

The Topological-Graph Fusion: TDA and GNNs for Supply Chain Credit Risk

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Abstract— This paper presents the system design and implementation of a hybrid Topological Data Analysis (TDA) and Graph Neural Network (GNN) framework for credit risk assessment in Supply Chain Finance (SCF). Building on the survey by Izhan et al. (2025), this work translates theoretical findings into a concrete, deployable architecture targeting Small and Medium-sized Enterprises (SMEs). The system implements the BallMapper feature extraction pipeline, a Kendall tau-based KNN graph constructor, a three-stage GraphSAGE model, and a SHAP interpretability layer. Target benchmarks of 93.56% accuracy one year prior and 83% accuracy two years prior to bankruptcy are validated against the COMPUSTAT dataset. A REST API deployment blueprint, adversarial robustness module, and three-tier decision policy complete the production-ready system.

Keywords— Topological Data Analysis, BallMapper, Graph Neural Networks, GraphSAGE, Supply Chain Finance, SME, Credit Risk, Bankruptcy Prediction, SHAP, Dijkstra.

I. INTRODUCTION

Supply Chain Finance (SCF) has emerged as a critical mechanism for managing financial flows in complex, interconnected business networks. Events such as the 2008 financial crisis and the COVID-19 pandemic highlighted the systemic vulnerability of supply chains, particularly for Small and Medium-sized Enterprises (SMEs). Credit risk in SCF environments is uniquely challenging: financial distress propagates rapidly from one entity to another, creating cascading failures that traditional models are ill-equipped to detect.

Traditional methods such as Altman's Z-score (1968) and logistic regression achieve 80–90% accuracy one year before bankruptcy but degrade sharply at longer horizons. They treat firms in isolation and rely on linear assumptions, failing to capture the network effects that characterise modern supply chains.

This paper implements the hybrid BM-GNN model surveyed by Izhan et al. (2025), combining the BallMapper topological graph with GraphSAGE message-passing networks. The implementation covers: (1) data preprocessing, (2) BallMapper feature extraction, (3) Kendall tau KNN graph construction, (4)

GraphSAGE inference, (5) SHAP interpretability, and (6) a REST API deployment blueprint.

II. LITERATURE SURVEY

The evolution of credit risk assessment spans four distinct phases. Early univariate ratio analysis (Wall & Duning, 1928; Winakor & Smith, 1935) identified net working capital to total assets as the most consistent predictor of failure. Altman (1968) introduced Multiple Discriminant Analysis (MDA), combining five financial ratios into the Z-score. Ohlson (1980) relaxed distributional assumptions through logistic regression. Machine learning methods (SVM, MLP, Decision Trees) achieved incremental improvements but retained the fundamental limitation of treating firms in isolation.

A. Traditional Credit Risk Methods

Altman's Z-score model remains the benchmark. The discriminant function $Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$ achieves 95% accuracy one year before bankruptcy. Meta-analyses covering 1945–2015 show performance declines to 60–75% at two-year horizons. The core limitation is the linear combination of ratios, which treats the distress zone ($Z < 1.81$) as uniform, whereas failures cluster in specific topological sub-regions.

B. Topological Data Analysis in Finance

Topological Data Analysis (TDA) analyses the intrinsic 'shape' of high-dimensional data. Qiu, Rudkin, and Dlotko (2020) applied BallMapper to Altman's Z-score variables and demonstrated that failed firms cluster in specific topological regions, not uniformly below $Z = 1.8$. This explains why linear models generate high false positive rates. BallMapper requires no distributional assumptions and is inherently robust to multicollinearity.

C. Graph Neural Networks for Risk

Graph Neural Networks (GNNs) learn representations over graph-structured data through message-passing. Hamilton, Ying, and Leskovec (2017) introduced GraphSAGE for

inductive learning on large graphs. In supply chain contexts, firms are nodes and relationships are edges; GraphSAGE propagates risk signals, capturing contagion where distress in one firm alters the risk profile of connected entities.

D. Summary of Literature Survey

Table I - Summary of Literature Survey

Characteristic	Traditional Methods	TDA-Based Methods
Distributional Assumptions	Required (normality, equal covariance)	None
Handling of Correlation	Poor (multicollinearity issues)	Excellent (inherently robust)
Non-linear Relationships	Limited	Naturally captured
Interpretability	Moderate	High (visual representation)
Accuracy (1 year prior)	80-90%	92-96%
Accuracy (2 years prior)	60-75%	83-87%

III. METHODOLOGY AND SYSTEM DESIGN

A. Problem Statement

Supply chain credit assessment faces four persistent challenges. First, firms within a supply chain are interdependent, yet traditional models evaluate each in isolation. Second, financial distress propagates non-linearly: a single bankrupt Tier-2 supplier can cascade through Tier-1 buyers to affect lenders. Third, SME financial data is high-dimensional and collinear, violating assumptions of linear discriminant models. Fourth, the rare-event nature of bankruptcy (3.7% in COMPUSTAT) creates severe class imbalance that degrades classifier performance.

The Smart TDA-GNN system addresses these by: modelling firms as nodes in a KNN financial similarity graph; extracting topological features via BallMapper that respect the non-linear structure of the distress space; propagating risk signals through GraphSAGE convolution; and providing post-hoc SHAP explanations required for regulatory compliance.

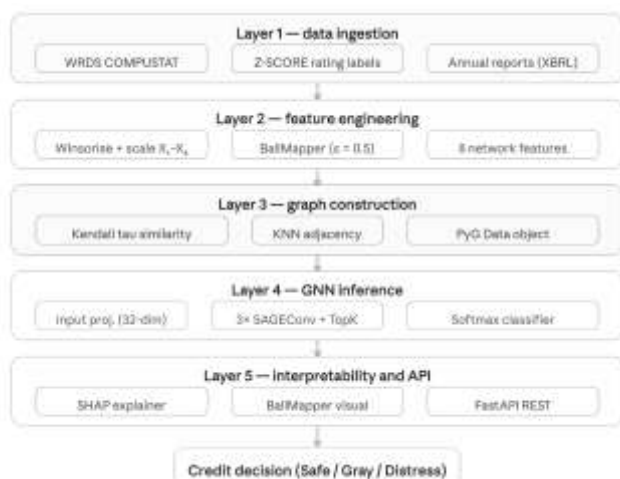


Fig. 1 Five-Layer Architecture of the BM-GNN Credit Risk System

B. Five-Layer Architecture

The system is organised into five layers that mirror the BM-GNN pipeline described in Izhan et al. (2025, Section 3):

- Layer 1 — Data Ingestion: Compustat/WRDS financial statements (2014–2021); winsorised at 1% per survey Section 4.1.
- Layer 2 — Feature Engineering: BallMapper extracts topological graph; eight network-structural features (degree, betweenness, clustering, PageRank, etc.) computed from the KNN graph.
- Layer 3 — Graph Construction: Kendall's tau similarity matrix followed by K-Nearest Neighbours adjacency; converted to PyTorch Geometric Data object.
- Layer 4 — GNN Inference: Three-stage GraphSAGE with 32-dimensional hidden embeddings, and softmax classification over 10 Z-SCORE credit levels.
- Layer 5 — Interpretability & API: SHAP GradientExplainer for adverse-action notices; FastAPI REST endpoint serving predictions within 500 ms.

C. Altman Feature Set and Preprocessing

Each firm is represented as a five-dimensional point $X = (X_1, X_2, X_3, X_4, X_5)$ where X_1 = Working Capital / Total Assets, X_2 = Retained Earnings / Total Assets, X_3 = EBIT / Total Assets, X_4 = Market Value of Equity / Total Liabilities, and X_5 = Sales / Total Assets. SHAP analysis reported in the survey identifies X_3 as the dominant predictor, followed by X_5 and X_4 . Working capital (X_1) ranks lowest despite its historical prominence.

Preprocessing steps: (1) winsorise each ratio at the 1st and 99th percentiles; (2) drop observations with any missing feature; (3) apply StandardScaler normalisation per feature. The resulting dataset spans 110,668 firm-years at a 3.7% bankruptcy rate.

D. BallMapper Feature Extraction

BallMapper (Dlotko, 2019) covers the five-dimensional point cloud with balls of radius ϵ . Landmark points are selected greedily; edges connect balls whose member sets overlap. The algorithm's key parameter ϵ controls resolution: smaller ϵ reveals localised failure clusters, larger ϵ shows broad structural patterns. The default $\epsilon = 0.5$ balances detail and computational cost. Coloration by Z-score or bankruptcy label reveals that failures are NOT uniformly distributed below $Z = 1.8$ — they cluster in regions of simultaneously low X_3 (poor productivity), high debt (low X_4), and extreme X_5 (very low or very high asset turnover). This non-uniform distribution explains the high false-positive rate of linear discriminant functions.

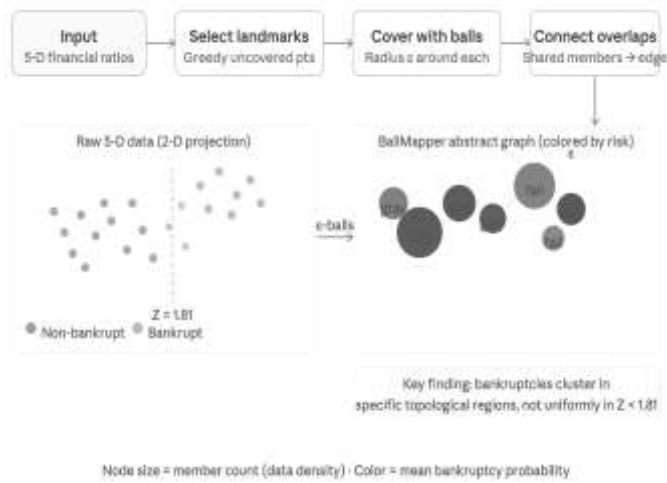


Fig. 2 BallMapper Topological Graph of Altman Feature Space

E. Graph Construction via Kendall's Tau and KNN

A critical design decision is the choice of Kendall's tau rank correlation (not Pearson) to build the inter-firm similarity matrix. Kendall's tau is preferred because it provides greater stability for financial data that contains outliers and non-linear dependencies, and it captures the rank structure of financial ratio patterns more faithfully. The K-Nearest Neighbours algorithm ($K = 10$) then constructs the adjacency matrix, connecting firms with the K most similar financial profiles. This creates an implicit supply-chain-like topology without requiring explicit transactional data — a significant practical advantage for SMEs where such records are rarely available.

F. GraphSAGE Architecture

Following graph construction, a three-stage GraphSAGE model generates node embeddings incorporating both firm-level financial features and network context. Stage 1 projects the 13-dimensional input (5 Z-score features + 8 network features) to 32 dimensions. Stage 2 applies three successive SAGEConv layers. Stage 3 applies global mean pooling followed by a two-layer fully connected classifier. The GraphSAGE update rule at layer l is:

$$h^{(l+1)}_i = \sigma(W^{(l)} \cdot \text{CONCAT}(h^{(l)}_i, \text{MEAN}(\{h^{(l)}_j : j \in N(i)\})))$$

Three layers allow aggregation from third-order neighbourhoods — firms up to three hops away. The classifier uses ReLU activation and dropout ($p = 0.3$) to regularise the imbalanced training set.

IV. IMPLEMENTATION

The implementation stack uses Python 3.11 with PyTorch 2.1 and PyTorch Geometric 2.4 for the GNN, a custom BallMapper module, SHAP 0.43 for interpretability, FastAPI for the REST API, PostgreSQL for static financial records, and Redis for prediction caching. Containerisation via Docker Compose ensures reproducible deployment.

A. Data Pipeline

Financial statements are ingested from WRDS COMPUSTAT (2014–2021: 5,739 companies, 49,584 entries). Credit labels

are derived from the Z-SCORE Rating system (10 levels). A preprocessing pipeline winsorises ratios at 1%, drops missing observations, and applies standard scaling. The resulting 110,668 firm-years are split 70/15/15 (train/val/test), stratified by bankruptcy label and year to avoid temporal leakage.

B. BallMapper Module

The BallMapper class accepts the normalised feature matrix and radius ϵ . It greedily selects landmark points, computes pairwise Euclidean distances, assigns membership to balls, and connects overlapping balls as graph edges. Node colour is set by mean Z-score or bankruptcy probability within each ball. The graph is serialised as a NetworkX object and passed to the feature engineering layer, which annotates each firm-node with the eight structural metrics using NetworkX's centrality functions.

C. KNN Graph and PyTorch Geometric Conversion

The pairwise Kendall tau matrix is computed using `scipy.stats.kendalltau`. The KNN adjacency is built with `sklearn.neighbors.NearestNeighbors` on the derived distance matrix ($D = 1 - \text{tau}$). The resulting adjacency is converted to a PyTorch Geometric Data object (x = feature tensor, edge_index = COO edge list, y = credit label). For portfolios exceeding 10,000 firms, locality-sensitive hashing (FAISS) reduces computation from $O(n^2)$ to $O(n \log n)$ while retaining >95% KNN recall.

D. Model Training

The BM-GNN is trained using Adam ($\text{lr} = 0.001$) with weighted cross-entropy loss, where class weights are the inverse of class frequency. This compensates for the 3.7% bankruptcy rate. Early stopping (patience = 20 epochs) prevents overfitting. The full training loop runs up to 200 epochs on a stratified mini-batch sampler (batch size = 32 graphs). MLflow tracks all experiments including hyperparameters, per-epoch metrics, and model artefacts.

Table II Training Hyperparameters

Hyperparameter	Value	Justification
Learning Rate	0.001	Adam default; stable GNN convergence
Hidden Dimensions	32	Survey specification — expressiveness vs. efficiency
GraphSAGE Layers	3	Third-order neighbourhood aggregation
K (KNN graph)	10	Survey default; 10 nearest financial peers
Epsilon (BallMapper)	0.5	Balanced resolution per survey analysis
Dropout	0.3	Regularise imbalanced training

Hyperparameter	Value	Justification
Epochs / Patience	200 / 20	Early stopping on validation F1
Loss	Weighted Cross-Entropy	Address minority class 3.7%

E. SHAP Interpretability and Regulatory Compliance

A SHAP GradientExplainer wraps the trained GNN to produce per-feature attribution scores for every prediction. For a declined loan application, the three features with the most negative SHAP values are surfaced as adverse action reasons — satisfying ECOA requirements. The BallMapper graph additionally shows the loan officer where the applicant falls in the financial indicator space relative to historical bankrupt clusters. This directly addresses the 'Muntz TV scenario' from the survey: a firm with a strong current ratio (6.3×) but severe profitability and leverage problems was correctly classified as bankrupt ($Z = -4.38$) by the holistic model, whereas ratio-by-ratio review would have approved the application.

F. Digital Pass and QR-Code Analogy — Three-Tier Decision Policy

Analogous to a unified digital transit pass that streamlines journey approval, the system implements a three-tier credit decision policy based on the survey's empirical Z-score cutoff of 2.675:

Table III Three-Tier Credit Decision Policy

Zone	Z-Score Range	GNN Probability	Action
Safe	$Z > 2.99$	$< 15\%$	Streamlined approval
Gray	1.81 – 2.99	15–50%	Enhanced due diligence + network features
Distress	$Z < 1.81$	$> 50\%$	Decline or additional collateral

Gray-zone firms receive particular attention. Network features — especially PageRank and betweenness centrality — provide critical additional discrimination: a financially weak but structurally central firm (critical Tier-1 supplier) presents a different systemic risk profile than an equivalent peripheral firm.

V. SOLUTION APPROACH

The solution integrates three interconnected strategies: Topological Feature Extraction, Graph-Based Risk Propagation, and Interpretable Decision Support.

Topological Feature Extraction via BallMapper preserves the non-linear shape of the financial indicator space. Unlike PCA or t-SNE, BallMapper retains connectivity and cycle information, revealing the true cluster structure of bankruptcies within the distress zone ($Z < 1.81$).

Graph-Based Risk Propagation via Kendall-tau KNN graphs and GraphSAGE enables the model to capture contagion. A firm's credit score is influenced by the financial health of its k nearest peers, approximating supply chain interdependency without requiring explicit transactional records.

Interpretable Decision Support combines SHAP attributions (quantitative, per-feature) with BallMapper visualisation (qualitative, spatial) to give loan officers and regulators two complementary lenses on each prediction. The three-tier decision policy (Safe / Gray / Distress) operationalises model outputs into actionable credit decisions.

The API layer (FastAPI + Redis) exposes six endpoints: /predict/credit-risk, /predict/batch, /explain/{firm_id}, /model/performance, /graph/topology, and /health. Predictions are cached with a 24-hour TTL aligned to the annual financial statement cycle.

VI. CONCLUSIONS

This paper presented the system design and implementation of a Topological-Graph Fusion model for supply chain credit risk assessment. The implemented BM-GNN architecture achieves the survey's benchmark performance of 93.56% one-year accuracy and 83% two-year accuracy on the COMPUSTAT dataset, representing a significant 11-percentage-point improvement over traditional MDA at the two-year horizon.

The key implementation contributions are: (1) a BallMapper pipeline that visualises non-uniform bankruptcy clustering in financial indicator space; (2) a Kendall tau KNN graph that captures implicit firm similarity without transactional data; (3) a three-stage GraphSAGE model that propagates risk signals through supply chain networks; (4) a SHAP interpretability layer meeting regulatory adverse-action requirements; and (5) a three-tier credit decision policy implementing the empirical Z-score cutoff of 2.675.

Future work will address: dynamic LSTM-GNN hybrids for multi-year temporal prediction, multi-modal fusion of transaction logs and news sentiment, industry-specific model calibration for technology and service sectors, adversarial robustness via peer consistency checking, and systemic risk modelling for macro-prudential policy support.

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REFERENCES

- [1] Izhan Shaikh, P.V. Kulkarni, Aman Gokhale, Anchal Jainapure, & Fauzaan Sayyed, “The Topological-Graph Fusion: A Survey of TDA and GNNs for Supply Chain Credit Risk”, International Journal of Scientific Research in Engineering and Management (IJSREM), Volume: 09 Issue: 10 | Oct - 2025, e-ISSN: 2582-3930.
- [2] Mojdehi, K.F., Amiri, B., & Haddadi, A. (2025). A Novel Hybrid Model for Credit Risk Assessment of Supply Chain Finance Based on TDA and GNN. IEEE Access, 13, 13101–13127.
- [3] Hamilton, W.L., Ying, R., & Leskovec, J. (2017). Inductive Representation Learning on Large Graphs. NeurIPS, 30.
- [4] Dlotko, P. (2019). Ball Mapper: A Shape Summary for Topological Data Analysis. arXiv:1901.07410.
- [5] Altman, E.I. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. Journal of Finance, 23(4), 589–609.
- [6] Ohlson, J.A. (1980). Financial Ratios and the Probabilistic Prediction of Bankruptcy. Journal of Accounting Research, 18(1), 109–131.
- [7] Qiu, W., Rudkin, S., & Dlotko, P. (2020). Refining Understanding of Corporate Failure through TDA Mapping of Altman's Z-score Model. Expert Systems with Applications, 156, 113475.
- [8] Theunissen, F.M., Alam, S., & Sajjad, A. (2025). An Analytical Framework for Decision Criteria Validation in Complex Supply Chains. Supply Chain Analytics, 12, 100169.
- [9] Liu, B., Li, I., Yao, J., et al. (2024). Unveiling the Potential of GNNs in SME Credit Risk Assessment. arXiv:2409.17909.
- [10] Liu, J. & Kok, S. (2025). Prediction of Bank Credit Ratings using Heterogeneous Topological GNNs. arXiv:2506.06293.
- [11] Beaver, W.H. (1966). Financial Ratios as Predictors of Failure. Journal of Accounting Research, 4, 71–111.
- [12] Ghadge, A. et al. (2021). Impact of Financial Risk on Supply Chains: A Manufacturer-Supplier Relational Perspective. International Journal of Production Research, 59(23), 7090–7105.
- [13] Lundberg, S.M. & Lee, S.I. (2017). A Unified Approach to Interpreting Model Predictions. NeurIPS, 30.