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Student Loans in America

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Introduction

As a prospective university student, student loans will probably account for a significant portion of one's financial obligations throughout higher education. Its primary objective is to enable students to focus exclusively on their studies without having to work part-time to cover living costs, tuition, books, or other needs necessary to sustain a modest lifestyle.

Tuition (annual fees), particularly for universities, has been increasing globally, most notably in developed countries, with a statistical estimate of a 1.6-times increase in actual monetary value (Digest of education, 2006), owing to the fact that university personnel salaries and benefits typically increase faster than inflation ('Baumol's cost disease'). As a result, education plays a critical part in society's functioning. It allows individuals with the ability to nurture new members of the labor force to tolerate higher prices with the aim of financing a solid education investment. With postsecondary education playing such an essential role in many people's lives, it's unsurprising that many students take out loans without giving it much thought. Student loans impact nearly 44.7 million debtors in the United States alone, with a total outstanding debt of over USD 1.71 trillion.

As a result, it is essential for students and their family members to be informed of the amount of money owing to specific organizations (whether public or private) to make sound financial choices while taking out loans, which would include the payback component after they graduate. Even though the majority of student loans are subsidized, it may feel frightening to owe such large sums of money as a student without a sizable net worth, particularly for first-generation students like myself.

This piqued my curiosity in the mathematical aspect of student loans, which is relevant given that banks are economically profit-driven organizations that often find a balance between revenue and loan attractiveness. By examining loans mathematically, we may better understand and ultimately justify the sort of loan that is desirable and perhaps apply the mathematical nature of loans to future spending. I would also use the mathematics learned here to analyze and assess existing student loan programs in Singapore. To begin, we'll examine how interest is computed on a certain quantity of borrowed money.

Aim: Finding the best way to borrow and repay student loans in America

Basic Concepts of Interest:

When any amount of money is borrowed, along with the Principal sum borrowed, a percentage of the original sum borrowed is repaid. The percentage amount over the original sum is called the Interest. For example, if the amount borrowed is \$10,000 and the interest is 5%, then the amount repaid in \$10,000 + \$10,000× 5% = \$10000 + \$500 = \$10500. Interest can be compounded annually, semi-annually, monthly, or daily depending upon the frequency and the terms and conditions of the banks. Generally, the interest is compounded monthly in the banks.

The interest rate depends on a variety of factors, mainly being the borrower's credit score. If the credit score and the borrower's net worth is good, the interest rate will be lower in comparison to the interest rate of the borrower, whose credit score is not that good. Compound interest is the interest on the loan, which is calculated on the total of the original principal plus the accumulated previous interests.

The formula for calculating the compound interest is as follows,

$C.I = P(1+i)^n$ - P (i)

Where,

P is the Principal amount borrowed

i is the nominal interest rate

n is the number of compounding in the year.

Suppose the loan is \$10000 taken for a period of 5 years with the interest rate being 10% compounded semi-annually in a year.

$C.I = \$10000(1+0.10)^{5\times2}$ - \$10000

= \$15937.42

In actuality, annual tuition costs may fluctuate owing to curriculum changes and other factors, but for the sake of simplicity, we may suppose they remain constant. While funding a complete education ahead may be convenient, it is seldom done due to the uncertainty of loan eligibility at a specific point in time. Additionally, even if tuition fees are fixed annually throughout the student's candidature, it is frequently financially unwise to defer payments after borrowing more funds than would be utilized at the time, as interest will be added (compounded) to the original amount borrowed even if it is not used.

Taking a general example of the loan amount being \$50,000 × 80% = \$40,000, which is to be repaid in 10 years, and interest rate being 4.5 % compounded monthly.

The amount owed in the first year = \$40,000

Interest owed in the second year,

$C.I = \$40000\ (1+\frac{0.045}{12})^{12}$ - \$40000

Interest = $1837.59

Therefore amount owed by the second year of the loan would be $41837.59 + 40,000 = $81837.59

Now, interest owed in the 3rd year,

$$C.I = \$81837.59\,(1+\frac{0.045}{12})^{12} - \$81837.59$$

= $3759.51

Therefore amount owed by the third year of the loan would be $81837.59 + $3759.51 + $40000 = $125597.10

Now, interest owed in the 4th year,

$$C.I = \$125597.1(1+\frac{0.045}{12})^{12} - \$125597.1$$

= $5769.91

Therefore amount owed by the fourth year of the loan would be $125597.1 + $5769.91 + $40000 = $171367.01

Considering courses that may be longer or shorter than four years, we use an arbitrary integer value for the number of years, n:

nth year =

$$\$40000\,(1+\frac{0.045}{12})^{12\,(n-1)} + \$40000\,(1+\frac{0.045}{12})^{12\,(n-2)} + \$40000\,(1+\frac{0.045}{12})^{12\,(n-3)} + \ldots\ldots\ldots + \$40000\,(1+\frac{0.045}{12})^{12\,(n-(n-1))} + \$40000\,(1+\frac{0.045}{12})^{12\,(n-n)}$$

Rewriting, amount owed in the nth year =

$$\$\,40000\,[\,(1+\frac{0.045}{12})^{12\,(n-1)} + (1+\frac{0.045}{12})^{12\,(n-2)} + (1+\frac{0.045}{12})^{12\,(n-3)} + \ldots + (1+\frac{0.045}{12})^{12\,(n-(n-1))} + (1+\frac{0.045}{12})^{12\,(n-n)}\,]$$

Which becomes a Geometric series:

Where $(1+\frac{0.045}{12})^{12}$ becomes the common ratio and

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$ 40000 [$(1 + \frac{0.045}{12})^{12\,(n-1)}$ becomes the first term,

Therefore, the amount owed in the nth year,

$$= \$\,\frac{40000[1-(1+\frac{0.045}{12})^{12n}}{1-(1+\frac{0.045}{12})^{12}}$$

By generalizing the equation, it gives:

Amount due in the nth year of college = $\$\,\frac{p - p(1+i)^{12n}}{1-(1+i)^{12}}$ (ii)

Where,

P denotes the principal amount owed;

I is the annual interest rate,

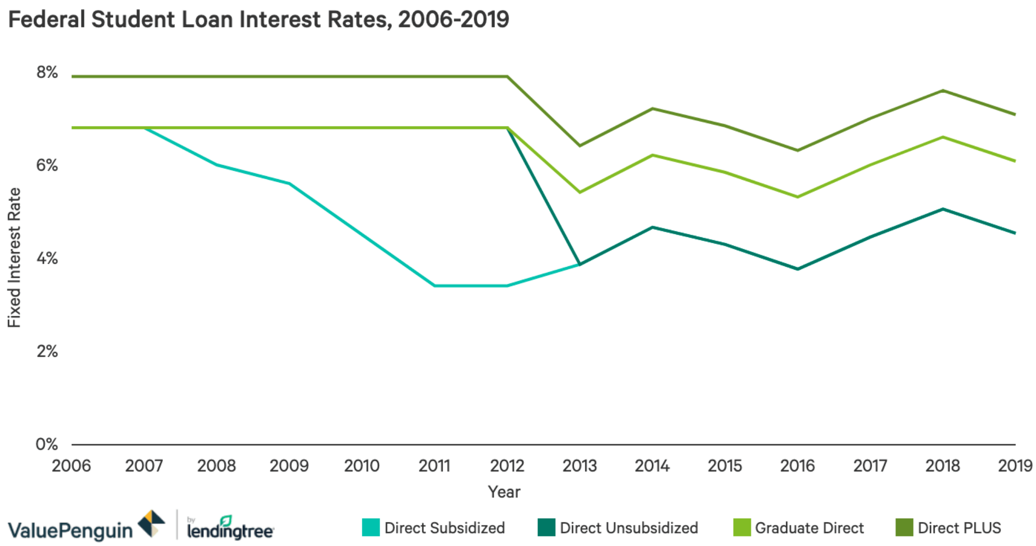
and n represents the number of years of college.

After n years after graduation, when loan disbursement ceases, the amount above is utilized as principle P (in equation 1), and the compound interest accumulation procedure begins.

The amount owed after Nth year of education

$$= \frac{p\,(1+i)^{12n} - p}{1-(1+i)^{12}}(1+\frac{i}{12})^{12N}$$

There are two types of federal student loans in the United States. The first is Direct subsidized loans, and the second is Direct Unsubsidized student loans. For Direct subsidized student loans, the government pays the interest until the student is in college; hence the loan is interest-free throughout the time of the study, and therefore the repayment process starts once the student has finished his study. This system just uses the standard compound interest formula. After graduation, however, the monthly compounded interest rate is calculated using a variable average prime rate set by the Department of Education. The interest rate is published here as follows:



(Figure 1: Average loan interest rate for student loans in America)

The Repayment process/The amortization process:

During the loan repayment period, students are given multiple choices to pay back the loan and the interest amount, but the two most relevant and prevalent choices in the United States are the Standard repayment plan and the Graduated repayment plan. For the Standard repayment plan, students pay a set monthly payment. Typically, the principal is compounded monthly at an annual percentage rate of ($\frac{APR}{12}$), although in certain countries, banks may compound it daily. Consistent loan repayment enables the student to repay the loan over more manageable time periods, such as ensuring that a lesser amount is compounded after that, resulting in a lower total interest accumulated.

The monthly installment, abbreviated as EMI (equated monthly installment), is the amount that will be taken from the student's or guarantor's account every month, often after a six-month grace period. This figure is typically set by the Department of Education or the third party that has lent the loan. This number is essential since it enables individuals to set aside a part of their monthly income to pay off debt and is also referred to as the amortization plan.

To better understand how much money the student would owe and repay each month based on interest, we may construct a formula from the procedures used to calculate the EMI. The reasoning behind this formula is that it allows the buyer to pay modest sums over time rather than paying the whole balance immediately, provided the interest rate stays constant.

Assume that a student borrows D dollars on a student loan at a rate of i %. This implies that the financial institution/department of education charges an interest rate of $(\frac{1}{12})$% percent on a monthly basis. At the completion of the first month, the student owes the original amount of D dollars plus $(\frac{1}{12})$% interest, which is as follows:

$$D\,(1+\frac{i}{12})$$

To simplify,

let $(1+\frac{i}{12})$ = P,

the EMI = M,

and D_n equal the money due after the nth month.

Assuming the EMI is followed correctly, the borrower will repay M dollars at the end of the first month and will thus still owe a total amount of $D_1 = D_0P - E$ at the end of the first month. The following will be owed by the student at the conclusion of the second month (n = 2):

$$D_1 + D_1\,(\frac{i}{12}) = D\,(1+\frac{i}{12})$$

$$= D_1P$$

After repaying E dollars for the second time in the second month, the individual owes the following:

$$D_2 = D_1P - E = (D_0P - M)P - M$$

= $D_0P^2 - M(1 + P)$

The student owes the following amount in the third month:

$D_3 = D_2P - M$= $[D_0P^2 - M(1 + P)]$ P - M

= $D_0P^3 - M(1 + \mathrm{P} + P^2)$

In the fourth month, the student owes the following amount:

$D_4 = D_3P - M$ =$[D_0P^3 - E(2 + P)]$ P - M

= $D_0P^4 - M(1 + \mathrm{P} + P^2 + P^3)$

Induction shows that at the conclusion of the nth month, or the loan duration, the student will owe an amount equal to D_n:

$D_n = D_0P^n - M\sum_{t=0}^{n-1} \quad P^t$= D_0R^n - E (1 + $P^2 + P^3 + \ldots$

We can see from the above expression that 1 + $P^2 + P^3 + \ldots + P^{n-1}$ is a geometric series, with P as the common ratio and 1 as the first term. Therefore,

1 + $P^2 + P^3 + \ldots + P^{n-1}$ = $(\frac{P^n - 1}{P-1})$, where $P \neq 1$

Therefore, D_n= $D_0P - M(\frac{P^n - 1}{P-1})$ - (iii)

P will never be one or lesser, pragmatically speaking, due to the interest rate $(\frac{i}{12})$. Additionally, if P - 1 is negative, it is conceivable that E $(\frac{P^n - 1}{P-1})$will be less than zero, implying that the loan would increase after a payment, which makes no sense and is unworkable.

If it is anticipated that the loan will be repaid entirely by the nth month, $D_n = 0$ is And from equation iii, we will therefore get:

$M\ (\frac{P^n - 1}{P-1})$ = $D_0P^n = E$

$$= \frac{D_0 P^n (P-1)}{P^n - 1}$$

Into which the following may be transformed by inserting: $(1+\frac{i}{12}) = P$:

$$M = \frac{D_0 (1+\frac{i}{12})^n (\frac{i}{12})}{(1+\frac{i}{12})^n - 1}$$

Ultimately, by substituting standard terms for the equation:

$$M = \frac{Pr\,(1+r)^n}{(1+r)^n - 1}$$

Where,

r is the set monthly interest rate, interest is calculated by dividing it by 12 because of 12 months in a year,

n denotes the number of monthly installments,

P is the initial loan amount from the department of education or a third-party institution and

M denotes the equated monthly installment or EMI.

The above formula calculates the Standard repayment plan or EMI for a student loan or any loan with a similar repayment mechanism(for a fixed rate of interest and equal monthly payments).

Upon additional investigation of the amortization formula, I discovered an intriguing feature of the Equated Monthly Installment (EMI) or Standard repayment plan: a part of the monthly EMI payment is deducted from both the principal and interest. Students who have taken out a student loan may be aware of this since the repayment amount covers a more significant part of the interest accumulated initially than the principal. Because the interest component of mortgage loans may be tax-deductible and may be claimed promptly on tax returns to reduce taxable income in the future, this information may be helpful in the future when acquiring mortgage loans. This may be shown mathematically by recalling the interest (per annum), i, at the finish of the first month, where total original principal, $D_0$and $(1+\frac{i}{12})$, denoted by P, which is:

$D_0(\frac{i}{12})$ = D_0(P - 1)

The above expression's value is deducted from the Equated Monthly Installment (EMI), M, and the remainder is subtracted from the principal. This means that after the first month, the amount paid toward repaying the principal debt will be M - D (P - 1).

The second month's interest is as follows:

$(D_0P - M)(\frac{i}{12})$

The amount that will be applied to the payment of the new principal DP - E for the second month is as follows:

M - $[(D_0P \ - M)\ (P - 1)]$ = P[M - D_0(P - 1)]

The following is the interest rate for the third month:

$[D_0P^2 - M(1 + P)]\ (\frac{i}{12})$= $[D_0P^2 - M(1 + P)$(P - 1)

As a result, the amount of money that is applied to the principal in the third month's EMI is as follows:

M - $[D_0P^2 - M(1 + P)$(P - 1) = P^2[M - $D_0(P \ - \ 1)$]

To generalize, one may see that by continuing the preceding techniques, a portion of the *x*th month's APR is allocated toward principal repayment, where it grows exponentially (since x is the sole variable):

Principal paid of k in a month

x = P^{x-1}[M - $D_0(P \ - \ 1)$] - (iv)

In contrast, the remainder will be applied to the interest owed for the xth month. Additionally, the entire principal after the xth month's EMI is now shown in the following way:

$P^xL_0 - M(\frac{P \ \ ^{x}-1}{P \ -1})$

Equation iv also indicates that, because the interest rate is expressed as a percentage over the principal, the amount contributed to the principal increases each month until it is completely repaid. As a result, the amount due to interest added each time after each payment naturally decreases as well.

Additionally, by substituting numerical values into equation iv, a graph may be produced that indicates whether the payment (EMI) is applied to the principal or interest portion. We assume the loaned amount to be \$171367.01(after four years of tuition, with three years interest-free; calculation on page 5) with an annual interest rate of 4.5 % or a monthly rate of $\frac{0.045}{12}$ = 0.00375 or 0.375% (to 3 significant figures). The tenure will be 30 years, the maximum allowed tenure for Direct subsidized student loans issued by the Department of Education. The EMI will then be:

$$M = \frac{Pr(1+r)^{n}}{(1+r)^{n}-1}$$

$$= \frac{171367.01(0.00375)(1+0.00375)^{30(12)}}{(1+0.00375)^{30(12)}-1}$$

= \$868.29 (2 d.p.)

As previously stated, there are two primary kinds of loans: those with a higher interest rate begin accruing interest only after graduation, while those with a lower interest rate begin accruing interest as funds are delivered (Maybank). To compare the two loans quantitatively, we may do so by swapping the numbers from the first example for a fifteen-year term

For standard repayment plan, interest servicing arrangement, interest is paid in monthly installments for the first four years/period of study (thus, no interest is charged to the principal), and thereafter the normal EMI is applied: [r is the monthly rate; P is the principal, and n is the loan term in months]

Total amount paid (1st four years) = \$40,000(4.5%) + 2(\$40,000)(4.5%) + 3(\$40,000)(4.5%) + 4(\$40,000)(4.5%) = \$18,000

$$\text{EMI (last 12 years)} = \frac{4(\$40000)(\frac{4.5\%}{12})(1+\frac{4.5\%}{12})^{12(12)}}{(1+\frac{4.5\%}{12})^{12(12)-1}} = \$1440.01 \text{ (v)}$$

Total amount paid (full-term) = \$1440.01(12 ×12) + \$180000 = 387361.44\$

For the standard repayment scheme, where monthly installments are paid towards the principal and interest once money is being disbursed.

Since the average prime rate has changed only once every few years since the turn of the twenty-first century and has been relatively stable (hovering below 6.0 percent since the 2000s), and interest rates generally increase or decrease by a small amount due to small policy changes, etc., around 0.1 percent, we will use the average of the average prime rate for the last ten years as a gauge, beginning in 2009-2010 and continuing through 2018-2019. (all values derived from the Department of Education) This technique of calculating the average works only if there are no significant swings in the average prime rate throughout the year the loan is taken.

From 2008-2018 all months inclusive, the prime rate has been constant at 5.05% so:

$$M = \frac{Pr(1+r)^{n}}{(1+r)^{n-1}}$$

Total amount paid (1st four years) = \$40,000(4.5%) + 2(\$40,000)(4.5%) + 3(\$40,000)(4.5%) + 4(\$40,000)(4.5%) = \$18,000

Total amount paid (full term) = EMI × Tenure

$$= \frac{4(40000)\frac{(5.05\%)}{12}(1+\frac{5.05\%}{12})^{12(12)}}{(1+\frac{5.05\%}{12})^{(12)(12)-1}}$$

= \$1483.84

Total amount paid (full term) = \$1483.84(12 ×12) + \$180000 = \$393672.96

Intuitively, we can see that the higher rate with no interest for the first four years will be chosen, since the total amount paid at the end will also be less, followed by the usual installment and finally the interest servicing program. However, whether choosing merely the interest servicing plan or the ordinary installment, one must evaluate the amount of money available each month to pay off the loan, since any balance remaining at the end of each month is simply compounded. The tenure begins in the fifth year since the program I am considering doing is four years in length.

Conclusion:

As illustrated above, as the loan tenure increases, the difference between the normal installment scheme and the DBS education loan becomes smaller, until a particular tenure at which point the DBS education loan overtakes the normal installment and continues to grow until it intercepts the interest scheme total payment around year twenty. Thus, even if the usual installment plan demands the largest initial payment, even throughout school, if the term is long, it may be more advantageous to take out the normal installment loan if funds are limited due to other obligations. To pay off the initial EMI during education, I may have to take up part-time work to offset the excess EMI. However, if I am eligible for the MOE's education grant to take the education loan, the ten-year tenure would be the better option. Meanwhile, the interest plan may be beneficial only if the DBS education loan's total payment exceeds that of a standard installment loan, since the amount paid over the years is more spread out, as opposed to the DBS loan, where the first three years are added to later years. However, one should keep in mind that in the case of severe economic changes, interbank lending rate (variable rate) loans, such as by the Department of Education tuition price loan, which is now the most appropriate and cost-effective loan available, may become less cost-effective over time. Thus, if interest rates increase in the future, the expenses must be reassessed to guarantee that personal debt is maintained to a minimum.

However, one practical constraint of this research is that, in order to factor in early repayment possibilities for particular components of a bank loan, the full balance must be recalculated and the EMI values must be reevaluated since early payback includes both principal and interest. In the event that the monthly installment is unable to be paid on time, the main part will be carried forward and added to succeeding months and split again, increasing the EMI and total amount paid during the duration.

Overall, the multiple variables that can be adjusted according to the type of loan, as well as the formula's flexibility in a variety of applications, such as student or even future mortgage loans, effectively utilize this amortization formula in conjunction with formula (2), and also the foresight it provides into projecting a certain amount of money, make it a powerful personal finance tool for many.

Bibliography

Introduction: What is a student loan? (n.d.). Retrieved from https://psu.instructure.com/courses/1806581/pages/introduction-what-is-a-student-loan?module item id=21491871

Tragakes, E. (n.d.), Economics 2nd edition for the IB Diploma.

T, M. (2012, November 23). Agam. Retrieved from http://tmuthukumar.blogspot.com/2012/11/deriving-formula-for-emi-and.html

Digest of Education Statistics, 2006. (2019). Nces.ed.gov. Retrieved 12 March 2019, from https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2007017

ONEAL, ANTHONY . “How Do Student Loans Work? | RamseySolutions.com.” *Ramsey Solutions*, Www.ramseysolutions.com, 2021, https://www.ramseysolutions.com/debt/how-do-student-loans-work.

“Federal Student Aid.” Federal Student Aid, Studentaid.gov, https://studentaid.gov/understand-aid/types/loans/interest-rates.

Zinn, Dori. “When To Choose The Graduated Repayment Plan – Forbes Advisor.” *Forbes Advisor*, Www.forbes.com, 7 September. 2020, https://www.forbes.com/advisor/student-loans/graduated-repayment-plan/.