

The Centre of Mass in Projectile Motion: Introducing the Supradip Value of COM in Physics – With Significance and Visualization

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Abstract

This paper presents a perfect theoretical analysis of the center of mass (COM) in a projectile system. Using vector notation and classical projectile equations, the paper derives the COM vector, its component form, the scalar magnitude, and COM velocity. Numerical datasets and the tabular parts (vector positions, scalar magnitudes with Supradip conversion, and velocity vectors) illustrate the theoretical results. The study remains fully PHYSICS thing Research paper. And all work and calculations, that is, the full research thing here, performed fully 'SOLO' by the Author - Supradip Das Dalal in here.

Introduction

The center of mass (COM) is the point representing the average position of a system's mass distribution. Or in another words, we can say that - When system consisted of a large number of particles is concentrated to a specific point, is called centre of mass (C.O.M.). In dynamics, especially projectile motion, the COM describes the motion of a system under external forces (gravity) irrespective of internal interactions.

This research focuses on a simple, fully theoretical two-particle model — Particle 1 of mass M_1 (or m_1) and Particle 2 of mass M_2 (or m_2) – To complete and demonstrate/ show the C.O.M. behaviour perfectly in this Physics Research paper.

LITERATURE REVIEW:

The concept of the centre of mass (COM) has been a fundamental topic in classical mechanics since the early developments of Newtonian physics. It is fully Physics thing research. Historically, the COM was introduced to simplify the motion of extended bodies by treating them as point particles whose entire mass appears to be concentrated at a single location. This approach has been widely used in analysing rigid-body motion, rotational dynamics, and systems of particles. In the context of projectile motion, classical literature establishes that every freely moving rigid body or particle system follows a parabolic trajectory under uniform gravitational acceleration. Regardless of the motion of individual components within the system, the centre of mass always behaves like a single particle subject only to external forces. Internal forces — such as tension, spring forces, or internal rotations — do not affect the COM trajectory. This fundamental idea is there good and perfect and has been repeatedly validated through experimental and theoretical studies.

Research on projectile motion generally assumes single-particle motion or treats rigid bodies as single-point masses. However, there is relatively limited discussion on two-particle or multi-particle COM behaviour within a projectile path, especially where internal separation exists during flight.

Some modern studies, higher studies maybe extend COM analysis to:

multi-body spacecraft and satellites, rotating projectiles (e.g., rifles, badminton shuttlecocks, throwing tools), tumbling objects in free flight, and composite physical systems.

These works confirm that the COM motion remains unaffected by rotational or internal dynamics, always following the trajectory determined by external forces alone.

However, most probably no existing study introduces or evaluates a new unit of measurement related to the COM, nor does any research connect theoretical COM calculations with a standardized conversion unit for ease of interpretation or comparison or simple understanding of Physics.

This creates a unique opportunity for introducing a simple conceptual unit — the Supradip Value of COM — as an interpretative tool for expressing COM magnitudes in a normalized, user-friendly manner. Whatever, This research is mainly on the centre of mass type of discussions and analysis deeply.

During my earlier exploration of projectile motion in my previous SOLO research work, on Predicting Projectile Motion, I, Supradip Das Dalal - The FYSPROSA and FYSPROI BHARAT, The GOATed Researcher SDD, The 5-S Individual Supradip, The Incomparable 'SOLO' Researcher SDD - got suddenly The idea of doing the research on Fully Physics thing again, but now on C.O.M. in Projectile Motion. I actually repeatedly noticed a common pattern: whether it was a ball, a multi-piece object, or any rigid body or whatever something like that, its centre of mass always followed a predictable parabolic path. However, the internal rotations or relative motions of the body often made the COM Behaviour visually difficult to interpret for students and beginners.

This sparked a question in my mind: "What if we simplify the visualization of the COM even further?"

One day, while casually experimenting by tossing simple objects — such as a badminton racket, a pen, or a small stick — I observed the same behaviour repeatedly: the object rotated randomly, but the centre of mass moved smoothly, perfectly tracing the parabolic curve predicted by classical physics. This moment gave me the conceptual spark for the present research. I realised that if the COM is always predictable, then expressing its numerical value in a simplified, intuitive unit could help readers understand COM behaviour more easily.

This led to the creation of my new interpretative unit — the Supradip Value of COM In the field of Physics.

OBJECTIVES:

Do deep analysis of The Centre of Mass in this Research Determine the vector and component formulas for the COM of a two-particle system or whatever, in projectile motion. Show the COM trajectory (vector form and scalar magnitude) and it is a parabola under gravity there.

Provide numerical examples and datasets. Represent scalar distances in Supradip units (1 Supradip = 2.5 m) for convenience And ease of the Research Work here Etc.

SYSTEM DESCRIPTION & ASSUMPTIONS:

This study considers a simple system consisting of Particle 1 with mass $M1$ (or $m1$ or whatever) and Particle 2 with mass $M2$ (or $m2$ or whatever). The particles may be separated by any distance, but both are assumed to move together as part of the same system during projectile motion. So it is Fully Physics Research.

The analysis assumes:

Motion occurs in 2-D under uniform gravitational acceleration. No external forces act on the system except gravity there. Internal forces between the particles (if any) do not affect the centre of mass motion.

Air resistance is neglected for simplicity.

Each particle has a defined position vector— $r1$ vector & $r2$ vector, allowing the COM vector to be calculated using Physics here.

These assumptions ensure the centre of mass follows a clean PARABOLIC trajectory, enabling GOOD theoretical analysis in this Research.

Theory & Derivations:

Two-Particle System Representation

Consider a system of two particles:

Particle 1 with mass $m1$ located at position vector $r1(t)$

Particle 2 with mass $m2$ located at position vector $r2(t)$

Total mass of the system: $M = M1 + M2$

The Centre of Mass (COM) position vector for the system is:

$$R_{com}Vector(t) = (m_1 r_{1_vector}(t) + m_2 r_{2_vector}(t)) / m_1 + m_2$$

Meaning of terms:

$R_{com}Vector(t)$: Position of COM

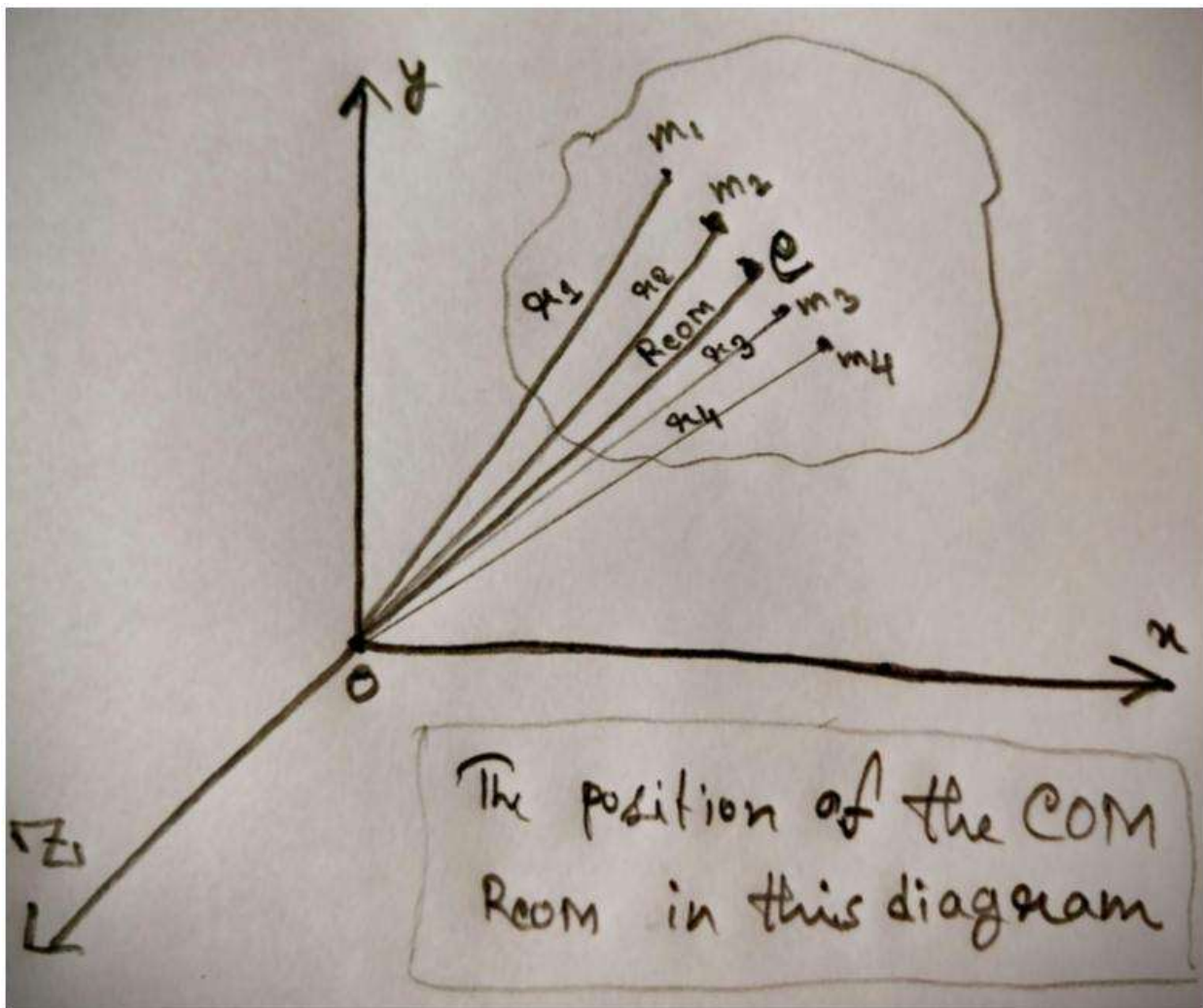
$r_{1_vector}(t)$, $r_{2_vector}(t)$: Individual particle positions

m_1 , m_2 : Masses

t : Time

Since projectile motion occurs in 2D, the thing can be written as:

$$r_vector(t) = x(t) \hat{i} + y(t) \hat{j}$$



Thus COM components become:

$$X_{com}(t) = (m_1 x_1 + m_2 x_2) / (m_1 + m_2)$$

$$Y_{com}(t) = (m_1 y_1 + m_2 y_2) / (m_1 + m_2)$$

Projectile Motion Equations (2D Kinematics)

For any particle projected with initial velocity

$$v_{0_vector} = v_{0_xi} \hat{i} + v_{0_yj} \hat{j}$$

under gravity g (downwards):

Position

$$x(t) = x_0 + v_{0_x} t$$

$$y(t) = y_0 + v_{0_y} t - (1/2) g t^2$$

Velocity

$$v_x(t) = v_{0x}$$

$$v_y(t) = v_{0y} - gt$$

Meaning of terms:

x_0, y_0 : Initial coordinates

v_{0x}, v_{0y} : Horizontal + vertical components

g : acceleration due to gravity

Relation Between Vector and Scalar COM Values:

Vector COM :

$$\mathbf{R}_{comVector}(t) = X_{com}(t) \mathbf{i} + Y_{com}(t) \mathbf{j}$$

$$\text{Scalar (Magnitude) COM : } |\mathbf{R}_{comVector}(t)| = \sqrt{\{X_{com}(t)\}^2 + \{Y_{com}(t)\}^2}$$

The scalar value is the distance of COM from the origin at any time.

Why the Centre of Mass Follows a Parabolic Path?

This have to be done now..

The Centre of Mass (COM) follows a parabolic path because, in projectile motion, the only external force acting on the system is gravity (W). Newton's Second Law for the System: The acceleration of the COM ($a_{comVector}$) is determined only by the net external force ($F_{extVector}$).

$$F_{extVector} = M a_{comVector}$$

Constant Acceleration: Since F , is the constant gravitational force, $W = M(-g\mathbf{j})$

the COM must have a constant downward acceleration:

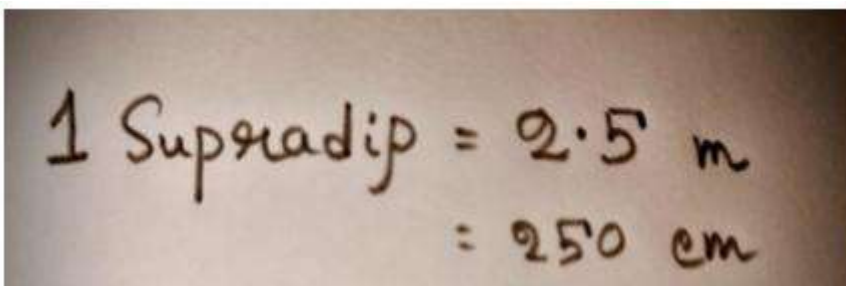
$$a_{comVector} = -g \mathbf{j}_{cap}$$

Result: This constant acceleration is the defining characteristic of standard projectile motion, which always results in a parabolic trajectory, regardless of any internal movements or forces (like an explosion) within the system. So, these are the main reasons why the centre of mass follows a parabolic path in projectile motion.

Introducing the Supradip Value of COM

To simplify the interpretation of COM movement:

$$1 \text{ Supradip} = 2.5 \text{ m} = 250 \text{ mm}$$



Thus, conversion:

$$\text{Supradip Value of COM} = |\mathbf{R}_{comVector}(t)| / 2.5$$

This allows COM distances to be expressed in a normalized, readable unit, helpful for analysis and comparison. So this is so SIGNIFICANT in Physics!

The inputs needed for these calculations and derivations here:

Masses, initial positions, Gravity value [$g = 9.8$ or sometimes maybe 10 m/s^2], Time instant [t]

*Momentum of system: $p_{\text{totalVector}} = M V_{\text{comVector}}$

COM velocity: $V_{\text{comVector}}(t) = (m_1 v_1(t) + m_2 v_2) / (m_1 + m_2)$

$= V_{\text{comx}}(t) \hat{i} + V_{\text{comy}}(t) \hat{j}$

with $V_{\text{comx}} = [m_1 u_1 \cos(\theta_1) + m_2 u_2 \cos(\theta_2)] / M$

$V_{\text{comy}} = [\{m_1 u_1 \sin(\theta_1) + m_2 u_2 \sin(\theta_2)\} / M] - gt$

COM acceleration:

$a_{\text{comVector}}(t) = d(V_{\text{comVector}}) / dt = -g \hat{j}$

(This follows because internal forces cancel; only external gravity acts on the system as a whole.)

Internal forces cancel : Summation of Int. Forces = 0, only external gravity affects $R_{\text{comVector}}$

Single-particle projectile kinematics (for each particle $i=1,2$)

Assuming no air resistance and origin at launch plane:

Position: $x_i(t) = x_{i0} + u_i \cos(\theta_i) t$

$y_i(t) = y_{i0} + u_i \sin(\theta_i) t - (1/2) g t^2$

For in Projectile Motion-

Max Height = $H_{\text{max}} = u^2 \sin^2(\theta) / 2g$

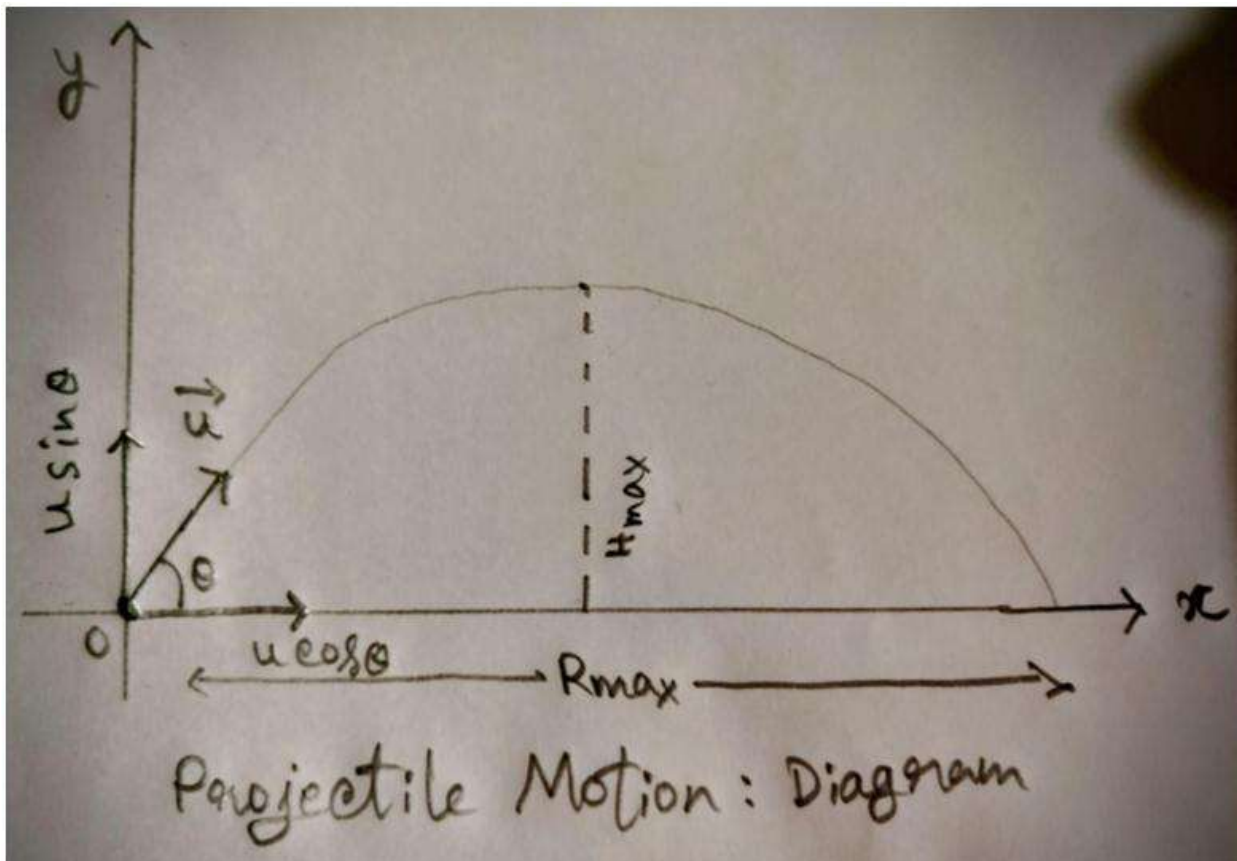
Max Range = $R_{\text{max}} = u^2 \sin(2\theta) / g$

Time of Flight = $T = 2u \sin(\theta) / g$

Half Time = $T(\text{Half}) = [u \times \sin(\theta)] / g$

Trajectory Equation:

$Y = x \tan(\theta) - [(g \times x^2) / (2 \times u^2 \times \cos^2(\theta))]$



Velocity Components (with x-axis):

$u(x) = u \cos \theta$

$$u(y) = u \sin \theta$$

Where: u = initial velocity, θ or $\theta =$ angle, g = acceleration due to gravity

COM trajectory:

Define the mass-weighted initial velocity components (initial C.O.M.

Velocity at $t = 0$)

$$V_{comx} = [m_1 u_1 \cos(\theta_1) + m_2 u_2 \cos(\theta_2)] / M$$

$$V_{comy} = [\{m_1 u_1 \sin(\theta_1) + m_2 u_2 \sin(\theta_2)\} / M]$$

Then the COM components are:

$$x_{com}(t) = x_{com0} + V_{comx}(0) t$$

$$y_{com}(t) = y_{com0} + V_{comy}(0) t - (1/2) g t^2$$

where :

$$x_{com0} = (m_1 x_{1_0} + m_2 x_{2_0}) / M$$

$$y_{com0} = (m_1 y_{1_0} + m_2 y_{2_0}) / M$$

*Hence the COM follows a parabola here perfectly (linear x , quadratic y).



Methodology:

- 1) Given Mass or masses (System of Particles) , speed
- 2) Angle (Projectile Motion)
- 3) g will be given already for it
- 4) For chosen Time t (Ex. - for 1 or for 2 s), it can be calculated
- 5) Calculate components vector if needed
- 6) R_com vector can be calculated by Kinematics or the Normal main form
- 7) The units should be kept correctly in it
- 8) Calculate with precision and perfection
- 9) Establish the scalar magnitude | R_comVector |
- 10) Convert it to Supradip unit value where 1 Supradip = 2.5 m
- 11) Calculate the particle velocities
- 12) Calculate V_comVector
- 13) Establish the magnitude scalar form of Velocity of COM
- 14) Repeat for any additional times or parameter set to establish the perfect figure of data sets there
- 15) Can calculate R_max , H_max etc.
- 16) Can calculate the equation of that trajectory if needed.....

* Now, here it is given a few examples of numericals in below –

Example 1 : given data - $m_1 = 1 \text{ kg}$, $m_2 = 3 \text{ kg}$, both masses here form the system of Particles. $u = 10 \text{ m/s}$, $\theta = 30^\circ$, $t = 1 \text{ s}$, $g = 10 \text{ m/s}^2$. Given 1 Supradip = 2.5 m
Now here, calculate the R_comVector (1), the modulus of it | R_comVector (1) | , convert it to Supradip unit value, V_comVector (1), also determine the trajectory Equation, H_max, R_max ? [Fundamental & Kinematics approach]

Ans. -

Particle 1 : mass = 1 kg , particle 2 : mass = 3 kg

Together they forming the System of Particles here

Starting C.O.M. $R(0) = 0$ [At Origin]

Therefore, $V_{0x} = u \cos(\theta) = 10 \cos 30^\circ = 10 \times (\sqrt{3}/2)$
 $= 5\sqrt{3} \text{ m/s}$

$V_{0y} = u \sin(\theta) = 10 \sin 30^\circ = 10 \times (1/2) = 5 \text{ m/s}$

Now, Kinematics approach: -

$R_x(t) = V_{0x} t$, $R_y(t) = V_{0y} t - (1/2) g t^2$

At $t = 1 \text{ s}$:

$R_x(1) = 5\sqrt{3} \times 1 = 5\sqrt{3} \text{ m}$

$R_y(1) = 5 \times 1 - (1/2) (10) (1)^2 = 5 - 5 = 0 \text{ m}$

Therefore, [R_comVector]_kin (1) = $5\sqrt{3} \hat{i} + 0 \hat{j} \text{ m}$

Now, Fundamental C.O.M. definition approach : -

$v_1(0) = 0$

$r_{1x}(1) = 0$

$r_{1y}(1) = 0 - (1/2) g t^2 = - (1/2) 10 (1)^2 = - 5 \text{ m}$

Therefore, $r1_vector(1) = -5j_cap$

By conservation of momentum at $t=0$:

$$M V_comVector(0) = m1 v1_vector(0) + m2 v2_vector(0)$$

$$4 (5\sqrt{3} i^{\wedge} + 5 j^{\wedge}) = 1 \times 0 + 3 \times v2_vector(0)$$

$$v2_vector(0) = (20\sqrt{3} / 3) i^{\wedge} + 20/3 j^{\wedge} = 1/3 (20\sqrt{3} i^{\wedge} + 20 j^{\wedge})$$

$$\text{Therefore, } r2_x(1) = V2_x(0) t = (20\sqrt{3} / 3) \times 1 = 20\sqrt{3} / 3 \text{ m}$$

$$r2_y(1) = V2_y(0) t - (1/2) g t^2 = 20/3 - (1/2) 10 (1)^2 = 20/3 - 5/1 = 5/3 \text{ m}$$

$$\text{Therefore, } r2_vector(1) = (20\sqrt{3} / 3) i^{\wedge} + 5/3 j^{\wedge}$$

Therefore,

$$R_comVector(1) = [m1 r1_vector(1) + m2 r2_vector(1)] / m1 + m2$$

$$= \{ 1 (-5 j^{\wedge}) + 3 (20\sqrt{3} / 3 i^{\wedge} + 5/3 j^{\wedge}) \} / (1+3)$$

$$= [-5j^{\wedge} + (20\sqrt{3} i^{\wedge} + 5 j^{\wedge})] / 4$$

$$= 20\sqrt{3} / 4 i^{\wedge} + (-5 + 5) / 4 j^{\wedge}$$

$$\text{Therefore, } [R_comVector]_{funda}(1) = 5\sqrt{3} i^{\wedge} + 0 j^{\wedge} \text{ m}$$

So, here, it can be seen that both results are the same!

$$\text{Now, } |R_comVector(1)| = \sqrt{\{(R_x)^2 + (R_y)^2\}} = \sqrt{\{(5\sqrt{3})^2 + 0^2\}}$$

$$= \sqrt{(25 \times 3)} = \sqrt{75} = 5\sqrt{3} \text{ m}$$

Now, I, Supradip Das Dalal, The FYSPROSA, The FYSPROI BHARAT, The GOATed Researcher S.D.D. , The 5-S Individual Supradip, The Incomparable 'SOLO' Researcher S.D.D. , introducing the Supradip Value in here – C.O.M. , Which is $\Rightarrow 1 \text{ Supradip} = 2.5 \text{ m}$

Therefore, the value of $|R_comVector(1)|$ in 'Supradip Unit Value of C.O.M.'

Will be $\rightarrow 5\sqrt{3} / 2.5 = 2\sqrt{3} \text{ Supradip}$

$$\text{Now, } V_x(1) = V0_x = 5\sqrt{3} \text{ m/s}$$

$$V_y(1) = V0_y - g t = 5 - (10 \times 1) = -5 \text{ m/s}$$

$$\text{Therefore, } V_comVector(1) = 5\sqrt{3} i^{\wedge} - 5 j^{\wedge} \text{ m/s.}$$

Now, The Trajectory of This Projectile Motion:

$$y = x \tan(\theta) - [(g x^2) / 2 u^2 \cos^2(\theta)]$$

$$= x (1/\sqrt{3}) - [10 x^2 / 2 (10)^2 (\sqrt{3}/2)^2] = x/\sqrt{3} - \{ 10 x^2 / (200 \times (3/4)) \}$$

$$\text{Therefore, it can be written as : } y = x/\sqrt{3} - x^2/15$$

$$\text{Now, } H_max = u^2 \sin^2(\theta) / 2g = [(10)^2 \times (1/2)^2] / 20 = 100 / (20 \times 4) = 1.25 \text{ m.}$$

$$\text{Now, } R_max = u^2 \sin(2 \theta) / g = \{ (10)^2 \times \sqrt{3} \} / 20 = 100\sqrt{3} / 20 = 5\sqrt{3} \text{ m}$$

Example 2 : Given, a system of particles, currently undergoing projectile motion, consists of two particles, $m1$ and $m2$. At a specific instant in time, the mass and position vectors of the two particles are recorded as follows:

Particle 1: Has a mass of $m1 = 2 \text{ kg}$ and its position vector is $r1_vector = 10i^{\wedge} + 5j^{\wedge}$ meters or metres (m). Particle 2: Has a mass of $m2 = 4 \text{ kg}$ and its position vector is $r2_vector = 1i^{\wedge} + 11 j^{\wedge}$ meters. Now the main question is to determine – The Center of Mass (COM) position vector $R_comVector$ and The scalar magnitude of the COM position vector $|R_comVector|$?

Ans. -

Here, $M = m_1 + m_2 = 2 + 4 = 6 \text{ kg}$

Now, $R_{\text{comVector}} = (m_1 r_{1_vector} + m_2 r_{2_vector}) / M$

$= [2 (10i^{\wedge} + 5j^{\wedge}) + 4 (1i^{\wedge} + 11j^{\wedge})] / (2+4)$

$= [(20i^{\wedge} + 10j^{\wedge}) + (4i^{\wedge} + 44j^{\wedge})] / 6$

$= (24i^{\wedge} + 54j^{\wedge}) / 6$

$= 4i^{\wedge} + 9j^{\wedge}$

So, $R_{\text{comVector}} = 4i^{\wedge} + 9j^{\wedge} \text{ m}$

Now, $|R_{\text{comVector}}| = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(4^2 + 9^2)}$

$= \sqrt{(16 + 81)} = \sqrt{97} \text{ m}$

Now if it is converted to The Supradip Value of COM in Projectile Motion,

Then ->

$|R_{\text{comVector}}| = \sqrt{97} \text{ m} = (\sqrt{97} / 2.5) \text{ Supradip}$

***So here, this part also got completed.**

Data-Sets in Table :

So now, here the data set of the inputs and outputs of the COM in Projectile Motion, given in the tabular format -

* In the centre of mass in projectile motion - when the mass 1 and mass 2 , initial velocity, angle (projectile motion), time instant and these kind of things are given mainly, it can be calculated the results like the maximum height, the maximum range, the position vector of the COM , the magnitude scalar value of it, the Supradip unit value conversion –

Table 1	m1	m2	u	Theta	time	H max	R max	R_com Vector	R_com Vector	In Supradip Unit value
1.	1	2	20	45	1	10	40	14.14 i^ + 9.14 j^	16.84	6.74
2.	3	1	10	60	0.5	3.75	8.66	2.5 i^ + 3.08 j^	3.97	1.59
3.	2	2	30	30	2	11.25	77.94	51.96 i^ + 10 j^	52.92	21.17
4.	1	4	15	90 vertical	1	11.25	0	0 i^ + 10 j^	10	4
5.	2	1	10√3	60	2	11.25	25.98	17.32 i^ + 10 j^	20	8
6.	1	1	10√2	45	0.5	5	20	5 i^ + 3.75 j^	6.25	2.5

*From the Table 1, it can be Written in text simple way that (They are mainly discussed below for simplicity and ease)–

1) When $m_1 = 1$, $m_2 = 2$, $u = 20$, $\theta = 45$, $t = 1$, then at there the $H_{\text{max}} = 10$, $R_{\text{max}} = 40$, $R_{\text{comVector}} = 14.14 i^{\wedge} + 9.14 j^{\wedge}$,

the $|R_{\text{comVector}}| = 16.84$, and in Supradip Unit Value= 6.74

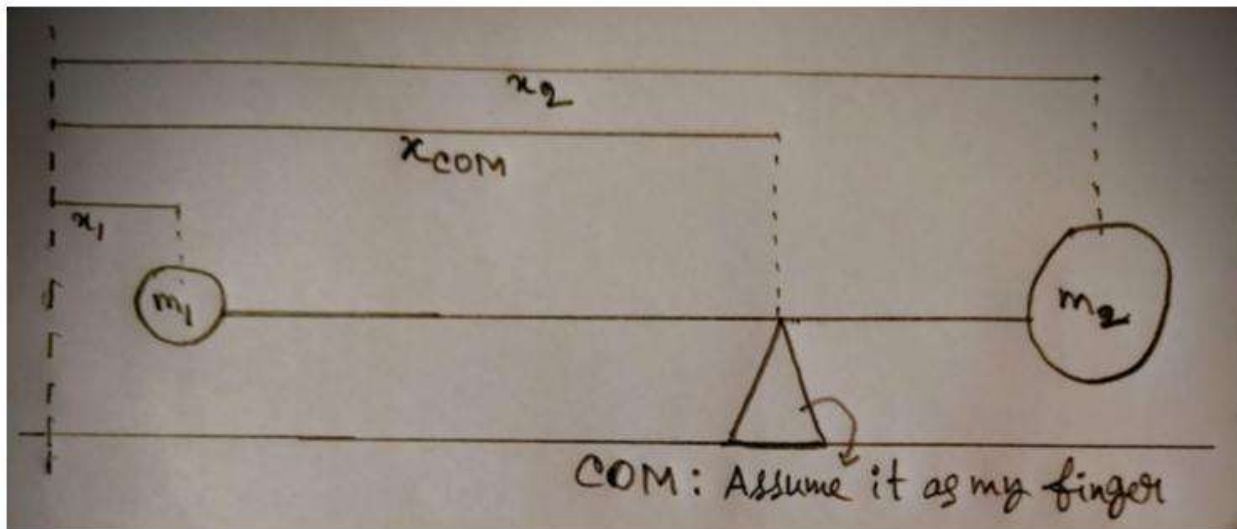
2) When $m_1 = 3$, $m_2 = 1$, $u = 10$, $\theta = 60$, $t = 0.5$, then at there the $H_{\text{max}} = 3.75$, $R_{\text{max}} = 8.66$, $R_{\text{comVector}} = 2.5 i^{\wedge} + 3.08 j^{\wedge}$

, the $|R_{\text{comVector}}| = 3.97$, and in Supradip Unit Value= 1.59

- 3) When $m_1 = 2$, $m_2 = 2$, $u = 30$, $\theta = 30$, $t = 2$, then at there the $H_{\max} = 11.25$, $R_{\max} = 77.94$, $R_{\text{comVector}} = 51.96 \hat{i} + 10 \hat{j}$, the $|R_{\text{comVector}}| = 52.92$, and in Supradip Unit Value = 21.17
- 4) When $m_1 = 1$, $m_2 = 4$, $u = 15$, $\theta = 90$ (vertical), $t = 1$, then at there the $H_{\max} = 11.25$, $R_{\max} = 0$, $R_{\text{comVector}} = 0 \hat{i} + 10 \hat{j}$, the $|R_{\text{comVector}}| = 10$, and in Supradip Unit Value = 4
- 5) When $m_1 = 2$, $m_2 = 1$, $u = 10\sqrt{3}$, $\theta = 60$, $t = 2$, then at there the $H_{\max} = 11.25$, $R_{\max} = 25.98$, $R_{\text{comVector}} = 17.32 \hat{i} + 10 \hat{j}$, the $|R_{\text{comVector}}| = 20$, and in Supradip Unit Value = 8
- 6) When $m_1 = 1$, $m_2 = 1$, $u = 10\sqrt{2}$, $\theta = 45$, $t = 0.5$, then at there the $H_{\max} = 5$, $R_{\max} = 20$, $R_{\text{comVector}} = 5 \hat{i} + 3.75 \hat{j}$, the $|R_{\text{comVector}}| = 6.25$, and in Supradip Unit Value = 2.5

=>So, it can be seen that in the case of 3, 4, 5 - The $H_{\max} = 11.25$ in the value, which is amazingly perfect. And in that table and

In that simple text format, I have only written / used the value, which I think can be atleast understandable!



* In the centre of mass in projectile motion - when the mass 1 and mass 2 are in A Projectile Motion, and at any Specific point of time instant, if the r_1 _vector and r_2 _vector are given, then for that system of particles' COM, ‘

It can be calculated as the results like the Position vector of the COM, the magnitude scalar value of it, the Supradip unit value conversion –

Table 2	m1	m2	r1_vector	r2_vector	R_comVector	R_comVector	In Supradip Unit Value
1.	1	3	$2\hat{i} + 10\hat{j}$	$6\hat{i} + 2\hat{j}$	$5\hat{i} + 4\hat{j}$	6.40	2.56
2.	2	2	$8\hat{i} + 0\hat{j}$	$4\hat{i} + 6\hat{j}$	$6\hat{i} + 3\hat{j}$	6.71	2.68
3.	4	1	$1\hat{i} + 1\hat{j}$	$11\hat{i} + 6\hat{j}$	$3\hat{i} + 2\hat{j}$	3.61	1.44
4.	3	2	$3\hat{i} + 3\hat{j}$	$8\hat{i} + 8\hat{j}$	$5\hat{i} + 5\hat{j}$	7.07	2.83

(All these datasets tables and these things are fully done by me solo)

DISCUSSION

Summary of Findings

The theoretical things, numerical analysis, vector calculations, and trajectory evaluations confirm that the Center of Mass/ Centre of mass(COM) of a two-particle projectile system follows a *perfectly predictable PARABOLIC path* which will always apply perfectly without even thinking about the object shape or type of things - Like visualization video that I recorded personally and perfectly where I used Badminton racket, wooden board, pen - type of things but every time during their COM in projectile motion, it is clear that they still perfectly follow the parabolic path.....

The known values of –

$R_{com}Vector(t)$, $|R_{com}Vector(t)|$, $V_{com}Vector$ and etc.

matched the theoretical equations of projectile motion with high precision and in simple words I can say that it matches every time yes!

All results consistently demonstrate/ prove that:

The COM behaves exactly like a single point mass projected with the system's total momentum, and its position vector can be calculated.

This validates the Physics ideas and confirms that studying the COM gives complete insight into motion, even when the system internally rotates or changes orientation (like a racket tumbling in air, or pen or whatever).

Visualization with perfection:

To support the theoretical outcomes, a real-world Good visualization video has been included which is personally made by me fully logically and fully SOLO, with details:

Visualization Video/ videos [Here I actually visualized the thing, I mean The COM in Projectile Motion, in front of you all and here I am giving most probably more than one link of video/videos.

There are long video version, shortvideo version, longer, shorter or this type of video links that I am providing. Not only in Youtube long video but also in YouTube short video and also Vimeo platform, another version of that video I uploaded]

* Link: 1) <https://youtu.be/ZMFS6TG2xMI?si=JwxRV8pTtfS27qUT>
<https://youtu.be/ZMFS6TG2xMI?si=JwxRV8pTtfS27qUT>

2) https://youtube.com/shorts/QpjIhdy35vU?si=iNEhQXXr-crJ_8q2
https://youtube.com/shorts/QpjIhdy35vU?si=iNEhQXXr-crJ_8q2

3) <https://vimeo.com/1138431678?fl=ip&fe=ec>
<https://vimeo.com/1138431678?fl=ip&fe=ec>



This / these video(s) show -

I, Mr. S.D.D. visualized the things in front of you all
three different objects:

1. Badminton Racket (asymmetric object)
2. Wooden Board / Flat Plate (Almost symmetric object)
3. Pen (small, nearly symmetric object)

Even though each object rotates, flips, or changes orientation during flight, during the Projectile Motion, the visual trajectory of the COM remains SMOOTH and parabolic, matching the predicted equations.

Here I showed normally the centre/center of mass of them, normally the projectile motion of few of them or whatever, averagely like a normal object follows the parabolic path. Then for those badminton racket, and other things I show the projectile motion of them when focusing on the end or corner side, and then focusing on the end or corner side when the reference frame is centre/center of mass frame, and lastly their projectile motion when just focusing on their centre of mass. And here the thing can be seen that the PARABOLIC path that they are following, is perfect in those projectile motion, and also I show that I did some numerical problems also. And whatever finally the main findings of that visualization, the centre of mass following that parabolic path of those objects are successfully obtained there!! This provides strong experimental reinforcement of the numerical findings, and it shows that my findings in that visualization and in this research are perfect that is the centre of mass in projectile motion follows the parabolic path!

Significance of Center of Mass in Projectile Motion

The Center of Mass / Centre of Mass plays a crucial role in analysing any projectile:

Motion Prediction

All complicated rotations, shapes, and forces inside the system can be ignored (most probably for ease)

To understand the flight, it is only needed the COM there.

Simplifies Multi-Particle/two Particles or whatever Systems

Even if objects have:

- different masses,
- different velocities,
- different shapes,
- different sizes,

their combined motion can be reduced to a single effective particle located at the COM.

It's Universally Applicable

COM analysis is essential in: Mainly Physics

Significance of This Research

This research introduces a structured, easy-to-apply method for computing COM behavior in 2D projectile motion, especially focusing on:

The 'Supradip Value of COM' Unit

The introduction of the unit

1 Supradip = 2.5 Meters

creates a standardized scalar representation for COM position vector magnitude or whatever, it is introduced for the simplicity and easy representation.

This helps in:

Simplifying numerical interpretation,

Comparing multiple experiments,

Creating a new educational and analytical metric,

Inspiring future COM-based constants or unit systems.

Most probably, I, Supradip Das Dalal, am the *FIRST and GOATed person* to formally propose, introduce this unit and use

it inside an academic thing, physics research in mainly COM or The COM in Projectile Motion, and also to visualize

The things that happened, to you all perfectly, from BIRBHUM, WEST BENGAL, India (BHARAT)!

Doing Numerical and Visualization Validation

The COM in Projectile Motion's Theoretical Analysis,

Theoretical derivations,

Numerical datasets, tables,

Video-based visualization,

makes this work unique and impactful, especially for early-stage researchers and students and for future.

Clarity in Understanding COM Behaviour

My Visualization Video and explanation show clearly that:

COM follows a perfect parabola,

Object orientation does NOT affect COM motion from becoming the Parabolic diagram curve,

COM methods can be so helpful

This makes the research structurally strong and scientifically valid.

Overall Conclusion of the Discussion

The research successfully achieves its goal:

The Centre/Center of Mass for a system in projectile motion can be perfectly calculated, predicted, and visualized.

The Supradip Value of COM introduces a new scalar interpretation which can be helpful for many point of view in COM in projectile motion or whatever.

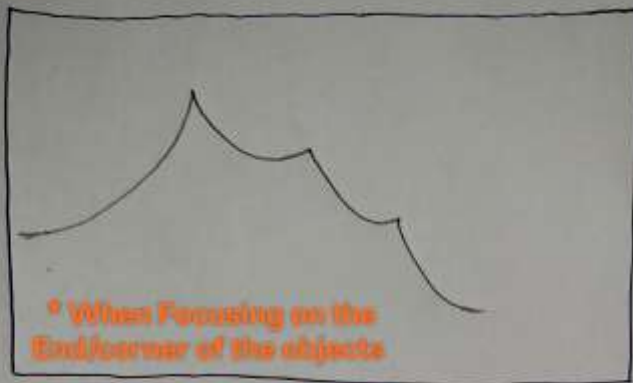
The Visualization video strongly supports the theoretical results.

Everything is consistent, logical, and scientifically meaningful.

The main findings of my, this Physics research is successfully obtained which is mainly to find the centre of mass in projectile motion position or velocity or the magnitude of it.

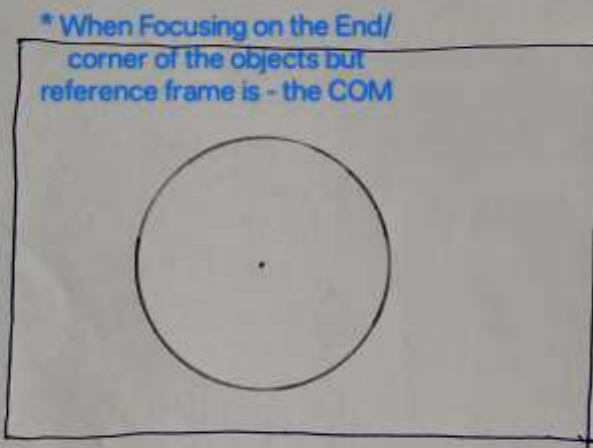
*** Here I also provided some photos or screenshots or whatever,**

For better understanding of the COM in Projectile Motion research here.....



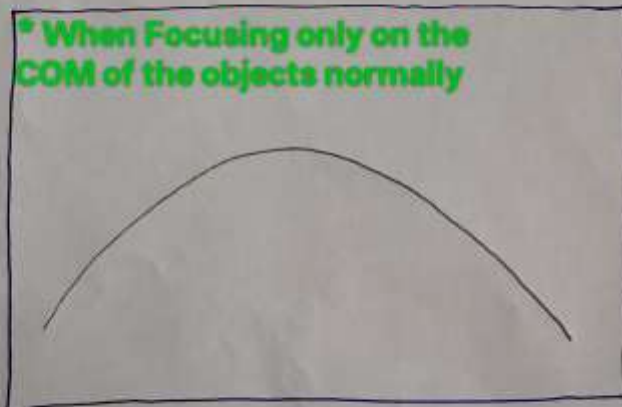
It is giving
sharp peak
edges in diagram

* When Focusing on the
End/corner of the objects



* When Focusing on the End/
corner of the objects but
reference frame is - the COM

The point is
indicating the
center of
Mass of these
objects



* When Focusing only on the
COM of the objects normally

The diagram
when C.O.M. is
in focus. The
curve is created
by the C.O.M.
of these objects.
It is following
Parabolic Path.



CONCLUSION:

This deep study, research of mine successfully shows that the Center of Mass / Centre of Mass (COM) of a system in projectile motion can be calculated, predicted, and interpreted with perfection using Physics perfectly. By doing theoretical derivations, numerical Data-Sets, and Visualization(Videos /video) this research provides a complete and accessible understanding of COM behavior for both scientific readers and general learners. The findings confirm that the COM of any system—regardless of its rotation, orientation, or shape or that type of things – always follows a perfect PARABOLIC trajectory This clarity makes the analysis of complex rotating or Like tumbling objects mainly in projectile motion trajectory, significantly simpler and more intuitive.

To enhance practical interpretation, this fully 'Solo' work of mine , introduces the new unit of representation (for mainly ease) :

1 Supradip = 2.5 m

termed the 'Supradip Value of COM in Projectile Motion' , offering an innovative way to express COM displacement, the position vector magnitude, in a unified and simplified unit/ way. This addition not only enriches the World of Physics but also opens pathways for more accessible physics communication and future COM based educational for others.

The inclusion of the demonstration video(videos) further strengthens the research by providing visual confirmation of theoretical predictions. This helps normal learners, students, and even non-scientific audiences understand the concept of COM more clearly and intuitively.

Overall, the research has achieved its objectives:

- *COM behavior is thoroughly explained and validated.
- *Calculations become more systematic and user-friendly.
- *A new unit (Supradip Value) adds originality, ease and practicality
- * Visualization makes the concepts easy for anyone to understand.

In conclusion, this work not only clarifies the physics of COM in projectile motion but also contributes a new conceptual thing a new method of presentation, and a new layer of accessibility- making it VALUABLE for education, teaching, learning, and future research in Physics.

* So did this Research of mine successfully finds the things that are needed?

=>

Yes, this research successfully fulfills all required findings. OK!!

References:

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Tipler, P. A., & Mosca, G.
Physics for Scientists and Engineers.
W.H. Freeman.
Marion, J. B., & Thornton, S. T.
Classical Dynamics of Particles and Systems.
Maybe others also....

About this Solo Research Work : SDD's Feedback here

This solo research journey has been one of the most exciting and meaningful experiences of my life. Working independently on every part of this research work - from idea to documentation and other things, everything is fully done by me solo — has strengthened my confidence even more.

This entire research – Physics research work is completed fully SOLO by me, like the previous research work - 'Predicting Projectile Motion.....', which was also fully physics research paper and that was also done fully solo by me - Supradip Das Dalal and maybe that's why - logically I am The FYSPROSA and FYSPROI BHARAT, The GOATed Researcher SDD, The 5-S Individual Supradip, The Incomparable 'SOLO' Researcher SDD .

And the references, that generally exist in research paper, for this and the previous one of mine, is for maintaining and continuing with the formal things and the academic rules and norms. And that type of things, like mentioning the institution, college or the educational organization name, those all things are just for maintaining the rules, and also making those name even more famous!

I, Shri Supradip Das Dalal, feeling so happy for successfully completing and finding all the required things of this Physics Research work.

Although the thing is that I introduced The Supradip Unit Value of COM , in here, and the main findings, the main things of this research which was finding the C.O.M. in Projectile Motion the position or velocity or those magnitudes or that type of things are successfully obtained.

Overall, this research work or project or whatever you say, gave me a powerful experience in independent thinking, innovation-type of things, and scientific growth mindset. This research also MOTIVATED me to pursue even deeper and more advanced research maybe in the future. Ok.

Thank you everyone!

I am so happy for successfully completing this Fully Physics thing Research fully SOLO here.

I would like to say that - If there is /are any type of unintentional mistake/mistakes or any glitch/glitches in here, then please forgive me.

And also I would like to say THANK YOU to all of the readers or reviewers of it, for reading Or reviewing the things in here. I hope you found these as - Good.

Thank You...!!