The Study of Five Integral Equation Involving Inverse Mellin Transforms

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Abstract:- *In this paper, we have considered the solution of the five integral equation involving inverse Mellin Transforms .*

Key Words **:-** *Integral equation ,series equation, Integral theorems, Inverse theorem.*

1. INTRODUCTION

We consider the following five integral equations

$$M^{-1}\left[\frac{\Gamma\left(\xi+\frac{\nu}{\delta}\right)}{\Gamma\left(\xi+\beta+\frac{\nu}{\delta}\right)}\Psi(\nu),x\right]=\begin{Bmatrix} f_1(x) & 0\le x<a \\ f_3(x) & b<x<c \\ f_5(x) & d<x<\infty \end{Bmatrix} \tag{1.1}$$

$$M^{-1}\left[\frac{\Gamma\left(1+\eta-\frac{\nu}{\sigma}\right)}{\Gamma\left(1+\eta+\alpha-\frac{\nu}{\sigma}\right)}\Psi(\nu),x\right]=\begin{Bmatrix} g_2(x) & a<x<b \\ g_4(x) & c<x<d \end{Bmatrix} \tag{1.2}$$

Where α, β, ξ, δ, η >0, σ >0 are real parameter$g_2(x) and\ g_4(x)$ are known function $\Psi(\nu)$ is to be determined.

2. PRELIMINARY RESULTS

1. The inverse operators are given by

$$I_{\eta,\alpha}^{-1}(a,x;\sigma)f(x)=I_{\eta+\alpha,-\alpha}(a,x;\sigma)f(x) \tag{2.1}$$

$$K_{\eta,\alpha}^{-1}(x,b;\sigma)f(x)=K_{\eta+\alpha,-\alpha}(x,b;\sigma)f(x) \tag{2.2}$$

2. $$M\left[I_{\eta,\alpha}(0,x:\sigma)f(x);\nu\right]=\frac{\Gamma(1+\eta-\frac{\nu}{\sigma})}{\Gamma(1+\eta+\alpha-\frac{\nu}{\sigma})}M[f(x);\nu] \tag{2.3}$$

$$M\left[K_{\eta,\alpha}(x,\infty:\sigma)f(x);\nu\right]=\frac{\Gamma(\eta+\frac{\nu}{\sigma})}{\Gamma(\eta+\alpha+\frac{\nu}{\sigma})}M[f(x);\nu] \tag{2.4}$$

3. THE SOLUTION

The five ranges of the variable x are defined as:

$I_1 = \{x: 0 \leq x < a\}$

$I_2 = \{x: a < x < b\}$

$I_3 = \{x: b < x < c\}$

$I_4 = \{x: c < x < d\}$

$I_5 = \{x: d < x < \infty\}$ (3.1)

and we shall write any function $f(x), x \geq 0$in the form

$$f(x) = \sum_{i=1}^{5} f_i(x) \quad (3.2)$$

Where,

$$f_i(x) = \begin{Bmatrix} f(x) & x \in I_i, \; i = 1,2,3,4,5 \\ 0 & otherwise \end{Bmatrix} \quad (3.3)$$

The five integral equations (1.1) & (1.2) as

$$M^{-1}\left[\frac{\Gamma\left(\xi+\frac{\nu}{\delta}\right)}{\Gamma\left(\xi+\beta+\frac{\nu}{\delta}\right)}\Psi(\nu), x\right] = f(x) \quad (3.4)$$

$$M^{-1}\left[\frac{\Gamma\left(1+\eta-\frac{\nu}{\sigma}\right)}{\Gamma\left(1+\eta+\alpha-\frac{\nu}{\sigma}\right)}\Psi(\nu), x\right] = g(x) \quad (3.5)$$

Where $f_1, g_2, f_3, g_4, f_5,$ are prescribed functions while$g_1, f_2, g_3, f_4,$ are unknown functions to be determined

$$\Psi(\nu) = M[\emptyset(x), \nu] \quad (3.6)$$

Use results (2.3) & (2.4)we find the equations (3.4) & (3.5) assume the operational form

$$I_{\eta,\alpha}(0, x; \sigma)\emptyset(x) = g(x) \quad (3.7)$$

$$K_{\xi,\beta}(x, \infty; \delta)\emptyset(x) = f(x) \quad (3.8)$$

Using results (2.1) & (2.2) solving the about equation for $\emptyset(x)$we obtain

$$\emptyset(x) = I_{\eta+\alpha,-\alpha}(0, x; \sigma)f(x) \quad (3.9)$$

$$= K_{\xi+\beta,-\beta}(x,\infty;\delta)g(x) \quad (3.10)$$

Now we proceed to determine $\emptyset$. The subscripts on all the operators I`s will be supposed to have subscript $(\eta,\alpha;\sigma)$ understood and all K`s To have subscript $(\xi,\beta;\delta)$.Evaluating (3.9) onI_1,$I_{2,}I_{3,}I_4$ we get

$$\emptyset_1 = \binom{x}{0} I^{-1} f_1 \quad (3.11)$$

$$\emptyset_2 = \binom{a}{0} I^{-1} f_1 + \binom{x}{a} I^{-1} f_2 \quad (3.12)$$

$$\emptyset_3 = \binom{a}{0} I^{-1} f_1 + \binom{b}{a} I^{-1} f_2 + \binom{x}{b} I^{-1} f_3 \quad (3.13)$$

$$\emptyset_4 = \binom{a}{0} I^{-1} f_1 + \binom{b}{a} I^{-1} f_2 + \binom{c}{b} I^{-1} f_3 + \binom{x}{c} I^{-1} f_4 \quad (3.14)$$

Equating (3.1) on $I_{1,}I_{2,}I_3, I_4\&I_5$

$$\emptyset_1 = \binom{a}{x} K^{-1} g_1 + \binom{b}{a} K^{-1} g_2 + \binom{c}{b} K^{-1} g_3 + \binom{d}{c} K^{-1} g_{4+} \binom{\infty}{d} K^{-1} g_5 \quad (3.15)$$

$$\emptyset_2 = \binom{b}{x} K^{-1} g_2 + \binom{c}{b} K^{-1} g_3 + \binom{d}{c} K^{-1} g_4 + \binom{\infty}{d} K^{-1} g_5 \quad (3.16)$$

$$\emptyset_3 = \binom{c}{x} K^{-1} g_3 + \binom{d}{c} K^{-1} g_4 + \binom{\infty}{d} K^{-1} g_5 \quad (3.17)$$

$$\emptyset_4 = \binom{d}{x} K^{-1} g_4 + \binom{\infty}{d} K^{-1} g_5 \quad (3.18)$$

$$\emptyset_5 = \binom{\infty}{x} K^{-1} g_5 \quad (3.19)$$

Since g_5 is known , $\emptyset_5$can be determined by equation(3.19) now we solve (3.11) for f_1

$$f_1 = \binom{x}{0} I\, \emptyset_1 \quad (3.20)$$

Substituting this in (3.12) to get

$$\emptyset_2 = \binom{a}{0} I^{-1} \binom{x}{0} I\, \emptyset_1 + \binom{x}{a} I^{-1} f_2 \quad (3.21)$$

By equations (3.17) & (3.20)

$$f_3 = \binom{x}{b} I \left[\emptyset_3 - \binom{a}{0} I^{-1} \binom{x}{0} I\, \emptyset_1 - \binom{b}{a} I^{-1} f_2\right] \quad (3.22)$$

Similarly from equation (3.18) we find out the value of $\emptyset_4$ in terms of $\emptyset_1$ & $\emptyset_3$with the help of equation (3.20) and (3.22)

$$\emptyset_4 = \binom{a}{0} I^{-1} \binom{x}{0} I\, \emptyset_1 + \binom{b}{a} I^{-1} f_2 + \binom{c}{b} I^{-1} \binom{x}{b} I \left[\emptyset_3 - \binom{a}{0} I^{-1} \binom{x}{0} I\, \emptyset_1 - \binom{b}{a} I^{-1} f_2\right] + \binom{x}{c} I^{-1} f_4 \quad (3.23)$$

Similarly forg_4

$$g_4 = \binom{d}{x} K \left[\emptyset_4 - \binom{\infty}{d} K^{-1} g_5\right] \quad (3.24)$$

Substituting this in equation (3.17) we find $\emptyset_3$ in terms of $\emptyset_4$

$$\emptyset_3 = \binom{c}{x} K^{-1} g_3 + \binom{d}{c} K^{-1} \binom{d}{x} K \left[\emptyset_4 - \binom{\infty}{d} K^{-1} g_5\right] + \binom{\infty}{d} K^{-1} g_5 \quad (3.25)$$

Now by equations (3.16) and (3.24)

$$g_2 = \binom{b}{x} K \left[\emptyset_2 - \binom{c}{b} K^{-1} g_3 - \binom{d}{c} K^{-1} \binom{d}{x} \left\{\emptyset_4 - \binom{\infty}{d} K^{-1} g_5\right\} - \binom{\infty}{d} K^{-1} g_5 \right] \quad (3.26)$$

Similarly from equation (3.15) we find the value of $\emptyset_1$ in the terms of $\emptyset_2$ and $\emptyset_4$ with the help of equation (3.24) and (3.26)

$$\emptyset_1 = \binom{a}{x} K^{-1} g_1 + \binom{b}{a} K^{-1} \binom{b}{x} K \left[\emptyset_2 - \binom{c}{b} K^{-1} g_3 - \binom{d}{c} K^{-1} \binom{d}{x} K \left\{\emptyset_4 - \binom{\infty}{d} K^{-1} g_5\right\} - \binom{\infty}{d} K^{-1} g_5\right] + \binom{c}{b} K^{-1} g_3 + \binom{d}{c} k^{-1} \binom{d}{x} K \left[\emptyset_4 - \binom{\infty}{d} K^{-1} g_5\right] + \binom{\infty}{d} K^{-1} g_5 \quad (3.27)$$

We have thus arrived at five simultaneous equations (3.27) (3.21) (3.26) and (3.19) associated with five unknown functions $\emptyset_1$, $\emptyset_2, \emptyset_3, \emptyset_4 and\ \emptyset_5$ respectively.

From these equations we can obtain the values of these unknowns and hence the solution of the problem will be determined by equations (3.6).

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