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The Interpretation of Complex Number Analysis by Hyperbolic Function

Manoj P. Khere [1]

Assistant Professor

Dr. Rajendra Gode Institute of Technology and Research, Amravati.

Abstract:

This work is intended to introduce the problem of complex and hyperbolic generalized complex numbers by easiest manner. The properties of the algebra are considered for this solution. Besides complex and hyperbolic-generalized complex valued functions are defined and different equation representations of these numbers are examined. Moreover, the relationship between hyperbolic and circular function are also explained.

Key Words:

Hyperbolic Function and Circular Function, Inverse Hyperbolic Functions.

Introduction:

The solution of the hyperbolic and inverse hyperbolic function of the complex number has been given in the book of engineering mathematics. They had explained the method by very lengthy and tedious way. The same problem has seen solved by us with less trigonometrically and hyperbolic formulas in a simple manner.

Inverse hyperbolic functions

The inverse function of hyperbolic functions is known as inverse hyperbolic functions. It is also known as area hyperbolic function. The inverse hyperbolic function provides the hyperbolic angles corresponding to the given value of the hyperbolic function. Those functions are denoted by $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, cosech^{-1}, sech^{-1}, and $\coth^{-1}$.

The inverse hyperbolic function in complex analysis is as follows:

$\text{Sinh}^{-1} x = \ln (x + \sqrt{[1+x^2]})$

$\text{Cosh}^{-1} x = \ln (x + \sqrt{[x^2-1]})$

$\text{Tanh}^{-1} x = (\tfrac{1}{2})[\ln(1+x) - \ln(1-x)]$

$\text{Sinh}\, x = \frac{e^x - e^{-x}}{2}$

$\text{Cosh}\, x = \frac{e^x + e^{-x}}{2}$

$\text{Tanh}\, x = [e^x - e^{-x}] / [e^x + e^{-x}]$

Explanation:

The simple way of solving the hyperbolic trigonometric function

Show that $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$

Solution: - $(\cosh x + \sinh x)^n = \left(\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}\right)^n$

$$= \left(\frac{e^x + e^{-x} + e^x - e^{-x}}{2}\right)^n$$

$$= \left(\frac{2e^x}{2}\right)^n$$

$$= (e^x)^n$$

$$= e^{nx} \text{ ----- (1)}$$

$$\text{Coshnx + sinhnx} = \frac{e^{nx} + e^{-nx}}{2} + \frac{e^{nx} - e^{-nx}}{2}$$

$$= \frac{e^{nx} + e^{-nx} + e^{nx} - e^{-nx}}{2}$$

$$= \frac{2e^{nx}}{2}$$

$$= e^{nx} \text{ ----- (2)}$$

From the equation no. 1 & 2 this problem is Prove.

Conclusion:

This method explains the hyperbolic function in easiest way. I have used only one trigonometric formula in the whole problem statement. I have also used hyperbolic formula and logarithmic formulae to prove the solution of this problem.

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