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Thermoelastic Analysis of Circular and Rectangular Objects with Moving Heat Source: A Review of Recent Literature

Kishor Hadke[1], M S Warbhe[2]

[1]P.G.T.D of Mathematics,
Gondwana University, Gadchiroli, India
[2]Department of Mathematics,
Sarvodaya Mahavidyalaya,
Sindewahi, Chandrapur, India

Abstract:
The article investigates the present review of thermoelastic theories of circular and rectangular objects with moving heat sources published on paper after 1950. The main theme of thermoelastic problems on circular and rectangular structures is to establish operational methods to solve governing differential equations. In the thermoelastic problems, we have considered a few practical problems of technical interest, taking into account the circular and rectangular shape geometry with moving heat sources by using integral transform techniques.

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1. Introduction

Temperature changes affect a material's properties, and those temperature-related properties, such as thermoelasticity and thermal stresses, need to be investigated. The non-isothermal difficulties associated with the theory of elasticity have captured the interest of a great number of scholars. The temperature-dependent features are essential to focus on in many other domains, such as aerodynamics heating, which generates significant thermal stresses that reduce the structural strength of high-velocity aircraft, for the fundamentals of the movement of heat through a solid combined with the generation of thermal stresses in the material. It offers a wonderful reference that is particularly helpful for novices in properly comprehending the concepts that are dependent on temperature. Changes in the elasticity of the material are caused by the thermodynamics of the transmission of energy through the solid, and these changes can be evaluated by mathematical models employing integral transform methods [1].

2. A Survey of the Published Material Concerning Rectangular Plates

Rectangular plates are used in many technical applications as key elements in structures operating at elevated temperatures. Some examples of these structures are nuclear reactors, supersonic aircraft, and steam and gas turbines. However, almost all of the work dealing with thermal-stress problems in such plates has been restricted to the cases of insulated conditions or constant heat-transfer case coefficients on the upper and lower surfaces of the plates. The literature on thermal stress in isotropic and anisotropic rectangular plates is quite extensive. However, these solutions do not accurately explain the real temperature and thermal-stress distributions in the rectangular plates when the media around them are moving [2]. In most instances, the plates are required to support a variety of loads. For this reason, it is necessary for designers to have a comprehensive understanding of the properties of their mechanics. The thermal stresses manifest themselves on surfaces and plates that are either homogeneous or non-homogenous. There is always the possibility of either expansion or compression taking place as a result of differences in the amount of heat either from within or coming from the environment. The thermal stresses that occur in the transversely isotropic thick circular plates have the potential to be useful in a wide variety of applications within the field of engineering. Some examples of these applications include the design of nuclear reactors, geothermal engineering, advanced aircraft structure design, industrial engineering, submarine structures, high-energy particle accelerators, and a great deal of new technology. Many domains, such as aerodynamics and heating, have researched the qualities' dependence on temperature. This results in significant thermal stresses, which in turn result in a loss in the structure's overall strength in high-velocity aircraft. Over the course of the past few years, a significant amount of investigation into the use of thermoelasticity theories to predict deformation and heat transport in a continuous medium has been carried out. The structures of a thick plate, a thin plate, and a membrane are quite distinct from one another in terms of their differences. Plates and membranes that are round and thin are employed extensively in a wide variety of devices, including pressure sensors, microphones, loudspeakers, gas flow metres, optical telescopes, solar panels, radio and radar antennae, and a great deal more. Plate theories are useful tools that can be applied to the design and analysis of these devices. Because of the importance of their fundamental structures in a wide variety of applications spanning the fields of civil, mechanical, electrical, and computer engineering, researchers have never stopped investigating how to predict the stresses that occur in circular plates that have a distributed heat supply. Calculating the durability and reliability of material requires knowledge of the components of the thermo-stressed condition of structural elements, such as hollow cylinders, that have been subjected to certain power and thermal loads. Composite materials have been created for use in a wide variety of industrial applications, including pipes, pressure vessels, fluid reservoirs, aerospace components, and naval structures, to name just a few of these applications. Composite materials offer several beneficial features, including high strength-to-weight ratios and good corrosion resistance [3]. Composite materials also have many other advantageous properties. The thermoelastic theory investigates how elastic bodies respond when subjected to the impact of temperature fields that do not have a uniform distribution. This theory gained widespread interest as a result of its applicability in a variety of technical difficulties. For example, challenges relating to the structure of today's aeroplanes, the design of nuclear reactors, and the processes involved in shipbuilding all benefited from this theory. The temperature and strain fields are coupled in the classical theory of dynamic thermoelasticity. This is something that has to be taken into account. The fact that thermal disturbances can spread at an unlimited rate, on the other hand, raises an interesting conundrum. Generalized thermoelasticity theories have been proposed [4] in an effort to resolve this seeming contradiction. In order to design and address the problem of thermal stresses in circular, rectangular, and other types of plates, many authors have written extensive mathematical literature related to the flow of heat. He also maintained a balance between temperature gradient and heat flow. It emphasises that we simply cannot fathom what life would be like in this day and age without the support of numerous sorts of modern technology. As a result of recent developments in industrial technology, brittle materials such as engineering ceramics are increasingly being selected for use in critical structural elements or machines that are required to bear thermomechanical loads. Regarding the kinds of materials we're talking about here, the compressive strength is noticeably superior to the tensile strength. As a consequence of this, these are selected for usage in places that experience compressive stress specifically. The problem of fracture generated by a crack in a material has not yet been solved since two-dimensional stress fields become rather important, and stress changes with their position when subjected to the thermal load. One of the many reasons why there is an issue is because of this. Rectangular plates are commonly the material of choice when a robust base or surface is required for highways, airports, building walls, or bridge decks. This might be the case for highways, airports, building walls, or bridge decks. Because of the nature of these particular engineering applications, it is necessary to contend with both heat and mechanical loads. Rectangular plates are one of the shapes that are reviewed in the process of analysing thermal stress, temperature distribution, and displacement function using several transform techniques. Other shapes that are evaluated include:

Rectangular plates have been analyzed with the heat conduction equation, and the necessary integral transforms, such as the Fourier transform and the Laplace transform, have been applied in order to determine the thermal stresses that are induced by internally moving heat sources. The goal of this endeavour is to determine the thermal stresses that are caused by heat sources. When it comes to laminated composite plates, the attributes of durability, strength and high stiffness are the most important to seek for. Interlaminar stress is generated if there is a sudden change in the properties of the material. This stress has the potential to produce delimitation. The disparity that exists between these two kinds of material can be corrected in a few different ways. One of these ways is by making use of material that has been functionally graded. Materials that have been functionally graded are in great demand because of the exceptional capabilities that these materials provide in terms of withstanding the quick thermal shock load. These thermally functionally graded materials, which act as thermal barriers to considerable fluctuations in heat and are therefore essential in the construction of structures that can withstand extremely high temperatures, can be functionally graded. This quality makes them essential in the construction of structures that can withstand extremely high temperatures. Because these materials are a hybrid of ceramic and metal, the mechanical endurance of structures can be improved even when placed in high-temperature situations. The development of an adequate mathematical model based on the integral transformation approach has allowed for the analysis of solids with various geometrical shapes and sizes.

The parts of many different types of industrial machines, such as the central rod called the main shaft in the lathe, turbines, and the belt rolling in the mill, are continuously subjected to the difference in temperature that is generated by the mechanical processes that are occurring in the machine. The solid can generate heat and be traversed by it in a variety of various shapes, such as a structure resembling a pipe, a plate in the form of a circle, or a linear conductor like a rod that can transport heat. Additionally, the shape of the solid can change as the heat moves through it. A significant number of researchers in academic institutions are investigating the amount of heat produced and the amount of stress induced in the material. The phenomenon of heat transmission via hollow pipes is studied by examining it in two dimensions: along the axis of the pipe and along its radius. It was revealed that the stress increased close to the source of the change in temperature when only one surface of a circular plate was exposed to a changing temperature while the other surface was maintained insulated. However, the stress did not increase anywhere else on the plate. The current thesis examines the effects of a constant temperature field on the thermal properties of various materials by analyzing the behaviour of a thick circular plate that is kept in a constant temperature environment. The plate is examined with the goal of determining the stresses by first resting its outer edge on thermal insulation and then exposing its upper surface to an arbitrary heat flux while ensuring that the initial temperature is not altered. This is done in order to ensure that the stresses are not altered. The research presented in this study can be used for various purposes, such as laying the foundations of hot liquid and gas containers, furnaces, and other devices that are functionally analogous. In order to achieve this goal, the boundary value problem is first established, and then the heat conduction equation is used to solve the problem.

The internal transform method is applied in order to solve the heat conduction equation, and the temperature at the beginning of the operation is maintained at a fixed value throughout the entire procedure. Throughout the course of this research, the one-of-a-kind qualities of three distinct types of material were analyzed and researched. The significance of this work has been demonstrated by the fact that it is of the utmost relevance to evaluate and analyze the thermal stresses that are created in solids as a result of shifting boundary conditions on the terms that are involved in the heat conduction equation, boundary conditions, and important system parameters. This is because evaluating and analysing the thermal stresses created in solids due to shifting boundary conditions is of the utmost relevance. Sneddon [5] presented an integral transform technique that made use of a very practical and straightforward mathematical tool. The author provided further information on various transform techniques for the benefit of the reader, including the Laplace transform, the Fourier transform, the Mellin transform, the Hankel transform, and others. The heat conduction equation can be solved with the help of this book's straightforward answer. It has been demonstrated that Noda et al. [6] is a basic source of knowledge on thermal stresses, and numerous straightforward answers to a variety of issues have been presented for the reader's delight. The storyline of this book is still applicable to the study of mathematical analysis of problems formulated with regard to the differing ways in which various solids experience heat and temperature. The Lord-Shulman model was examined by Sarkar [7] in the context of a generalized thermoelastic half-space as a result of a moving heat source. Authors have analyzed a traction-free homogeneous, isotropic and thermoelastic half-spatial, one-dimensional problem to a moving plane of the heat source by considering the Lord-Shulman thermoelastic theory model. Both the Laplace transform and the Eigenvalue approach were used to solve the non-dimensional coupled equations successfully. The findings of the thermal stresses, temperature, and displacement functions were reported numerically and graphically. Gaikwad et al. [8] researched the thermal strains that generate heat in thin rectangular plates. The heat generation in a rectangular plate was used to calculate thermal stresses, and the author performed those calculations after the plate was subjected to a variety of temperatures. Every edge of the rectangular plate was maintained at a temperature of zero degrees, with the exception of the point. The double integral transform approach was utilized in order to resolve the mathematical models. In the form of a circular function, the numerical findings of thermal stresses and displacement were found to have been produced. Singru and Khobragade [9] evaluated thermal strains using a rectangular plate equipped with an internally moving heat source. The authors took into consideration a rectangular plate that was completely insulated against temperature, except for one edge, which was maintained at an arbitrary temperature with an initial temperature of zero. The solution to this problem allowed for the determination of the displacement function, thermal stresses, and temperature distribution within a thin rectangular plate that contained an internal moving heat source. The author used the integral transform method, and the findings we acquired were expressed in terms of Bessel's functions. Deshmukh et al. [10] examined thermal stresses in the plate that was simply supported and applied thermal bending moment utilizing heat generation. They found that there were no significant thermal stresses. The authors have conducted the necessary research to determine the thermal deflection, stress function, and temperature distribution caused by the thermal bending moment. The thermal stresses of long rectangular plates that were restricted to the shorter edge were investigated by Sundara and colleagues [11]. By clamping the long rectangular plate at one of its shorter edges, we were able to get the stresses that were induced by a temperature gradient that was constant across the plate. For a strip that is only partially infinite, a two-dimensional problem with mixed border values has been solved. In their study [12], Walde et al. looked at a direct problem involving induced stresses in rectangular plates brought about by moving heat sources. Calculations in mathematics have been carried out by the authors making use of the finite Marchi-Fasulo transform and the Fourier cosine transform. Ghume and Khobragade [13] looked at the stresses that were present in a three-dimensional thermoelastic problem involving a thin rectangular plate. The authors have computed an unknown temperature gradient, displacement function, temperature distribution, and stress function near a rectangular plate's edge. The mathematical modelling equations were solved using the Marchi-fasulo and Laplace transform methods. The boundary conditions of the model were already known. Kidwa-Kukla [14] investigated the effect of an oscillating heat stream on an annular plate and discovered that thermally induced vibration occurred. Using the Greens function method, the author was able to find a solution to the heat conduction problem. The author was able to establish analytical forms of transverse deflection and temperature distribution by using the features of Green's function. Calculation of the plate's thermally induced vibration was accomplished with the help of the solutions that were obtained. The researchers Manthena et al. [15] looked at a rectangular plate made of a nonhomogeneous material and used internal heat generation. They then measured the temperature distribution, thermal stresses, and displacement function. The integral transform method was applied in order to find a solution to the heat conduction equation. An evaluation was done on the temperature distribution as well as the thermal stress distribution for the inhomogeneous material. The solutions were a combination of zinc and copper in the proportion of seventy to thirty. Both numerical and graphical representations, including both homogeneous and nonhomogeneous materials, were reported in this study. Thermoelastic difficulties in semi-infinite rectangular beams were explored by Pardhi et al. [16] employing a moving heat source as the heating element. The authors utilised the finite Marchi-Fasulo transform and the Fourier cosine transform to determine the temperature distribution and thermal stresses. Bessel's function was applied to the data to make sense of the results. Both circular and rectangular plates were examined for thermoelasticity for Parihar and Patil's [17] study. Based on the application of the finite Hankel transform and the Laplace transform, the authors have made an effort to determine the temperature distribution, the stresses function, and the displacement function. Axisymmetric boundary conditions were imposed on the plate in this simulation. Two plane surfaces were given boundary conditions of the third sort by imposing a temperature of zero on the layer that was on the surface's surface. For the purpose of this analysis, the techniques of the finite Marchi-Fasulo transform and the Laplace transform were utilized. Functionally graded material plates were described by Tian and Jiang [18], who used changeable gradient values while being subjected to exponential heat generation. The heat balance equation served as the foundation for the hybrid numerical method theory. The authors solved the heat conduction equation by employing an exponential heat source load, utilizing a hybrid numerical method, which was then used to solve the equation. Utilizing both the Fourier transform and the inverse Fourier transform allowed for the determination of the temperature distribution throughout a plate of functionally graded material. It was discovered that the heat source had only a limited influence and that the temperature distribution gradually decreased as the distance from the source increased, reaching zero when the distance was infinite. Following the application of third-kind boundary conditions around the borders of the rectangular plate, Sugano [19] discovered transient temperatures as well as thermally induced strains in the plate. The solution to the problem that was investigated was found by taking into account the difference in the coefficient of heat transmission between the upper and lower surfaces. Tanigawa et al. [20] chose a plate made of a material that changed its properties depending on the temperature, and they calculated the thermal stresses that resulted from the constant conduction of heat. The issue was solved by generalizing a nonlinear heat conduction equation as the starting point for the investigation. A mathematical analysis was performed on a transient heat transfer that was non-uniform through the plate, and the temperature solution of the nonhomogeneous plate was discovered. Laguerre polynomials were the form that the solution for thermal stress and thermomechanical transformation had when it was presented by Hutsaylyuka et al. [21]. Graphical analysis is used to illustrate the dynamic stress stain state both under constant and high-speed deformation situations. There is a discernible slowing down in the rate of strain that occurs whenever there is a transfer of mechanical energy into heat. The investigated analysis was based on the theory that a material's strength depended on the pace at which it was being stretched. Tungikar and Rao [22] took into consideration easily supported finite rectangular plates when attempting to discover a solution to three-dimensional forms of temperature distribution and thermal stresses. By utilizing this methodology, the strength of the evaluated adhesive bond caused high normal stresses on the plate. In order to arrive at a thermoelastic solution, Walde et al. [23] utilized the Marchi-Fasulo transform, the finite Fourier cosine transform, and the finite Fourier sine transform techniques. An inverted approach was used when dealing with a rectangular plate that contained revolving heat sources on the inside. In order to complete the calculations, a series containing Bessel's functions were utilized. The relative accuracy of the finite element method was discussed in greater detail by Laura et al. [24], and the thermal stresses that were present in the rectangular plate were evaluated. The stress function's expansion was utilised to meet the boundary requirements, which ultimately led to the generation of an expansion coefficient. Jadhav [25] conceived of a three-dimensional thermoelastic inverse problem and a semi-infinite solid rectangular plate treated for the determination of unknown temperature distribution, displacement, and thermal stresses along the edge based on internal moving heat sources. His work was published in the journal Science. To do computations on the issue that was taken into consideration, the Fourier and Laplace transform methods along with third-kind boundary conditions, were utilized. Patil et al. [26] measured the stresses in the plates by drilling circular holes into the rectangular ones. Researchers have examined the distribution of membrane stress and the temperature in a plate containing a constant circular hole. Through the use of the point-matching approach, the free edge conditions that were present around the circular hole were successfully satisfied, while the boundary conditions were found at the plate's outer edges. Using the generalized thermoelasticity theory, Sharma et al. [27] have examined the propagation of free waves in homogeneous transversely isotropic plates. The secular equation was used to derive the wave mode propagation for both symmetric and skew-symmetric waves. We carried out numerical solutions for transversely isotropic plates made of zinc material and solid helium. In addition, we presented and graphically illustrated dispersion curves for symmetric and anti-symmetric modes. Pandita and Kulkarni [28] used the finite difference approach to analyse the thermal stresses placed on the rectangular plate. The authors made an effort to investigate the influence of temperature change on the thermal conductivity of the rectangular plate by using the temperature gradient. The displacement function, thermal stresses, and temperature distribution for the copper plate were demonstrated numerically and visually. Salve [29] sought to solve inverse transient quasi-static thermal stresses within a narrow rectangular plate. The author has found out the component of displacement, the thermal stresses function, the temperature distribution, and the previously unknown temperature. Under the conditions of an inverse steady-state thermoelastic issue, Thete and Ghadle [30] could determine unknown temperatures at any point along the thin rectangular plate. In order to solve the problem that was investigated, the finite difference method was utilized, and as a consequence, superior results were produced and graphically displayed. Tanigawa and Komatsubara [31] conducted research on the effects of non-uniform heat distribution on the thermal strains in a rectangular plate. An investigation of the compressive stress-strain relationship was carried out utilizing a plate that had cracks. An analysis of the compressive stress field in a transient condition was performed on the assumption that a fracture was positioned in an arbitrary position within the temperature field. In Airy's thermal stress analysis, which was created on the basis of the two-dimensional plane stress problem, the stress function method that was utilized was called "Airy's." Bagade et al. [32] investigated the inverse thermoelastic problem in a rectangular plate using the integral transform method to determine unknown values for temperature gradient, temperature distribution, thermal stress, and thermal deflection. Thakare et al. [33] investigated locating strains in a thin rectangular plate using the finite Marchi-Fasulo transform and the finite Fourier sine transform to identify the answer to the predicament. Vel and Batra [34] have done research on three-dimensional thermos-mechanical deformations in a rectangular plate made of functionally graded material. The plate was put on support with the condition of time-dependent thermal loads on either the top or the bottom. The authors solved the equation using the power series method after employing the Laplace transform technique to convert it into an ordinary differential equation with regard to thickness coordinates. In order to determine the values of the elastic constants, either the Mori-Tanaka method or the self-consistent scheme was utilized. On the top surface of the plate, the unsteady temperature, displacement, and thermal stresses can be characterized by either heat flux or time-dependent temperature. The displacement function was used to analyze the instantaneous temperature distribution, and the results were in good agreement with the ones that were obtained by using identical boundary conditions. Ahire and Ghadle [35] investigated a rectangular plate with an internal moving point heat source to evaluate thermal strains' effects. The integral transform method was utilized both in the formulation and investigation of this problem in order to find a solution to the governing heat conduction equation. The solution was obtained through this computation in the form of an infinite series and applied to an aluminium specimen. Cheng and Lin [36] have developed a mathematical model that considers and analyses the temperature field in a plate with a finite thickness. In this model, three-dimensional temperature fields are contained within the plate. To bring about the required heating of the plate, we used a mobile heat source with a Gaussian distribution. The authors assumed that the source of heat travelled at a constant speed along a path parallel to one edge of the rectangular plate.

3. The Behaviour of Thermoelasticity in a Circular Plate

The non-isothermal difficulties of the theory of elasticity were an increasing focus of attention during the second half of the twentieth century. This is a result of their widespread application across a variety of industries. The high speeds at which modern aeroplanes travel give birth to a phenomenon known as aerodynamic heating. This heating produces significant thermal stresses, which weaken the aircraft's structural robustness. A great number of reports can be found in the body of the study titled "induced thermal stresses and temperature profiles in materials with an internal moving heat source," and these reports detail a number of experiments that were carried out by a great number of scholars. Only a few of the studies discussed in the published research will be covered here. The thermoelastic problem of a thin circular plate that is subjected to a dispersed heat supply was examined and analyzed by Khobragade and Deshmukh [37]. The authors have conducted their investigation of the temperature profiles present in the circular plate using the integral transform approach. Using the temperature profiles, the authors of the study were able to determine induced thermal stresses, displacement function, and transverse deflection by applying a mathematical transform. This was done for the processes that were being researched. Mathematical modelling was carried out by Bhongade and Durge [38] based on the unsteady state behaviour of circular plates with the formation of heat internally. The thermal stresses, as well as the displacement function of a circular plate, have been established by the authors. The authors have provided an external arbitrary heat supply to the upper surface, whereas on the lower side, they have placed insulation. The consequences of radial thermal stresses, which lead to the creation of internal heat, have been modelled and estimated. The equation for heat conduction that was obtained was solved using the integral transform method. The results that were acquired of stress and temperature profiles have been determined numerically as well as graphically. The axisymmetric vibration that occurs at a micro-stretch in a circular plate that is subjected to thermomechanical sources was studied by Kumar et al. [39]. Analysis was carried out by applying the Laplace and Hankel transforms, which resulted in the acquisition of displacements, micro rotation, micro stretch, temperature distribution, and stresses. This study makes use of a methodology known as the eigen value method. The findings of this study can be applied to the development of a variety of homogeneous thermoelastic elements, which can then be employed to fulfil particular technical needs. An inverse method was used by Gaikwad et al. [40] in order to solve the problem of thermoelasticity in a circular plate. The Hankel transform method was used to evaluate the investigated problem with temperature within a steady-state field. This was accomplished by ensuring that the circular edge was unaffected by the temperature throughout the process. The authors made an effort to determine the unidentified temperatures, displacements, and stresses in the system. The writers have obtained the results in series form by using Bessel's function. The authors have also computed and presented graphically the results that they have obtained. The thermoelastic unstable variation in the circular plate was investigated by Ghonge [41] using an approach of the inverse method. Using the Marchi-Fasulo and Laplace Transform technique with boundary conditions for occupied particular regions, a report was made that provided a precise solution for the deflection problem. Because the thickness was so thin, a definitive convergent solution was brought to light. In [42], Kidawa-Kukla examined the vibrations caused by temperature changes in a thin circular plate. An application of Green's function approach was made on a circular heat source that was maintained by travelling in a circular trajectory on the surface of the circular plate. The author of this work is responsible for this. The author has worked on a circular plate in which transverse vibrations are produced as a result of internal mobile heat sources. Suitable boundary conditions have been applied for the study of this plate. Xiang-Yu et al. [43] investigated an external temperature source's effect on circular plates with functionally graded gradations. Three distinct varieties of boundary conditions were used to derive temperature fields for plates that were either simply supported or clamped. The authors have carried out numerical calculations in order to check the validity of the obtained solutions and to make a prediction regarding the heterogeneous models' applicability. The writers provided evidence that a unique way of analysis of a circular plate was responsible for the work's distinctive characteristics. The examination of this round plate turned out to be a benchmark for the additional research that was done. Thermal stresses were measured on a circular plate made of micropolar porous material and conducted the research by Kumar et al. [44]. Using techniques such as the Hankel transform and the Laplace transform, the authors were able to get a variety of expressions, including microrotation, displacements, and volume fraction, amongst others. The formulated models each have a graphical representation of the quantities that have been determined. Initially, the porosity of the material caused a drop in the temperature distribution of the oscillating and volume fraction field. However, following that, the temperature distribution increased and continued to oscillate. Tripathi et al. [45] estimated the stress, displacement, and temperature in a circular plate with traction-free surfaces. The temperature distribution was assumed to be axisymmetric. The material that is copper has been subjected to a mathematical treatment. The authors provided graphical representations of the temperature distribution, the stress function, and the displacement, and they reported on the effect of fractional order parameters. Lord-Shulman, Green Lindsay, and a unified system of equations have all been employed by Tripathi et al. [46] in the context of classical coupled thermoelasticity. A problem involving a plate with an infinite surface area and a finite thickness has been addressed using an extended thermoelasticity model and a temperature rate-dependent thermoelasticity model. The obtained findings have been compared with those obtained via classically linked thermoelasticity. For the purpose of investigating the rate at which heat travels through a plate, it was determined that extended thermoelasticity and temperature rate-dependent thermoelasticity models were superior to the more traditional method of classically coupled thermoelasticity. It has been demonstrated that this method helps resolve thermoelastic issues. It was stated that the differences in outcomes among the three theories that were applied were highly obvious for a short period of time since it took longer for heat transfer to reach the centre of the plane. Copper was singled out as its unique case, and its exhibited axisymmetric temperature distribution was investigated and examined. The work of Tripathi and colleagues [47] focused on the thermoelastic diffusion interaction in a thick circular plate with a finite thickness. This plate was subjected to an axisymmetric heat supply, and the associated equations were solved using the Lord Shulman theory. The load was not applied to either the top or the lower surface at any time. The integral transform technique was applied to the examination of copper material as a unique one-off case. It was reported that the incorporation of a heat source and the action of diffusion both had an impact on the body's ability to deform. Authors have confirmed that very little work has been reported in this field due to the complicated nature of the generalized form of the mathematical equation. Tripathi et al. [48] investigated an issue by subjecting a circular plate to the research of axisymmetric temperature distribution. This was accomplished by applying traction-free boundary conditions and periodically varying the heat source that was considered to acquire the desired results. An inverted variant of the

Laplace transform was utilized in order to determine the temperature, displacement, and stress functions. It was found that the material's conductivity affected the fractional order parameters, and this finding was reported. The authors have assessed the copper material's temperature distribution, displacement, and stress functions and have graphed the results. Studying micropolar porous circular plates using the three-phase lag model and using the Laplace and Hankel transform with the potential function, Kumar et al. [49] conducted their research. The authors have made an effort to analyze altered domains for factors such as displacement, microrotation, temperature distribution, and stress. For the utilization analysis, both normal and thermal sources were used. It was found that porosity affected the temperature distribution, the normal stress, the shear stress, and the volume fraction field. The usual stress levels differed across all of the cases that were examined. The inverse thermoelastic problem was studied by Khan et al. [50] in a circular plate with arbitrary heat supplied in the interior of the plate, while the upper surface and lower surface were maintained at a temperature of zero. The authors utilised the Hankel and Laplace integral transform approach with boundary conditions. Goodie's and Michell's functions were utilized in order to conduct the analysis that determined the component displacements and stresses. The examination was carried out using a plate that was positioned in a certain place. The unknown temperature was altered when the temperature changed from having a negative value to having a positive value; as a result, the unknown temperature was discovered to drop along the radial direction. The investigation led to the following conclusion: stress components and displacement were induced close to the heat source. Tikhe and Deshmukh [51] analyzed the force component as well as the thermal deflection. Additionally, they determined the previously unknown temperature as well as the temperature distributions along the upper surface of the plate. The answer, which includes the Bessels function, was obtained by the use of the mathematical tool known as the integral transform technique. The graphical representation of the numerical data was utilized to validate the thermal deflection temperature distribution and the angle of variation. By making use of the Hankel transform and the Fourier cosine integral transform, Patil and Ahirrao [52] were able to evaluate the solution so that it took the shape of an unknown temperature, displacement, and stress function. Within the scope of this work, an inverse problem of thermoelastic heat conduction with internal heat generation was investigated and explained. Bessel's function and an infinite series were the forms that the output took when it was obtained. Kulkarni and Deshmukh [53] examined the issue of fixed insulating edges of a circular plate by maintaining a temperature of zero on the lower surface of the plate while maintaining an arbitrary temperature on the top surface. The authors conceived of the problem as one in which the temperature of the plate would change and then evaluated it to determine the displacement and stress function. According to the findings, the radial displacement function was shown to increase along the circular region with regard to time, while it showed a decreasing trend along the annular direction over the course of the study. The axial displacement did not change over the annular part but became increasingly significant along the radial part. Kulkarni and Deshmukh [54] used a steady-state temperature field and a thick circular plate with insulated edges in their experiment. On the upper surface of the plate, an arbitrary temperature was maintained, while the lower surface was subjected to the conditions of the initial temperature field. At the lower boundary surface, there existed a heat transfer that allowed the heat to be exchanged. After the heat had been transported through a lower surface, the solution demonstrated the presence of the Bessel function. It was found that there was induced stress as well as displacement near the source of the heat. An analytical approach of variable separation and a numerical method known as the Finite Element Method was used in order to solve the problem that Nakajo and Hayashi [55] had developed, which involved a circular plate that was simply supported and clamped. The findings of this investigation showed that a traditional strategy is consistent with the findings. During this study, the effect of nonlinearity was noticed, and the researchers came to the conclusion that membrane forces were also effective in stress calculation; nevertheless, they discovered that these effects were quite minor. The conclusion that the membrane forces were insignificant as a result of the increased temperature was not supported by the experimental findings, which were inconveniently adequate. Gaikwad [56] investigated the arbitrary temperature that existed at the edges of a circular plate while the upper and lower surfaces of the plate were maintained at a temperature of zero. The quasi-static thermal stresses in thick circular plates were calculated by solving a heat conduction equation with the finite Fourier sine transform method. These calculations were made using a heat conduction equation. Compressive stresses were formed as a result of the evaluation of radial stresses using the direct technique, and radial displacement and stresses led to the production of these stresses. The graphic display demonstrated that the direction of heat flow and the body were identical and that proportionality had been established between them. Irie and Yamada [57] separated one side of the circular plate from the heat by applying a continuous sinusoidal heat source. The other side of the plate remained unaffected by the heat. The Fourier equation for heat conduction was utilized to carry out stress analysis and heat conduction. The thermal stresses and temperature distribution in a plate were found to be radially uniform and depending on the distance from the heated surface. This was discovered after the plate was subjected to a heating process. Ahmed et al. [58] measured thermal stresses in a thin circular plate, which maintained a zero temperature on the plate's lower surface while insulating the circular edge. The heat was dispersed not only on the top surface but also all the way around the plate. We employed boundary conditions of the radiation kind and discovered the answer to be expressed in terms of the Bessel function. Due to the circulation of heat in an annular pattern, Bhad and Varghese [59] analysed the thermal stresses that occur in circular plates when radiation-type boundary conditions are present. The formulas were produced in closed form with the assumption that the bottom surface had a temperature of zero and that the curved surfaces were insulated. In this scenario, it is assumed that the temperature field is axisymmetric along the circle. For the purpose of this study's analysis, the temperature function was selected since it satisfied all of the boundary conditions. Using a nonlinear differential equation and Galerkin's approach, Xuefeng et al. [60] investigated a linked temperature and stress field. These findings have been contrasted with those obtained under uncoupled settings by the authors. When the initial displacement was very little, an increase in frequency was seen; however, the opposite outcome was found when the first displacement was quite big. The thermomechanical coupling brought about a reduction in vibrating amplitude. Because the heat from the surface has been allowed to escape, this has come about. The authors Chauthale and Khobragade [61] investigated the effect of heat generation and generated pressures on the behaviour of thick circular plates. The plate defined in a particular location had the necessary boundary conditions applied to it. The finite transformation technique was utilised to achieve the answer expressed in the Bessel function. According to Jadhav [62], the heat generated in the thin circular plate dissipated according to Newton's law of cooling. Under inverse unstable conditions, thermoelastic deformation on the surface of the plate caused by the heat source was discussed. In order to obtain the solution, a nonhomogeneous third-kind boundary condition that included a finite Marchi-Fasulo and Hankel integral transform was employed. The stress of a transversely isotropic thermoelastic thin circular plate was evaluated by Kaur and Lata [63], and the rotation effect was analyzed by thermal insulation with isothermal boundary conditions. The Laplace and Hankel transform technique was applied to determine the displacement components and conductive temperature distribution with radial distances, which allowed for the problem's successful resolution. Internal strains, displacement, and temperature were measured by Gaikwad [64], who used a uniform heat-generating method. During the process, the upper and lower surfaces were insulated against heat loss, and the round edges were maintained at zero temperature. A technique known as the finite Hankel transform was utilized to solve the governing heat equation. Boley and Weiner [65] discovered a lot of highly appropriate materials for thermal stresses. This literature was presented in a good way with a pragmatic approach. This book provided a theoretical explanation of the relationship between changing temperatures and the stresses they cause, making it an excellent resource for studying mechanical and engineering structures that are close to high temperatures. Nowinski [66] provides a foundation for the study of thermal stresses for people who are just getting started. He covers the entirety of the relevant literature on the basis of applications in industry, the nuclear field, aeroplanes, and other areas. The understanding of heat flow and thermal stresses in mechanical structures that is provided by this book is based on the applications of such stresses. Nowacki [67] attempted to ascertain the steady-state thermal stresses present in circular plates. The upper surface of the circular plate is considered to have a temperature of zero degrees, while the lower surface is exposed to the temperature distribution, which is axisymmetric. The temperature distribution does not affect the circular edge, which keeps it thermally insulated.

4. A Comparative Analysis of the Annular and Circular Discs

The study of thermal strains caused by moving heat sources has always attracted many research researchers who want to work on solids with various geometrical shapes and dimensions. Welded thin-walled pipes are required for use in the piping system for the boiling water reactor, the piping system for steam, and the oil pipe transport. The expansion and compression of the material are inextricably linked to the heat transmission that occurs through the disc. The thermal stresses and strains present in the thick annular disc became the foundation in a container containing hot liquids and gases. High-temperature heat barriers are preferred in various industrial applications, such as aerospace, nuclear reactors, chemical plants, and other similar applications. The annular disc made of functionally graded material can withstand a high range of temperatures and is widely used in these applications. Numerous academics have produced a great deal of work on determining the thermal stresses present in the hollow disc. Yapici et al. [68] were able to measure the system's thermal stresses and transient temperature on a hollow disc with a uniformly moving heat source. The authors have investigated transient temperature and thermally induced strains using a moving uniform heat source within a stationary hollow steel disc from the disk's outer surface. The ambient conditions were constant throughout the research. The different thermal conductivities were each subjected to their own set of calculations. Gonczi and Ecsedi [69] conducted research on functionally graded hollow circular discs with arbitrary gradients and thermoelastic boundary value problems. The authors found the temperature distribution by assuming it to be a function of radial coordinates in conjunction with the temperature that was provided for the boundary surfaces. For the purpose of creating a smooth function of the radial coordinate, material qualities were assumed. In [70], Kordkheili and Naghdabadi addressed the thermoelastic problem of a revolving disc with a functionally graded surface. The author's primary focus is developing linear algebraic equations that can yield thermoelastic responses for each sub-domain as exponential functions of the radial coordinate. As a result of the centrifugal force and the heat loading, the components of stress and strain, displacement, and radius were provided. Khobragade et al. [71] used the integral transform technique to find thermal stresses after they evaluated a thick annular disc that generated heat. The authors could determine the thermal stresses, temperature distribution, and displacement function by applying radiation-type boundary conditions. The study that has been given has produced findings in the form of Bessel's functions. Thermal stresses in a circular plate with an axisymmetric heat supply were the subject of an investigation and analysis by Praveen and colleagues [72]. The authors have studied thermoelastic problems with radiation-type boundary conditions in circular plates on flat surfaces, and the circular edge was thermally insulated throughout the study. The authors utilised techniques, including Laplace and Marchi-Fasulo transform, to solve a specific situation's mathematical model. We were able to determine the stresses and temperature profiles of the aluminium material. Kulkarni and Deshmukh [73] conducted an analysis to determine the stresses that are placed on a thick annular disc when it is subjected to a constant temperature field. The authors took into consideration a thick annular disc in which heat was transported at the upper surface and dissipation of heat at zero temperature in the surrounding from the bottom surface, all while the borders of the circular plate were thermally insulated. The Hankel transform was used to solve the heat conduction equation, and at the end of the process, it produced Bessel's function. The Hankel transform was responsible for determining the displacement and stresses in the system. The governing heat equation was solved using the Marchi-Zgrablich transform and the Marchi-Fasulo transform procedures. Navlekar [74] analysed the displacement, thermal stresses, and temperature of the thick annular disc that was subjected to the particular space. A third type of boundary condition was utilised to investigate the formation of heat, and it was discovered that the variation rate was linearly related to the temperature. The unsteady state thermoelastic problem was taken into consideration, and the characteristics, including temperature distribution, thermal stress function, and displacement owing to internal heat generation, were discovered in terms of Bessel's function. Plastic deformation in an annular disc was the topic of discussion in Dastidar and Ghosh's [75] paper, which was based on deformation theory. It was expanded to include a variety of materials, each of which is dependent on a constant heat gradient. The problem of an annular disc made of functionally graded material was investigated by Kursun et al. [76], who did so with reference to a temperature distribution that decreased linearly and an inner surface that was under a constant amount of pressure. The authors have performed the analysis on the assumption of plane stresses to find elastic stresses in an annular disc with a variation of elastic modulus and thermal expansion coefficient radially by taking Poisson's ratio constant as a constant. This was done in order to determine whether or not the stresses were plane stresses. Additionally, it was solved for displacement, the elasticity modulus, the change in stresses, and the thermal expansion coefficient. Thermal stresses and temperature in an axisymmetrically heated functionally graded annular disc of thickness with spatially heat transfer coefficients have been investigated by Chiba [77]. The analytical solution was evaluated using the power function approximation for Young's modulus as the basis for the evaluation. It was determined that a random subject was the degree of irrationality in the heat transfer coefficients. In order to conduct an investigation into the topic of stochastic temperature, the annular disc was modelled as a multi-layered structure with stepwise variations in thickness. It was thought that each layer had consistent and deterministic cloth homes as well as unpredictable heat transfer coefficients. In the numerical results, the effects of the magnitude of the heat transfer coefficient mean, volume fraction distributions of the constitutive materials and thickness variation were shown to affect the temperature and thermal stress data. Zenkour [78] examined the effects of thermal pressures on a functionally graded rotating annular disc. Within the radial direction of the annular disc, it was anticipated that the material properties would not be affected by temperature and would exhibit continuous variation. The variation in material density, conductivity coefficients, Young's modulus, and thermal expansion was represented by a solitary exponential-regulation distribution throughout the radial route of the disc. On the other hand, Poisson's ratio was held constant throughout the entire process. At each and every location on the disc, the governing differential equations showed improved agreement with one another. In terms of exponential integral and Whittaker's functions, precise solutions for the temperature and stress fields were found. On this topic, we spoke about the effects that angular velocity, inner and outside temperature masses and the qualities of the fabric have on the pressure, stress, and displacement additions. Bhongade and Durge [79] investigated the thermoelastic behaviour of thick annular discs by employing a moving heat source from the inside of the discs. The authors have used an internal moving heat-generating technique to estimate the thermal stresses and displacement function in annular discs. Calculations were made to determine the effects of internal heat generation in a thick annular disc in terms of strains along the radial direction. We used the integral transform approach to solve the problems of temperature change, displacement function, and stresses, and the answers we got were expressed in terms of Bessel's functions. The mathematical modelling of a thermoelastic problem with heat generation in a circular disc was created by Gaikwad [80]. The author has determined the displacement function, thermal stresses, and temperature distribution in a circular disc that is subjected to a temperature field that is in a steady state and has an internal heat source that is moving. The heat conduction equation was solved by employing the Hankel transform in conjunction with the method of generalised finite Fourier transforms. The author used a disc made of aluminium in its purest form. Both a numerical and a graphical representation of the findings were produced. Thermal and mechanical strains caused by radially symmetric loads were analysed by Bayat et al. [81] in a revolving disc constructed of functionally graded material with varying thicknesses. The material grading index of the disc, as well as its geometry, were related to the thermoelastic solution and its weight. The authors discovered that the disc with parabolic and hyperbolic thickness convergent had less displacement and stress than the disc with uniform thickness. In addition, the authors discovered that a functionally graded disc with a concave thickness profile had the lowest overall weight compared to a disc with a uniform thickness profile. The weight of the disc that had been functionally graded was found to be somewhere in the middle of the weights of all metals and all ceramic discs. Based on the 3D thermomechanical coupling model, Belhocine and Bouchetara [82] examined vented disc brakes' thermal behaviour and temperature. Their findings can be found below. In order to solve the mathematical modelling for this research work, the present study's authors used the computing code known as ANSYS. An equation that describes the temperature distribution in a disc brake was used to determine all of the factors and entering parameters that were important during the process of applying brakes. These included the kind of braking applied, the material being used, and the geometric design of the disc. The numerical simulation for the coupled brief thermal and stress field was accomplished with the assistance of a sequentially thermal-structural coupled technique that was entirely based on ANSYS. This technique was used to evaluate the pressure fields, deformations established in the disc, and the contact strain on the pads. When compared to the results found in the specialised literature, the results obtained from the simulation stood out as very impressive. Yevtushenko et al. [83] conducted research on the thermal sensitivity of the pad and disc materials in relation to the temperature throughout the braking process. The authors took into consideration the momentary heating caused by friction in the pad-disk tribosystem during single braking. In order to ascertain the typical temperature of the friction surface, the authors devised a one-dimensional thermal problem of friction throughout the braking process. It has been taken into mind that the thermos-physical properties of the materials used for the disc and the pad have a linear relationship with the temperature. A model of materials with a simple nonlinearity has been developed. This model describes substances as having a simple nonlinearity if their coefficients of heat conduction and unique heat fluctuate on temperature while their ratio-coefficients of thermal diffusivity remain constant. The authors used the Kirchhoff transformation and the linearizing parameter approach in order to linearize the relevant boundary-value heat conduction problem. This was accomplished by performing linearization on the problem. The authors have utilised the integral Laplace transform in conjunction with the Newton-Raphson methods in order to locate the numerical and analytical solution to the problem. The temperature was measured before, and after the thermosensitive materials of a titanium pad were allowed to slide across the surface of a disc made of steel, aluminium alloy, or grey cast iron. The authors investigated the effect that these materials had on the temperature. In order to investigate the dynamic thermoelastic response of the functionally graded annular disc, Bagri and Eslami [84] generalised coupled thermoelasticity. This approach is mostly based on the Lord-Shulman version, and it allows for the second sound impact to be taken into consideration. The material of the disc was thought to be graded along the radial direction, and a strength regulation distribution was assumed to be the result of this thinking. Abrupt heat exposure was applied to the boundary in order for the authors to investigate both thermal and mechanical wave propagation. In order to solve the system of equations in the space domain, the Laplace transform and the Galerkin finite element method was utilised. Using the Laplace transform, the governing equations were first transformed into the Laplace domain. Plots were made of the time-established temperature, radial displacement, radial pressure, and hoop strain. Gangopadhyaya and Das [85] researched an analytical solution for an annular disc's axisymmetric one-dimensional thermomechanical response. The fundamental equations had been formulated as a vector matrix differential equation in the Laplace transforms domain, and the eigenvalue method had been utilised to find the solutions to them. Within the Laplace transform domain, we were able to derive closed-form solutions for the radial and hoop stresses and displacement and temperature. In the space-time domain, numerical inversions for these field variables have been made, and the results have been shown in graphical form. Kedar and Deshmukh [86] estimated stresses in a hollow disc in an unstable state temperature field as a result of a factor heat source placed at the centre along with radial and axial direction inside it. A somewhat thin and hollow disc was seen as having arbitrary preliminary status. Both the top and the bottom of the disc had the same temperature, which was zero degrees. The clamps were placed on both the disk's inner and outer edges. The integral transform technique was applied to find a solution to the governing equation for heat conduction. The findings were presented in terms of Bessel's functions, and graphical representations of the impacts were provided. The inverse unsteady state problem of thermoelastic deformation of a narrow annular disc in the plane state of stress was investigated by Patil et al. [87]. On the curved surfaces of the disc, homogeneous boundary conditions of the third sort were kept in place, and on the decrease aircraft surface, the heat flux was kept at $u(r, t) = 0$, which is a known function of radius and time. Both of these criteria were kept constant. Because of the conditions within, the flux was also defined for the plane that was used. The difficulty of mathematically estimating the temperature, displacement, and stress characteristics of a thin annular disc was investigated. Inverse transient thermoelastic issues of a thin annular disc were solved by employing the finite Marchi-Zgrablich and Laplace transform procedures, respectively. The surface temperatures of the friction that occurs between clutch plates have been estimated by Abdullah et al. [88] using the heat partitioning approach as well as the total heat generated approach. The authors measured the highest surface temperature when two plates were rubbed against one another in either a single cycle or multiple cycles. The authors utilised an axisymmetric finite element model to investigate the friction's thermal behaviour, and they compared the temperature distribution to demonstrate the techniques' accuracy. However, as time went on, it was discovered that the heat partitioning approach was no longer enough to calculate the disc's thermal parameter during a multicycle operation. By employing the Laplace transform, Kar and Kanoria [89] have studied a homogeneous transversely isotropic circular disc for the purpose of conducting an analysis of stresses that are dependent on step inputs of temperature. According to the findings of the authors, radial stress builds up along the edges of an annular disc as a function of an increase in the size of the structure. However, radial stress gradually decreases within the disc itself. Compared to an isotropic material, the amplitude of stresses in a transversely isotropic material was significantly higher. In their study, Sewalkar and Meshram [90] considered a partial and axisymmetric heat treatment in an annular disc with heat supplied from the upper surface. They also studied the cooling of the plate. By taking into account the thermoelastic displacement potential Loves function, the integral transform technique was applied to locate the answer to the problem. During both the heating and cooling processes, there was an increase in the radial distribution of temperature. After reaching the highest possible peak value, it was observed that the temperature began to drop. The finite Hankel transform and the modified finite Fourier transform were used by Gaikwad and Ghadle [91] for the determination of thermal deflection and temperature in thin hollow circular discs. In order to evaluate this, arbitrary temperatures were applied to circular discs, and varied temperatures were applied to the various boundary surfaces. The author found an inverse relation between their thermal conductivity and their thermal deflection. It has been utilised on a variety of discs made of metallic material.

5. Analysis of a Cylinder That Is Hollow

The unsteady conduction of heat in composite hollow circular cylinders is a class of problems that has attracted a lot of attention because it belongs to a category of problems that could be considered fundamental to the design of reactors, ordinance, and aerospace engineering, as well as other related fields. The prediction of the related thermal stresses, which are derived on the basis of the uncoupled equations of thermoelasticity, also requires knowledge of the non-steady temperature field. Today's most fascinating study issue is the convective movement of heat through a hollow cylinder, which emphasises thermal stresses in all directions of the cylinder. This phenomenon is receiving attention as the most intriguing research topic of today. The transfer of moving heat sources and the stresses that correspond to those transfers along the radius and axis of a hollow cylinder has many applications in a variety of industries, including the automotive industry, the power industry, civil engineering, the biomedical industry, thermodynamic and solidification processes, high-density microelectronics fuel cells, electrochemical reactors, building structures, sports goods, and so on. Recently, problems that are based on heat generation in hollow cylinders have been explored and solved numerically for the evaluation of thermal stresses. This has been done by examining the material with multi-bore in the circular cylinder, which is useful in nuclear reactor fuel rods. The method of heat transmission using moving heat sources has been researched and shown to play an important part in industrial applications. Kayhani et al. [92] worked on cylindrical multilayer composite laminates in order to determine heat conduction. They did this by applying the necessary boundary conditions, and the heat conduction equations were solved using the Sturm-Liouville theorem by transforming it into Fourier transform. The problem of heat conduction that was investigated in the aforementioned research publications consists of the inside and the outside of the cylinder, and there was a consistent temperature gradient between the different layers of the material. The Thomas algorithm proved successful in resolving the system of equations. The unsteady flow through a finite composite hollow cylinder was analysed by Olcer [93] using second-kind boundary conditions. The analysis was based on the imperfect thermal contact between two concentric radial layers of a hollow right circular cylinder and the distribution of heat traverses each and every layer of the cylinder. Bagri and Eslami [94] established a unified formulation for the coupled thermoelastic problem by combining the ideas of Lord Shulman, Green-Linsay, and Green-Naghdi. This was done in order to solve the problem. The answer was found for a general isotropic heterogeneous thermally elastic material, and then it was transformed into an equation system for isotropic heterogeneous materials. We presented generalised theories for functionally graded cylinders, and we derived thermoelastic waves as a result of our work. In order to find a solution for this issue, the inverting mode of the Laplace transform was utilised. At the beginning of the temperature shock treatment, both the Lord –Shulman and the Green –Naghdi models revealed an abrupt rise in temperature. Lu et al. [95] outlined an issue involving a one-dimensional hollow cylinder in which heat conduction occurs with a boundary temperature that varies with time. In order to solve a problem of this nature, an original approximation of the Laplace transform was utilised. The problem of calculating eigen value and residue could be sidestepped by using the transformation approach. Dalir and Nourazar [96] used an eigenfunction expansion method to determine the heat conduction rate caused by non-uniform internal volume heat sources in a cylinder constructed out of multipolar radial layers, and they acquired a transient temperature from their findings. First- and second-kind boundary conditions in the axial and radial directions of the cylinder were used to find the solution. Nonhomogeneous third-kind boundary conditions were used in the radial direction. The problem of heat conduction in a three-layer semicircular cylinder was the focus of the investigation's examination, and its ultimate goal was to address the problem. Mallik and Kanoria [97] investigated the effect of periodically changing heat sources in a functionally graded unbounded isotropic media. They came up with a solution for generalised thermoelasticity that did not include the loss of energy. The Green-Naghdi type II model was used in the development of the governing heat equation, which was then solved using the Laplace-Fourier double transform domain. The technique of inverting the Laplace transform was accomplished through the expansion of the Fourier series, and the inversion of the Fourier transform was accomplished through the residual approach in order to obtain numerical value through Laguerre's method. In order to calculate the amount of displacement and tension, a hypothetical medium was taken into consideration. In order to evaluate the thermal stresses, displacement function, and temperature distribution in a semi-infinite hollow cylinder, Barai et al. [98] took into account known boundary conditions. The Marchi-Zgrablich and Fourier cosine transforms were utilised in the work that was examined, and the stress level was determined. To determine the deformation amount in a functionally graded hollow cylinder, Nie and Batra [99] used the Airy stress function to arrive at an exact solution. This function was used to determine the amount of stress placed on the cylinder's inner and outer surfaces. When combined linear loads and hoop stresses were considered, the shear modulus changed depending on the radius. It was discovered that the radius had an effect on the hoop stress, which maintained its original value all the way along the thickness of the cylinder. Takeuti [100] researched thermal stresses in multi-bore circular and square cylinders to discover how temperature is distributed inside the structures. The conducting sheet analogue approach was utilised in order to solve the issue, which was determined by the presence of two dimensions. Laplace transform or the Poisson equation. whichever comes first. Valentin [101] worked on a method to cool the heat-generating circular cylinder by using temperature-independent boundary conductance. This method takes into consideration traction-free substrate in order to determine thermal stresses. It was reported that the plane strain solution overestimated the radial and circumferential stresses for a significant portion of the region's length. The plane strain solution became dominant over the axial stress. The plane strain estimate did not change significantly for lower Biot values. The thermal displacements and stresses in the right circular isotropic cylinder of a homogeneous elastic material with a known radial temperature distribution were solved by Valentin and Carey [102]. It was discovered that the solution was compatible with traction-free circumstances on the cylinder's plane ends as well as its lateral surfaces. A greater amount of radial displacement was seen in the displacement field as we got closer to the cylinder's terminus. Iyengar and Chandrashekhara [103] found the perfect three-dimensional solution to calculate thermal stresses by exposing a finite hollow cylinder to a steady-state axisymmetric temperature field on one of its surfaces. This caused the thermal stresses to be generated. The ratio of length to outer diameter has been set at 1, and the ratio of inner to outer diameter has been set at 0.75 for the purposes of this analysis. Radial and tangential stresses were altered as a result of the presence of relatively minor axial and shear loads. Using a generalised plane strain condition, Orcan [104] was able to identify the solution to the problem of the distribution of stress and the deforming state of an elastic-perfectly plastic cylindrical rod when it was subjected to internal heat generation. The Tresca yield function was utilised for the treatment, and the stress points of the inner and outer plastic regions were discovered on two separate corners of the Tresca prism. Takeuti [105] referred to the condition in which the holes were arranged in a symmetrical pattern around the centre of the circular cylinder, and the heat was dispersed evenly throughout the entirety of the structure as a result of the outer border being thermally insulated. It was also considered to have four circular holes in the square cylinder, and either three or four holes may have been placed in a circular cylinder. The findings were consistent with the situations when there were no weights on the border. With the integral transform technique, Walde et al. [106] attempted to model a solid circular cylinder exposed to internal heat sources that varied linearly with temperature and had radiation-type boundary conditions. After everything was said and done, the solution turned out to have an infinite series of Bessel's functions. It was implemented on a specific substance, and its graphical representation was shown. Yapici et al. [107] used a method to generate internal heat, resulting in thermal stress and uneven temperature distribution inside the hollow disc. The surface was rotated in sync with the angular speed while a moving heat source heated a specific angular segment. After two or three cycles, there was a discernible lessening in the maximum levels of the temperature gradient and the thermal stresses. When the object's angular velocity increases, the effective thermal stress ratio decreases. It was noticed that there was an exponential decrease in the maximum effective thermal stress ratio with a substantial reduction. Ozisik et al. [108] investigated the process of heating a circular pipe using periodically moving heat sources. The pipe was heated partially from its outer surface, and the conditions surrounding it were treated as though they were stationary. It was investigated by passing water through the pipe while the heat source shifted from one end of the pipe to the other and back to the first end. It has been reported that the thermo-physical properties do not alter as

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a result of the temperature change. The tangential component eventually came to have a greater influence than the radial stresses. Noda and Sugano [109] conducted an experiment in which they allowed a semi-infinite isotropic circular cylinder to experience convective heat loss on the end surface. After maintaining the cylinder in traction free, the generalised Fourier integral was applied to an exponential function in order to calculate the transient thermal stresses and displacements. A correlation was found between the anisotropy of thermal conductivity and the sensitivity of both the displacement and the maximum stress. Goshima and Miyao [110] treated an infinitely long composite hollow cylinder subject to heat change due to an internal source. The heat was declining exponentially at the wall thickness where x-ray radiation was being offended, and the cylinder was infinite in length. Cylinders made of stainless steel and carbon steel were treated similarly in the analytical process. The absorption coefficient, the Biot number, and the splicing radius were all considered during the research. Goshima and Miyao [111] researched the thermal strains that occur in a hollow cylinder when subjected to x-ray radiation heating and the heat loss owing to convection. Their study focused on the conditions that occur during temperature transitions. The cylinder cooled down directly from the heat loss that occurred at the inner and exterior surfaces. Because heat dissipates as it travels through the thickness of the cylinder, an analysis of tangential stresses was carried out. For the purpose of determining the temperature field in the circular cylinder, which was a transversely isotropic and finite solid cylinder, Sugano [112] used finite Fourier cosine transforms as well as finite Hankel transforms. Both of these transformations were finite. Within a circular cylinder, both unstable temperature fields and axisymmetrical temperature fields were investigated. An illustration of the effect that anisotropy has had on Young's Modulus and the linear thermal expansion coefficient, which in turn has had an effect on the stress and displacement distributions, was shown via graphical analysis. In their study, Haojiang et al. [113] analysed the behaviour of an orthotropic hollow cylinder subjected to axisymmetric conditions. They compared the effect of axial strain to that of plane strain. The results of this study led the researchers to conclude that the axial strain, rather than the plane strain condition, should be considered. The problem statement was broken down using the variable separation method, resulting in the thermoelastic issue being recast as the Volterra integral equation and applying the second boundary condition. The work that was presented included research on misplacement as well as stressors. The thermal and mechanical properties of a long circular cylinder were investigated by Noda and Daichyo [114], who found that these parameters were affected by temperature. A perturbation method was ultimately successful in resolving the issue. Additionally, the thermoelastic potential function, Love's function, and displacement function were all applied to the problem. Kirchhoff variable transformation was used by Manthena et al. [115] to convert the nonlinear form of heat conduction into the linear form. This was done to design a problem on a functionally graded thick hollow cylinder to determine temperature distribution and thermal stresses inside of a cylinder. The author obtained an unsteady state solution of two-dimensional heat equations while taking into account the effect of thermosensitivity on thermal and mechanical behaviour. Along both the axial and the radial directions, a sinusoidal pattern was seen in both the distribution and the displacement of the temperature. In the case of homogeneity, both the compressive and the tensile stresses along the axis and the radial direction were discovered. On the other hand, only compressive stress was detected in the event of nonhomogeneity. To generalise the equations of thermo-elasticity for the nonhomogeneous isotropic hollow cylinder, Abbas and Othman [116] have dealt with the variable modulus of elasticity and thermal conductivity that is based on Lord-Shulman's theory. This was done in order to make the equations as accurate as possible. The authors have performed an analysis of the problem that was investigated using the finite element method, and they have ascertained the radial stress, displacement, temperature, and hoop stresses. A comparison was made between the temperature-dependent and temperature-independent moduli of elasticity, using the findings from coupled theory and the theory of generalised thermo-elasticity. A graphical analysis demonstrated that factors such as time, space, and the coefficient influence parameters like as temperature, displacement, and stress. Kim and Noda [117] have performed mathematical analysis on a two-dimensional unsteady temperature field and corresponding thermal stresses by using an approach of Green's function that is based on laminate theory in an infinite hollow circular cylinder that is made up of functionally graded material with radial direction. For the purpose of conducting an analysis of the thermoelastic field, both the thermoelastic displacement potential function and Michell's function were utilised. The authors have created an unsteady state heat conduction equation in terms of an eigenvalue issue. This equation was formed using eigenfunction expansion theory and laminate theory. The authors demonstrated that it is not difficult to study and determine parameters such as thermoelasticity and displacement in functionally graded hollow cylinders. This was demonstrated through their work. Calculating thermal deflection in a semi-infinite hollow cylinder internal heat source, Jadhav and Ahirrao [118] selected an approach consisting of the March-Zgrablich and Fourier transform technique. After designing the governing heat equation for the plate with zero temperature at a lower surface and then solving it, a series of Bessel functions were obtained. The authors have considered two-dimensional nonhomogeneous boundary values as a problem for the conduction of heat and evaluation of temperature as well as the thermal deflection of a semi-infinite hollow cylinder as a result of internal heat generation within it. This problem is related to the problem of a problem for the conduction of heat. For the treatment of interactions of thermoelastic behaviour in an infinitely long hollow cylinder with a fractional heat conduction equation, Abouelregal [119] has used the Fourier expansion technique to determine the inverse of the Laplace transform. This was done by using the Fourier expansion technique to determine the inverse of the Laplace transform. The author subjected the traction-free inner surface of the hollow cylinder to thermal and mechanical shock, while the surface that faced the outside of the cylinder was subjected to continual heat flux. Graphical analysis was performed on the thermal stresses as well as the temperature and displacement components. According to the reports, there was an increase in the radial stress, distributions of displacement, and hoop stresses as the coefficient were increased. The distributions were wholly determined by the coefficient when it came to high conductivity. Chen and Chu [120] investigated the effects of variable heat sources on the unsteady thermal stresses induced in a hollow cylinder formed of two distinct materials using their findings. The authors have made the assumption that the inner boundary will be heated while the external surface will be cooled by convection, and the heat source will be located along the axis of the cylinder, where its intensity will diminish exponentially. In order to determine the end of the temperature distribution, both the eigenfunction expansion approach and the Laplace transform methodology were utilised. In the realm of thermal stresses, the stresses along the axis and the hoop stresses emerged as the most important. There was a correlation between the rise in the thermal conductivity ratio and an increase in both the temperature and the thermal stress. In order to determine the stress field and displacement function, Gaikwad and Ghadle [121] investigated expansion in a circular cylinder along axial and radial directions. It has been discovered through this that heat flow causes tensions to be exerted in a direction that is descending from the surface's uppermost point. Because of this, the cylinder was seen to bend in a concave direction. When the circular cylinder under consideration operated mathematically with a thermally insulated circular edge, and temperature varied between the upper and lower surface, the cylinder analysis led to a series of Bessel's functions. This occurred when the temperature difference between the upper and lower surface was varied. Chen [122] worked on a solution to the problem of the linear thermoelasticty of the transversely isotropic hollow cylinder. The Lanczos-Chebyshev method was applied to study a finite-length hollow cylinder so that the researchers could examine it. An approximation based on direct power series is utilised here. Collocation analysis was used at a number of Chebyshev points to determine the values of the economised series' acquired coefficients. It has been demonstrated how significant the final impacts will be. Manthena and Kedar [123] considered temperature-dependent features of functionally graded thick hollow cylinders in their research. They used specific coordinates for the investigation of thermal stresses and temperature distribution. The quantity known as Poisson's ratio does not change in response to changes in temperature, and the heat conduction equation can be solved using a number of different point heat sources. In addition to that, we looked at how inhomogeneity affected the parameters. Gahane et al. [124] discussed a finite hollow cylinder by producing sources as a linear function of temperature with radiation-type boundary conditions. This was done using a radiation-type boundary condition. In order to calculate the stress function, temperature distribution, and displacement in a material, the integral transform method is put into practice. The investigation is carried out on a particular piece of evidence in order to validate its veracity. Grysa and Kozlowski [125, 126] decided to develop a one-dimensional transient thermoelastic problem using an isotropic infinite slab. The goal of this problem was to determine the heating temperature as well as the heat flow on the surface. For the purpose of determining the relationship between temperature and the associated stresses, the author has taken into account a variety of dimensions, such as the shape of a sphere, a circular plate, and an infinite plate, respectively. In inverse issues, the dependence of boundary conditions on temperature and different stresses is applied rather than the direct technique of using known boundary conditions. This is done in place of the direct approach. Sierakowski and Sun [127] researched the deformation of a finite-length hollow cylinder that was subjected to mechanical and thermal load. Their findings were presented in the form of a paper. At both the surface and the end boundary conditions, we were able to find the same exact solution. Solanke and Durge's [128] research focused on the movement of heat sources at a constant velocity via a unidirectional rod that started out at zero degrees Celsius. A one-dimensional rod of isotropic material was carried out using Robin's boundary condition as the guiding principle. The author researched and analysed the time-dependent temperature distribution and thermal stresses in a single dimension. For the purpose of determining the temperature in any location along the circumference of a hollow cylinder, Marchi and Zgrablich [129] made use of a novel finite integral transformation, which is an extension of those provided by Sneddon. The author offered a suggestion for a technique that might be used to specify the mathematical tool that is given here.

6. Practical problems of technical interest

Applicability of circular and rectangular objects with moving heat sources holds for a wide range of fields, e.g. eddy currents, non-destructive testing, wave scattering, offshore dynamics, and minerals prospecting etc. Though it has been proved that ample cases of heat production in solids have led to various technical problems in mechanical applications in which heat produced is rapidly sought to be transferred or dissipated. For instance, gas turbine blades, engine walls, outer surface of a space vehicle and other factors depend on their durability on rapid heat transfer from their surfaces.

7. Discussion and Remarks

For boundary-value problems, however, there are only a few transforms known inspite of their potentially useful applications. After reviewing the aforementioned literature, it has been observed that the aforementioned researchers have not taken into consideration any thermoelastic problem expressed in rectangular and cylindrical coordinates with boundary conditions with moving type internal heat sources. In the case of transient thermoelastic problems, closed-form solutions are generally restricted to infinite or semi-infinite domains due to mathematical difficulties. Hence it is strongly desired that a precise study of the thermoelastic behaviour of real homogenous and nonhomogeneous behaviour of different bodies in circular and rectangular objects with moving heat sources is needed.

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