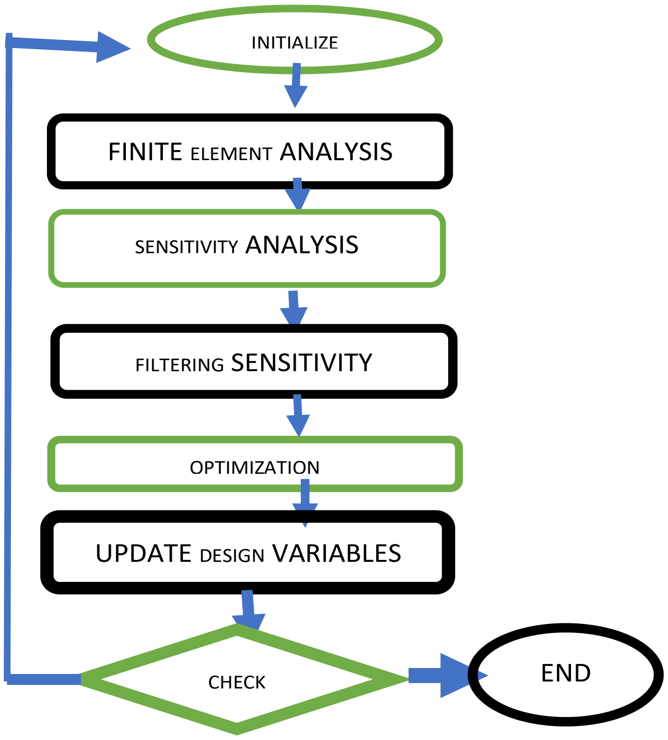
In paper, they generally explain the simply supported structure domain with only one point load at the centre. But in general the loads can be varied throughout the length (L/3, L/5, L/8 etc). Which was actually a challenging problem to optimize the structure. In practical, load on the structure may be inclined, vertical and horizontal which again was a major to solve this type of problems. Following are the three basics types of Structural topology optimization.

a. Size Structural optimization b. Shape Structural optimization c. Topology Structural optimization

2.2 Aim and objectives of the Paper: -

- To formulate and implement structural topology optimization problem.
- To obtain stiffness matrix and finite element formulation for topology optimization.
- To obtain topology optimization for multiple load cases.
- To obtain topology optimization for structures under uncertain loading.

2.3 Approaches for the topology structural optimization



SIMP stands for Solid Isotropic Microstructure with Penalization [2]. This method was developed by Rozvany and Zhou in 1992. However, the method is applied by Bendsoe in 1989. According to Bendsoe it is also known as direct "approach" and "power law approach". According to this approach, we assume a continuous structure with uniform densities as opposed to discrete form and implement the topology optimization. The resulting solution can be represented by solids and voids regions. Here even intermediate densities between 0 and 1 are considered while optimizing the structure in a definite pattern. Here solids mean 1 and voids mean 0. The solution of intermediate densities moving closer to 0 and 1 involves the penalization power that's why it is called power law approach. And because of the power-law approach, it gave more clear Solution.

3. LITERATURE REVIEW

Rong Y. et al (2022) [1] explain structural topology optimization by using adaptive design domain and they found this method very suitable and they developed it. Cornelius Lanczos from this book, I understood the basic of optimization and what the needs to optimize areand resource. The following optimization problem statements are given below Minimize objective function (optimization variable, state variable). Optimization variable (related to geometric features).

- Constraints on a resource (weight, cost, size etc.)
- Constraints on performance (stiffness, strength etc.)
- Limits on variable

Bendose & Sigmund [2] used optimization and its function. In the book, various methodologies are explained but most appropriate methodology is SIMP. Bendose also gave the penalization formula and explained its effect on the optimized structure. He also explained the sensitivity and its function that is how it participates in topology optimization. Bendose also gave the filtering criteria which depend on the filter radius and sensitivity. In this book, optimization is broadly divided into three categories (size, shape and topology) and this book only explained about topology optimization.

O. Sigmund (2001) [3] The paper explained the Matlab coding in topology optimization for the static external load. The total line of Matlab code is 99 that's why it's called 99 line codes. It also gives an explanation of all lines. O. Sigmund gives the particular point load that is in the middle, it does not give any generalize formulation. O. Sigmund solved the topology optimization based on the SIMP approach and this approach is based on material densities that means compliance is minimized with respect to the material densities. It also gave an example of a passive element with Matlab code.

Christian Gogu (2009) [4] Topology optimization of larger domain structure is expensive because of computational time taken by the computer at each iteration. Reduced orders models reduce the computational time because it uses reduce basis approach which increases the effectiveness of the topology optimization. In this method the difference between the reduced displacement fields and the original displacement field is calculated and compared with the threshold residue. A direct approach and reduced approach was tested and gave 2D and 3D comparisons results. In this method, vector concepts like orthogonal and orthonormal are used.

Ruben Ansola Loyola, et al (2018) [5] this paper explained the topology optimization with different MATLAB coding SERA (Sequential Element Rejection and Admission). In this paper optimization is based on the densities of material same as the Sigmund**.**

Peng Wei, et al (2018) [6] Gave the 88 line MATLAB code and compare the results came from this code and 99 line MATLAB code . Author gives the explanations of lines with numerical examples. It is also gave the passive element example and compare the results to 99 line code.

4. THEORY OF TOPOLOGY OPTIMIZATION: -

4.1 Theoretical formulation - It is based on the adjustable densities of materials and design criteria based on stiffness or flexibility. In topology optimization flexibility is synonymous of 'compliance'. Compliance means flexibility of structure that is inverse of stiffness. This design aims to optimize a structured domain having minimum flexibility/ minimum compliance or maximum stiffness ($c = k^{-1}$). The mathematical formulation of the material distribution problem may be represent as follows***:***

{Maximum Stiffness S t. V (X)< V_{max}}

Now replace stiffness by compliance for an optimization problem.

{Equilibrium KU=F, Compliance***:*** C = F^T U s.t-K U = F},

$Min: C(X) = U^T KU = \int_{e=1}^{N} (X_E)^P \, u_e k_0 u_e$

$s.t: \frac{V(X)}{V_0} = f$

$: KU = F$

$: 0 < X_{min} \leq X \leq 1$

: e=1,2,3,4………… N

F→ external forces vector, U→ Displacement vector , K→ global stiffness matrix of the structure V_x→ volume of an element, V→ maximum arranged volume of whole structure.

X →vector design variables c N→ dimension of the displacement and force vectors are denoted.

onsisting of the individual element (e) densities(X_e).

u_e →elemental displacement vectors and stiffness matrix.

$V_{(x)}$ and V_0 is the material volume and design domain volume.

N=*nelx* × *nely* Where *nel x* and *nely* is the no of the element in x and y-direction. f is the volume fraction, p is penalization power (generally, p=3).P≥ max { 2/1-µ , 4/1+µ }

4.2 Sensitivity analysis

Sensitivity means refining the element and it can be defined as the derivatives of the compliance (objective function) with respect to the densities of material (design variables). In this minimum objective function (compliance) is found. To find this minimum value, the derivatives are done on the elemental basis i.e computations is done over each element (e).

Sensitivity derivatives of compliance

$$\frac{\partial c}{\partial x_e} = F^T \frac{\partial U}{\partial X_e}$$

$$\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1} {U_e}^T k_0 u_e$$

Sensitivity is work for discretization the element whether zero or one required at that place.

- Sensitivity = 0 Hence no relative densities that is no material is required at that place.
- Sensitivity >0 Relatives densities value comes and material is required at that palace.

4.3 Volume fraction

It is the ratio the volume of an element to maximum prearranged volume. According to M. langelaar [13] maximum volume is restricted to 50% of solid.

Volume fraction 0.6 means in total volume 40% volume are extracted from the structural domain that is optimized structure have only 60% material. The aspect ratio 3 that is length to height gives more effective and clear results.

Topology optimization is dependent on volume fraction that is how much material is required to resist given load or feasible structure criteria.

5. FINITE ELEMENT ANALYSIS

In this chapter the MATLAB codes and formulation of the stiffness matrix are explained. In Topology optimization for curved domains while discretizing with rectangular element there may be discretization error, as we will not get the curved shape properly, there was a concept of iso parametric elements which maps rectangular elements with quadrilateral elements for discretizing a curved domain properly.

5.1 Finite element analysis

The design domain is discretized with square meshing. Each square element has four nodes with each node having 2° of freedom with horizontal and vertical respectively. Elements are numbered column-wise from left to right and also nodes are numbered from left to right. The meshing pattern is given below in the figure

Here for the element (1), elemental displacements 3 ,4, 11, 12, 9, 10, 1 and 2 are considered anticlockwise from left side lower end.

$$edfmt = \begin{bmatrix} 3 & 4 & 11 & 12 & 9 & 10 & 1 & 2 \\ 5 & 6 & 13 & 14 & 11 & 12 & 3 & 4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Considered element in x direction is two similarly in y direction is 2. All individual elements having 8×8 elemental stiffness matrix. If total nodes considered n here 9 then total no of degree of freedom is 2n here 18 and global stiffness matrix size is also 2n.

Formulation of elemental stiffness matrix in the iso Type equation here.parametric form

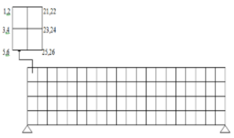


FIG 5.1

Given Data Thickness=1 unit Width=1 unit Length=1 unit

$$X = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4$$
$$Y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$$

6.TOPOLOGY OPTIMIZATION: RESULTS

Generalize load calculation and results

In paper, they generally explain the simply supported structure domain with only one point load at the center but in general the loads can be varied throughout the length (L/3, L/5, L/8etc). Which was actually a challenging problem to optimize the structure. In practical, load on the structure may be inclined, vertical and horizontal which again was a major to solve this type of problems. However, after changing some line, all the above problems were sorted and the code gave expected results.

In this case, user defined codes are

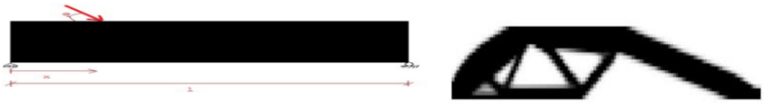
top (nelx, nely, vol frac, penal, r min, b1, n) Where, b1 is multiplication of length (L/3, L/5 etc.) and n is no. of loads acting on the structure. For inclined load, force coordinate is resolved in the vertical and horizontal planes respectively at that point. Where (in degree) is the angle between the horizontal surface and applied external force.

6.1 Test case 1- For generalize vertical load

The boundary conditions and loads of the beam for the 2D case are characterized in Figure 6.1and its optimized structure is given below. top (nelx, nely, vol frac, penal, r min , b_1 ,n)

FIG 6.1 (load beam & boundary condition)

6.2 Test case 2- For generalize inclined load



The boundary conditions and loads of the beam for the 2D case are represented in Figure 5.2 and

FIG 6.2 (a)Load condition (b) boundary condition

its optimized structure is given below top (nelx, nely, vol frac, penal, r min, b1, n)

6.3 Multiple load calculation and results

In the paper (a 99 line topology optimization) by Sigmund in 2001, he explained the simply supported structure domain with an only point load at the center. In multiple loads, it becomes quite challenging to optimize the structure domain. I optimized the structure domain of simply supported with external two and three-point load for the specific number of an element. The main program is called from the Matlab prompt: -top (nelx, nely, vol frac, penal, r min)

6.3.1 Test case 1- For MBB (the Messerschmitt-Bolkow-Blohm) problem

The boundary conditions of the beam for the 2D case are represented in Figure 5.3 (the Messerschmitt-Bolkow- Blohm/MBB problem).

6.3.2 Test case 2 – For two-point problem

The boundary conditions and loads of the beam for the 2D case are represented in Figure 5.4 and its optimized structure is given below.

Figure 6.3 (a) Two point load structure domain (b) optimized structure

7.CONCLUSION AND FUTURE SCOPE

7.1Introduction

Topology optimization can be done with several solution methods. However, in this report, the SIMP methods with penalization value have been used. This project aims for the computation of the structure topology optimization of structural continuum domain.

7.2 Based on the results and analysis, the following conclusions/summary can be drawn from this study

1.Develop the code, iso parametric mapping for actual quadrilateral element (not square or rectangle) and from this mapping defined the stiffness matrix of actual element on the basis of iso parametric formulation.

2.Develops the code, for generalization of load that means load is vertical, horizontal, inclined and at any point of the structure i.e. multiple of its length (L/3, L/5, and L/9 etc) code gave expected result.

3.Generated the code for structure domain having multiple loads (two point, three point & more) and gave optimal solution of that structure domain.

4.Topology optimization of structures domain under uncertain loading.

7.3 Future scope

Extensive work included following

•Formulation of stiffness matrix of curved structure (arch, complicated structure etc) with the help of iso parametric mapping.

•Structure topology optimization of curved structures domain (arch, complicated structure etc) using iso parametric mapping and ROM.

•Structure topology optimization for composite structures domain using ROM.

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