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# Temporal Regularized Matrix Factorization for High-Dimensional Time Series Forecasting

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## Introduction

### A. Background on Time Series Forecasting

Time series forecasting plays a critical role in numerous domains, including finance, economics, climatology, and retail. The ability to predict future values based on historical patterns enables better decision-making, resource allocation, and risk management. Traditional approaches to time series forecasting include statistical methods such as autoregressive integrated moving average (ARIMA) models, exponential smoothing, and vector autoregression (VAR)[1].

### B. Challenges in High-Dimensional Time Series Data

Modern applications of time series forecasting increasingly involve high-dimensional data, where multiple interdependent series must be analyzed simultaneously. For example, a retail store might need to forecast demand for thousands of items, or climate scientists might analyze data from numerous sensors collected over several years[1]. This high dimensionality introduces several challenges:

1. The curse of dimensionality makes traditional methods computationally expensive or infeasible.
2. Data often contains missing values due to sensor malfunctions, human errors, or other collection issues.
3. Complex dependencies exist between different time series.
4. Scalability becomes a critical concern with large datasets.
5. Data heterogeneity requires flexible modeling approaches.

### C. Problem Statement

Given these challenges, there is a pressing need for forecasting methodologies that can effectively handle high-dimensional time series data with missing values. Specifically, we need approaches that are:

1. Scalable to handle very large numbers (n) of possibly interdependent time series and/or have a large time frame (T).
2. Robust to missing values and noise.
3. Capable of capturing complex temporal dependencies.
4. Effective at forecasting future values[1].

Traditional time series methods such as autoregressive (AR) models or dynamic linear models (DLM) fall short in handling these issues. For example, an AR model of order L requires $O(TL^2n^4 + L^3n^6)$ time to estimate $O(Ln^2)$ parameters, which is prohibitive even for moderate values of n[1].

### D. Importance of Handling Missing Values

Missing values are pervasive in real-world time series data. In retail datasets like Walmart E-commerce data, missing values can constitute over 50% of the data due to stock-outs and other factors[1]. Similarly, sensor data in climate applications often has gaps due to malfunctions or maintenance. Effective handling of missing values is therefore critical for accurate forecasting.

### E. Paper Organization

The remainder of this paper is organized as follows: Section II reviews related work on time series forecasting.

Section III presents the Temporal Regularized Matrix Factorization framework. Section IV discusses applications in financial forecasting and climate prediction. Section V describes our experimental evaluation. Finally, Section VI concludes the paper and discusses future research directions.

## Related Work

### A. Traditional Time Series Forecasting Methods

#### 1) ARIMA Models

Autoregressive Integrated Moving Average (ARIMA) models are among the most widely used approaches for time series forecasting. They combine autoregressive (AR) and moving average (MA) components with differencing to handle non-stationary data[2]. However, ARIMA models face significant challenges when applied to high-dimensional data, particularly in terms of computational complexity and parameter estimation.

#### 2) Exponential Smoothing

Exponential smoothing methods forecast future values by giving exponentially decreasing weights to past observations[3]. While effective for short, volatile time series, these methods struggle with high-dimensional data and complex dependencies between series.

#### 3) Vector Autoregression (VAR)

VAR models extend univariate autoregressive models to capture linear interdependencies among multiple time series[1]. However, they suffer from computational inefficiency for high-dimensional data, requiring $O(Tn^2)$ computation cost to update parameters, where n is the dimensionality and T is the time frame.

### B. Matrix Factorization Approaches for Time Series

#### 1) Matrix Factorization Techniques

Matrix factorization (MF) has been applied to time series data, particularly for handling missing values. In standard MF, a matrix $Y \in \mathbb{R}^{nxT}$ representing n time series over T time points is factorized into a product of two low-rank matrices: $Y \approx FX$, where $F \in \mathbb{R}^{nxk}$ contains latent features for each time series, and $X \in \mathbb{R}^{kxT}$ contains latent temporal embeddings[1].

#### 2) Regularization Methods

Regularization in matrix factorization helps prevent overfitting and can incorporate domain knowledge. Graph-based regularization has been used to model temporal dependencies, but such approaches often fail to capture negative correlations between time points and struggle with learning the weights of dependencies[1][4].

### C. Limitations of Existing Approaches

Existing approaches face several limitations:

1. Scalability: Traditional methods like ARIMA and VAR become computationally infeasible for high-dimensional data[1].
2. Missing Values: Many methods cannot handle datasets with significant proportions of missing values[1].
3. Temporal Dependencies: Standard matrix factorization approaches do not account for the temporal ordering of data[5].
4. Forecasting Ability: While some methods excel at imputing missing values, they often perform poorly at forecasting future values[1].

The Temporal Regularized Matrix Factorization framework presented in this paper addresses these limitations.

## Temporal Regularized Matrix Factorization

### A. TRMF Framework Overview

Temporal Regularized Matrix Factorization (TRMF) is a novel approach designed to incorporate temporal dependencies into matrix factorization models for time series data[1]. TRMF offers several advantages over existing methods:

1. It handles high-dimensional time series data efficiently.
2. It naturally accommodates missing values.
3. It captures temporal dependencies through well-designed regularization.
4. It enables effective forecasting of future values.
5. It supports data-driven learning of dependency structures.

The key innovation in TRMF is the incorporation of a temporal regularizer that encourages the latent temporal embeddings to follow specific time series patterns[1].

## B. Mathematical Formulation

### 1) Problem Definition

Let $Y \in \mathbb{R}^{n \times T}$ be the matrix representing n time series over T time points, with $Y_{it}$ being the observation at the t-th time point of the i-th time series. We aim to factorize Y into a product of two low-rank matrices $F \in \mathbb{R}^{n \times k}$ and $X \in \mathbb{R}^{k \times T}$, where k is the latent dimensionality, such that $Y_{it} \approx f_i^T x_t$. Here, $f_i \in \mathbb{R}^k$ is the latent embedding for the i-th time series, and $x_t \in \mathbb{R}^k$ is the latent temporal embedding for the t-th time point[1].

### 2) Matrix Factorization Component

The standard matrix factorization objective function is given by:

$$\min_{F,X} \sum_{(i,t)\in\Omega} (Y_{it} - \mathbf{f}_i^T \mathbf{x}_t)^2 + \lambda_f \mathcal{R}_f(F) + \lambda_x \mathcal{R}_x(X)$$

where Ω is the set of observed entries, R_f(F) and R_x(X) are regularizers for F and X respectively, and $\lambda_e$ and $\lambda_x$ are regularization parameters[1][4].

### 3) Temporal Regularization Component

The key innovation in TRMF is the design of the regularizer R_x(X) to incorporate temporal dependencies. Instead of using the standard Frobenius norm, we define a temporal regularizer T_M(X|Θ) based on a time series model M_Θ[1]:

$$\mathbf{x}_t = M_\Theta(\{\mathbf{x}_{t-l} : l \in \mathcal{L}\}) + \boldsymbol{\epsilon}_t$$

where L is a set of lag indices, Θ captures the weighting information of temporal dependencies, and $\varepsilon_t$ is a Gaussian noise vector.

The temporal regularizer is defined as the negative log-likelihood of observing the latent temporal embeddings under the time series model[1]:

$$\mathcal{T}_M(X|\Theta) = -\log \mathbb{P}(\mathbf{x}_1, \ldots, \mathbf{x}_T|\Theta)$$

The complete TRMF objective function is[1]:

$$\min_{F,X,\Theta} \sum_{(i,t)\in\Omega} (Y_{it} - \mathbf{f}_i^T \mathbf{x}_t)^2 + \lambda_f \mathcal{R}_f(F) + \lambda_x \mathcal{T}_M(X|\Theta) + \lambda_\theta \mathcal{R}_\theta(\Theta)$$

## C. Learning Algorithm

### 1) Optimization Procedure

The TRMF objective function can be optimized using alternating minimization over F, X, and Θ[1]. The optimization consists of three main steps:

a) Update F with X and Θ fixed:

$$\min_{F} \sum_{(i,t)\in\Omega} (Y_{it} - \mathbf{f}_i^T \mathbf{x}_t)^2 + \lambda_f \mathcal{R}_f(F)$$

b) Update X with F and Θ fixed:

$$\min_{X} \sum_{(i,t)\in\Omega} (Y_{it} - \mathbf{f}_i^T \mathbf{x}_t)^2 + \lambda_x \mathcal{T}_M(X|\Theta)$$

c) Update Θ with F and X fixed:

$$\min_{\Theta} \lambda_x \mathcal{T}_M(X|\Theta) + \lambda_\theta \mathcal{R}_\theta(\Theta)$$

### 2) Parameter Estimation

For the specific case of an autoregressive (AR) temporal regularizer, the parameter estimation step involves solving ridge regression problems for each dimension of the latent space[1]. This makes the learning procedure efficient and scalable.

## D. Handling Missing Values

One of the key advantages of TRMF is its ability to handle missing values effectively[1]. Since the objective function only considers observed entries (i,t) ∈ Ω, TRMF naturally accommodates incomplete data. For missing entries, TRMF can impute the values as $f_i^T x_t$ once the model is learned.

Furthermore, the matrix factorization approach enables information sharing across different time series, allowing for more accurate imputation of missing values compared to univariate methods that treat each time series independently[1].

## E. Scalability Considerations

TRMF is designed to be highly scalable, making it suitable for high-dimensional time series data[1]. The alternating minimization procedure can be implemented efficiently, with updates for F and X requiring $O(|\Omega|k^2)$ and $O(|L|Tk^2)$ time respectively, where $|\Omega|$ is the number of observed entries, |L| is the size of the lag set, T is the number of time points, and k is the latent dimensionality.

For practical high-dimensional applications, such as forecasting demand for 50,000 items, TRMF has been shown to be two orders of magnitude faster than competing methods[1].

## Applications

### A. Financial Forecasting

#### 1) Market Prediction Applications

Financial time series forecasting is one of the most challenging and important applications of time series analysis[3]. TRMF has been successfully applied to predict stock prices, exchange rates, and other financial indicators.

The high dimensionality of financial data, with thousands of stocks and other securities, makes traditional methods computationally infeasible[1]. TRMF's scalability and ability to capture dependencies between different financial instruments make it particularly suitable for this domain.

#### 2) Challenges in Financial Data

Financial data presents several unique challenges:
a) High volatility and non-stationarity
b) Complex interdependencies between different financial instruments
c) Effects of external events and news
d) Seasonal and cyclical patterns

TRMF addresses these challenges through its flexible lag structure, which can incorporate domain knowledge about financial cycles, and its ability to learn complex dependency structures from data[1].

### B. Climate Prediction

#### 1) Weather Pattern Analysis

Climate data typically involves readings from numerous sensors over extended periods, resulting in high-dimensional time series with complex spatial and temporal dependencies[1]. TRMF can be applied to analyze and predict weather patterns, temperature variations, and precipitation levels.

The ability to handle missing values is particularly valuable in climate applications, where sensor malfunctions or maintenance can lead to gaps in the data[1].

#### 2) Long-term Climate Forecasting

Long-term climate forecasting requires capturing complex cyclical patterns and trends. TRMF's flexible lag structure allows it to incorporate domain knowledge about climate cycles, such as annual seasons, El Niño–Southern Oscillation (ENSO), and other periodic phenomena.

By selecting an appropriate lag set L, TRMF can model both short-term dependencies and long-term cyclical patterns, leading to more accurate long-term forecasts[1].

## Experimental Evaluation

### A. Datasets

We evaluate TRMF on five datasets[1]:

1. Synthetic: A randomly generated dataset with n=16 time series and T=128 time points, following an AR process with lag set L={1,8}.
2. Electricity: Data from the UCI repository containing electricity consumption measured for 370 customers over 26,304 time points.
3. Traffic: Data from the UCI repository containing occupancy rates of 963 car lanes over 10,560 time points.
4. Walmart-1: A proprietary dataset from Walmart E-commerce containing weekly sales information for 1,350 items over 187 weeks, with 55.3% of entries missing.
5. Walmart-2: Another proprietary dataset from Walmart E-commerce containing weekly sales information for 1,582 items over 187 weeks, with 49.3% of entries missing.

### B. Evaluation Metrics

We evaluate the performance of different methods using two metrics[1]:

#### 1) Normalized Deviation (ND)

ND measures the relative deviation of the predicted values from the true values:

$$\mathrm{ND} = \frac{\sum |y_{\text{true}} - y_{\text{pred}}|}{\sum |y_{\text{true}}|}$$

### 2) Normalized RMSE (NRMSE)

NRMSE measures the root mean squared error normalized by the magnitude of the true values:

$$\text{NRMSE} = \frac{\sqrt{\sum (y_{\text{true}} - y_{\text{pred}})^2}}{\sqrt{\sum (y_{\text{true}})^2}}$$

For both metrics, lower values indicate better performance.

## C. Experimental Setup

We compare TRMF with several baseline methods[1]:

1. TRMF-AR: Our proposed method with an autoregressive temporal regularizer.
2. SVD-AR(1): Rank-k approximation using SVD followed by learning an AR(1) model on the latent temporal embeddings.
3. TCF: Temporal Collaborative Filtering, a matrix factorization approach with a simple temporal regularizer.
4. AR(1): n-dimensional AR(1) model.
5. DLM: Dynamic Linear Model.
6. Mean: Baseline that predicts everything to be the mean of the observed portion of the data.

For TRMF-AR, we use the following lag sets[1]:

- Synthetic: L={1,2,...,8}
- Electricity and Traffic: L={1,...,24}∪{7×24,...,8×24-1}
- Walmart-1 and Walmart-2: L={1,...,10}∪{50,...,56}

## D. Results and Discussion

### 1) Prediction Accuracy

Table 1 shows the forecasting results in terms of ND/NRMSE for each method. TRMF-AR outperforms all other methods on most datasets[1]. In particular, on the Walmart datasets with missing values, TRMF-AR achieves significantly better performance than competing methods.

On the synthetic dataset, TRMF-AR achieves an ND of 0.373 and an NRMSE of 0.487, which are substantially better than the next best method, SVD-AR(1), which achieves an ND of 0.444 and an NRMSE of 0.872[1].

### 2) Computational Efficiency

Fig. 1 shows the scalability of different methods as the number of time series n increases from 500 to 50,000. TRMF-AR is significantly faster than competing methods, especially for large n[1]. For n=50,000, TRMF-AR is two orders of magnitude faster than the AR(1) method.

### 3) Handling of Missing Values

Table 2 shows the results for missing value imputation with different percentages of observed data. TRMF-AR consistently outperforms other methods across different observation percentages[1]. For example, with 20% observed data on the synthetic dataset, TRMF-AR achieves an ND of 0.467 and an NRMSE of 0.661, while TCF achieves an ND of 0.713 and an NRMSE of 1.030.

Fig. 2 demonstrates that TRMF maintains superior performance even as the percentage of missing values increases, showing its robustness in real-world scenarios with incomplete data[1].

## E. Comparison with Traditional Methods

Our results demonstrate that TRMF-AR outperforms traditional time series methods in several aspects[1]:

1. Prediction Accuracy: TRMF-AR consistently achieves better forecasting performance across multiple datasets.
2. Handling Missing Values: TRMF-AR effectively handles datasets with significant proportions of missing values.
3. Computational Efficiency: TRMF-AR is significantly faster than traditional methods for high-dimensional data.
4. Scalability: TRMF-AR can handle time series data with dimensions up to 50,000, which is infeasible for traditional methods.

# Conclusion and Future Work

## A. Summary of Contributions

In this paper, we presented Temporal Regularized Matrix Factorization (TRMF), a novel framework for high-dimensional time series forecasting[1]. Our main contributions include:

1. A general framework that incorporates temporal dependencies into matrix factorization models.

2. A novel autoregressive temporal regularizer that can capture both positive and negative dependencies.
3. An efficient learning algorithm that scales to high-dimensional time series data.
4. Empirical evidence of TRMF's superiority over traditional methods in terms of prediction accuracy, handling of missing values, and computational efficiency.

TRMF addresses the limitations of existing approaches, providing a flexible and scalable solution for high-dimensional time series forecasting with missing values[1].

## B. Limitations

Despite its advantages, TRMF has some limitations:

1. Like many matrix factorization methods, TRMF assumes that the underlying data has a low-rank structure[1].
2. The autoregressive temporal regularizer assumes linear dependencies between time points, which may not capture all complex temporal patterns.
3. The current formulation does not explicitly model uncertainty in the forecasts.

## C. Future Research Directions

Several directions for future research emerge from this work[1]:

1. Incorporating more complex temporal dependencies, such as nonlinear relationships or long-range dependencies.
2. Extending TRMF to handle streaming data for online forecasting applications.
3. Developing methods to quantify uncertainty in TRMF forecasts.
4. Incorporating additional external factors or covariates that may influence the time series.
5. Applying TRMF to new domains beyond financial forecasting and climate prediction.

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