THERMOELASTIC BENDING ANALYSIS OF SINGLE LAYER (0^0) PLATE USING TRIGONOMETRIC SHEAR DEFORMATION THEORY

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ABSTRACT

This paper presents the thermal flexure analysis of single layer plates subjected to sinusoidal thermal load linearly varying across the thickness. Analytical solutions for thermal displacements and stresses are investigated by using a trigonometric shear deformation plate theory which includes different functions in terms of thickness coordinate to represent the effect of shear deformation. The theory presented is variationally consistent, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions. Governing equations of equilibrium and associated boundary conditions of the theory are obtained using the principle of virtual work. The Navier solution for simply supported orthotropic plates has been developed. The validity of the present theory is verified by comparing the results with various Shear Deformation Theory.

Keywords:
Single layer plate, orthotropic material, Trigonometric shear deformation theory, thermal load.

1. INTRODUCTION

Orthotropic materials are widely used, particularly in aerospace engineering. By virtue of their high strength to weight ratios and because of their mechanical properties in various directions, they can be tailored as per requirements. Further they combine a number of unique properties, including corrosion resistance, high damping, temperature resistance and low thermal coefficient of expansion. These unique properties have resulted in the expanded use of the advance orthotropic materials in structures subjected to severe thermal environment.

These structures are usually referred to as high temperature structures. Examples are provided by structures used in high speed aircraft, spacecraft etc. The high velocities of such structures give rise to aerodynamic heating, which produces intense thermal stresses that reduces the strength of aircraft structure. Coefficients of thermal expansion in the direction of fibers are usually much smaller than those in the transverse direction. This results in high stresses at the interfaces. In order to describe the correct thermal response of orthotropic plates including shear deformation effects refined theories are required.

2. SIGNIFICANCE

Composite materials are widely used, particularly in aerospace engineering. By virtue of their high strength to weight ratios and because of their mechanical properties in various directions, they can be tailored as per requirements. Further they combine a number of unique properties, including corrosion resistance, high damping, temperature resistance and low thermal coefficient of expansion. These unique properties have resulted in the expanded use of the advance composite materials in structures subjected to severe thermal environment. These structures are usually referred to as high temperature structures. Examples are provided by structures used in high speed aircraft, spacecraft etc. The high velocities of such structures give rise to aerodynamic heating, which produces intense thermal stresses that reduces the strength of aircraft structure. Coefficients of thermal expansion in the direction of fibres are usually much smaller than those in the transverse direction. This results in high stresses at the interfaces. In order to describe the correct thermal response of plates including shear deformation effects refined theories are required.

3. THEORETICAL CONTENTS

Consider a square single layer plate as shown in Figure 1. The plate is assumed in Cartesian coordinate (x, y, z) system with origin $'O'$. It is convenient to take the xy−plane of the coordinate system to be the undeformed middle plane of the laminate. The $z-$ axis is taken to be positive in a downward direction from the middle plane. The plate thickness is denoted by $'h'$ while its dimensions, along the x and y directions, are denoted by $'a'$ and $'b'$ respectively. Perfect bonding between the orthotropic layers and temperature independent mechanical and thermal properties are assumed. The plate is subjected to a thermal load $T(x, y, z)$.

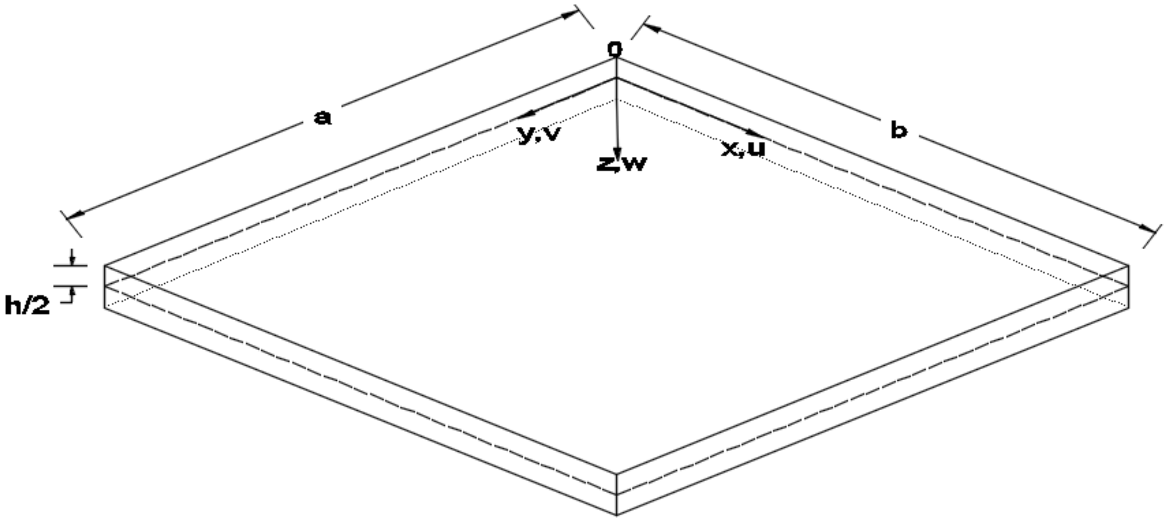

Fig.1 Plate Geometry

3.1 The Displacement Field

For the bending analysis, the displacement field of a trignometric shear deformation theory at a point in the single layer plate is expressed as:

$$U(x, y, z) = u_0(x, y) - z\frac{\partial w(x, y)}{\partial x} + f(z)\phi(x, y)$$
$$V(x, y, z) = v_0(x, y) - z\frac{\partial w(x, y)}{\partial y} + f(z)\psi(x, y)$$
$$W(x, y) = w(x, y) \tag{1}$$

Where Uand Vare the in-plane displacement components in the x and ydirections respectively, and W is the transverse displacement in the z direction. The Trigonometric function in terms of thickness coordinate in both the displacements U and V is associated with the transverse shear stress distribution through the thickness of the plate and the functions $\phi(x, y)$ and $\psi(x, y)$ are the unknown functions associated with the shear slopes. The trignometric function $f(z)$ is given as:

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$$

3.2 Strain-displacement Relation

For the small plate deformation, the six strain components (ε_x, ε_y, ε_z, γ_{xy}, γ_{xz}, γ_{yz})and three displacement components (U, V, W) are related according to the well-known linear kinematic relations.

$$\varepsilon_x = \frac{\partial U}{\partial x}, \varepsilon_y = \frac{\partial V}{\partial y}, \varepsilon_z = \frac{\partial W}{\partial z}$$
$$\gamma_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}, \gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}, \gamma_{yz} = \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \tag{2}$$

By applying the strain displacement relations of three dimensional elasticity to the displacement field given by Eq. (1), one obtains the following approximate strain field.

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x}$$

$$\varepsilon_y = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + f(z)\frac{\partial \psi}{\partial y}$$

$$\varepsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} - 2z\frac{\partial^2 w}{\partial x \partial y} + f(z)\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right)$$

$$\gamma_{xz} = \frac{df(z)}{dz}\phi$$

$$\gamma_{yz} = \frac{df(z)}{dz}\psi \qquad (3)$$

3.3 Stress-Strain Relationship

The stress-strain relationship for the single layerplate under thermal loading can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{pmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \qquad (4)$$

Where α_x and α_y are the thermal expansion coefficients in the common structural axis systems, $T= zT_1(x, y)$ is the thermal load and $Q_{ij}$are the transformed elastic coefficients.

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, Q_{12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}}, Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}$$

$$Q_{66} = G_{12}, Q_{55} = G_{12}, Q_{44} = G_{23} \qquad (5)$$

Where E_1, E_2 are the elastic moduli, μ_{12} and μ_{21} are Poisson's ratios and G_{12}, G_{23}, G_{13} are the shear moduli of the material.

3.4 Resultant forces and moments

The resultant forces and moments of a orthotropic plate can be obtained by integrating Eq. (4) through thickness, and are written as:

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$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz$$

$$\begin{Bmatrix} M_x^c \\ M_y^c \\ M_{xy}^c \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} zdz$$

$$\begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} f(z)dz$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} g(z)dz \tag{6}$$

Where (N_x, N_y, N_{xy}) are the resultant shear forces, (M_x^c, M_y^c, M_{xy}^c) are the resultant moments associated with the classical plate theory, (M_x^s, M_y^s, M_{xy}^s) are the resultant moments associated with the transverse shear effects and (Q_{xz}, Q_{yz}) are the resultant shear forces associated with the transverse shear effects. By substituting Eq. (4) into Eq. (6), following expressions of resultant stresses and moments are obtained.

$$N_x = A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w}{\partial x^2} + C_{11}\frac{\partial \phi}{\partial x} + A_{12}\frac{\partial v_0}{\partial y} - B_{12}\frac{\partial^2 w}{\partial y^2} + C_{12}\frac{\partial \psi}{\partial y} - (J_{11} + M_{12})T_1 \tag{7}$$

$$N_y = A_{12}\frac{\partial u_0}{\partial x} - B_{12}\frac{\partial^2 w}{\partial x^2} + C_{12}\frac{\partial \phi}{\partial x} + A_{22}\frac{\partial v_0}{\partial y} - B_{22}\frac{\partial^2 w}{\partial y^2} + C_{22}\frac{\partial \psi}{\partial y} - (J_{12} + M_{22})T_1 \tag{8}$$

$$N_{xy} = A_{66}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2B_{66}\frac{\partial^2 w}{\partial x \partial y} + B_{66}\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right) \tag{9}$$

$$M_x^c = B_{11}\frac{\partial u_0}{\partial x} - D_{11}\frac{\partial^2 w}{\partial x^2} + F_{11}\frac{\partial \phi}{\partial x} + B_{12}\frac{\partial v_0}{\partial y} - D_{12}\frac{\partial^2 w}{\partial y^2} + F_{12}\frac{\partial \psi}{\partial y} - (K_{11} + N_{12})T_1 \tag{10}$$

$$M_y^c = B_{12}\frac{\partial u_0}{\partial x} - D_{12}\frac{\partial^2 w}{\partial x^2} + F_{12}\frac{\partial \phi}{\partial x} + B_{22}\frac{\partial v_0}{\partial y} - D_{22}\frac{\partial^2 w}{\partial y^2} + F_{22}\frac{\partial \psi}{\partial y} - (K_{12} + N_{22})T_1 \tag{11}$$

$$M_{xy}^c = B_{66}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2D_{66}\frac{\partial^2 w}{\partial x \partial y} + F_{66}\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right) \tag{12}$$

$$M_x^s = C_{11}\frac{\partial u_0}{\partial x} - F_{11}\frac{\partial^2 w}{\partial x^2} + H_{11}\frac{\partial \phi}{\partial x} + C_{12}\frac{\partial v_0}{\partial y} - F_{12}\frac{\partial^2 w}{\partial y^2} + H_{12}\frac{\partial \psi}{\partial y} - (L_{11} + O_{12})T_1 \tag{13}$$

$$M_y^s = C_{12}\frac{\partial u_0}{\partial x} - F_{12}\frac{\partial^2 w}{\partial x^2} + H_{12}\frac{\partial \phi}{\partial x} + C_{22}\frac{\partial v_0}{\partial y} - F_{22}\frac{\partial^2 w}{\partial y^2} + H_{22}\frac{\partial \psi}{\partial y} - (L_{12} + O_{22})T_1 \quad (14)$$

$$M_{xy}^s = C_{66}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2F_{66}\frac{\partial^2 w}{\partial x \partial y} + H_{66}\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right) \quad (15)$$

$$Q_{xz} = I_{55}\phi \quad (16)$$

$$Q_{yz} = I_{44}\psi \quad (17)$$

Where $A_{ij}, B_{ij}, C_{ij}, D_{ij}, F_{ij}, H_{ij}, I_{ij}, J_{ij}, K_{ij}, L_{ij}, M_{ij}, N_{ij}, O_{ij}$ are the laminate stiffness coefficients which are defined in terms of the reduced stiffness coefficients Q_{ij}^k for the layers as given below:

$$\left\{A_{ij}, B_{ij}, C_{ij}, D_{ij}\right\} = \sum_{k=1}^{N} Q_{ij}^k \int_{h_k}^{h_{k+1}} \left\{1, z, f(z), z^2\right\} dz; (i = j = 1,2,6)$$

$$\left\{F_{ij}, H_{ij}\right\} = \sum_{k=1}^{N} Q_{ij}^k \int_{h_k}^{h_{k+1}} f(z)\{z, f(z)\} dz; (i=j=1,2,6)$$

$$\left\{I_{ij}\right\} = \sum_{k=1}^{N} Q_{ij}^k \int_{h_k}^{h_{k+1}} [f'(z)]^2 dz; (i=j=4,5) \quad (18)$$

$$\left\{J_{ij}, K_{ij}, L_{ij}\right\} = \sum_{k=1}^{N} Q_{ij}^k \alpha_x^k \int_{h_k}^{h_{k+1}} \left\{z, z^2, f(z)\right\} dz; (i=j=1,2)$$

$$\left\{M_{ij}, N_{ij}, O_{ij}\right\} = \sum_{k=1}^{N} Q_{ij}^k \alpha_y^k \int_{h_k}^{h_{k+1}} \left\{z, z^2, f(z)\right\} dz; (i = j = 1,2)$$

3.5 Governing Equation and boundary conditions

Using the expressions for strains, stresses, and principle of virtual work, variational consistent differential equations and boundary conditions for the plate under consideration are obtained. The principal of virtual work when applied to the plate leads to:

$$\int_v (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx} + \tau_{xy}\delta\gamma_{xy}) dv = 0 \quad (19)$$

Where the symbol δ denotes variational operator. In Eq. (7) mechanical load is taken as zero since the plate is subjected to pure linear thermal load. Inserting strains from Eq. (3) and stresses from Eq. (4) into Eq. (19), integrating by parts and setting coefficients of $\delta u_o, \delta v_o, \delta w_o, \delta\phi, \delta\psi$ to zero, the governing equations of the unified plate theory are obtained as follows,

$$\delta u_o : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\delta v_o : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\delta w_o : \frac{\partial^2 M_x^c}{\partial x^2} + 2\frac{\partial^2 M_{xy}^c}{\partial x \partial y} + \frac{\partial^2 M_y^c}{\partial y^2} = 0 \tag{20}$$

$$\delta \phi : \frac{\partial M_x^s}{\partial x} + \frac{\partial^2 M_{xy}^s}{\partial y} - Q_x = 0$$

$$\delta \psi : \frac{\partial M_y^s}{\partial y} + \frac{\partial^2 M_{xy}^s}{\partial x} - Q_y = 0$$

The following are the boundary conditions obtained at the edges $x = 0$ and $x = a$:

– Either $N_x = 0$ or u_0 is prescribed

– Either $N_{xy} = 0$ or v_0 is prescribed

– Either $V_x = 0$ or w is prescribed

– Either $M_x^c = 0$ or $\frac{\partial w}{\partial x}$ is prescribed

– Either $M_x^s = 0$ or ϕ is prescribed

– Either $M_{xy}^s = 0$ or ψ is prescribed

The following are the boundary conditions obtained at the edges $y = 0$ and $y = b$:

– Either $N_{xy} = 0$ or u_0 is prescribed

– Either $N_y = 0$ or v_0 is prescribed

– Either $V_y = 0$ or w is prescribed

– Either $M_y^c = 0$ or $\frac{\partial w}{\partial y}$ is prescribed

– Either $M_{xy}^s = 0$ or ϕ is prescribed

– Either $M_y^s = 0$ or ψ is prescribed

Inserting stress resultants in terms of unknown variables from Eqs. (7) to (16) into the Eq. (20), the five governing equations of unified shear deformation theory are taken from:

$$\delta u_0 : A_{11}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}-B_{11}\frac{\partial^3 w}{\partial x^3}-(2B_{66}+B_{12})\frac{\partial^3 w}{\partial x\partial y^2}$$
$$+C_{11}\frac{\partial^2 \phi}{\partial x^2}+C_{66}\frac{\partial^2 \phi}{\partial y^2}+(C_{12}+C_{66})\frac{\partial^2 \psi}{\partial x\partial y}-(J_{11}+M_{12})\frac{\partial T_1}{\partial x}=0 \tag{21}$$

$$\delta v_0 : (A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{22}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+-B_{22}\frac{\partial^3 w}{\partial x^3}-(2B_{66}+B_{12})\frac{\partial^3 w}{\partial x^2\partial y}$$
$$+(C_{12}+C_{66})\frac{\partial^2 \phi}{\partial x\partial y}+C_{22}\frac{\partial^2 \psi}{\partial y^2}+C_{66}\frac{\partial^2 \psi}{\partial x^2}-(J_{12}+M_{22})\frac{\partial T_1}{\partial y}=0 \tag{22}$$

$$\delta w_0 : B_{11}\frac{\partial^3 u_0}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 u_0}{\partial x\partial y^2}+B_{11}\frac{\partial^3 v_0}{\partial y^3}+(B_{12}+2B_{66})\frac{\partial^3 v_0}{\partial x^2\partial y}-D_{11}\frac{\partial^4 w}{\partial x^4}$$
$$-D_{22}\frac{\partial^4 w}{\partial y^4}-(D_{12}+4D_{66}+D_{12})\frac{\partial^4 w}{\partial x^2\partial y^2}+F_{11}\frac{\partial^3 \phi}{\partial x^3}+(2F_{66}+F_{12})\frac{\partial^3 \phi}{\partial x\partial y^2} \tag{23}$$
$$+F_{22}\frac{\partial^3 \psi}{\partial y^3}+(F_{12}+2F_{66})\frac{\partial^3 \psi}{\partial x^2\partial y}-(K_{11}+N_{12})\frac{\partial^2 T_1}{\partial x^2}-(K_{12}+N_{22})\frac{\partial^2 T_1}{\partial y^2}=0$$

$$\delta\phi : C_{11}\frac{\partial^2 u_0}{\partial x^2}+C_{66}\frac{\partial^2 u_0}{\partial y^2}+(C_{12}+C_{66})\frac{\partial^2 v_0}{\partial x\partial y}-F_{11}\frac{\partial^3 w}{\partial x^3}-(2F_{66}+F_{12})\frac{\partial^3 w}{\partial x\partial y^2}$$
$$+H_{11}\frac{\partial^2 \phi}{\partial x^2}+H_{66}\frac{\partial^2 \phi}{\partial y^2}+(H_{12}+H_{66})\frac{\partial^2 \psi}{\partial x\partial y}-(L_{11}+O_{12})\frac{\partial T_1}{\partial x}=0 \tag{24}$$

$$\delta\psi : (C_{12}+C_{66})\frac{\partial^2 u_0}{\partial x\partial y}+C_{22}\frac{\partial^2 v_0}{\partial y^2}+C_{66}\frac{\partial^2 v_0}{\partial x^2}+-F_{22}\frac{\partial^3 w}{\partial x^3}-(2F_{66}+F_{12})\frac{\partial^3 w}{\partial x^2\partial y}$$
$$+(H_{12}+H_{66})\frac{\partial^2 \phi}{\partial x\partial y}+H_{22}\frac{\partial^2 \psi}{\partial y^2}+H_{66}\frac{\partial^2 \psi}{\partial x^2}-(L_{12}+O_{22})\frac{\partial T_1}{\partial y}=0 \tag{25}$$

3.6 Thermoelastic analysis of plate

Bending solutions of Eq. (20) for a simply supported single layer square plate are obtained by using the Navier's approach. The plate is subjected to thermal load (*T*) only. The following simply supported boundary conditions are assumed

$$\text{At } x=0 \text{ and } x=a : N_x = v_0 = M_x^c = M_x^s = \psi = 0 \tag{26}$$

$$\text{At } y=0 \text{ and } y=b : N_y = u_0 = w = M_y^c = M_y^s = \phi = 0 \tag{27}$$

The orthotropic plate is subjected to sinusoidal thermal load linearly varying through the thickness $(T = zT_1)$ of plate as given below:

$$T_1 = T_0 \sin\alpha x \sin\beta y \tag{28}$$

Where $\alpha = \pi / a, \beta = \pi / b$ and T_0 is the maximum intensity of thermal load at center of plate. The following middle surface displacement functions are assumed which satisfies the boundary conditions and the governing equations of simply supported laminated composite plates:

$$
\begin{aligned}
u_0(x, y) &= u_1 \cos \alpha x \sin \beta y \\
v_0(x,y) &= v_1 \sin \alpha x \cos \beta y \\
\phi(x,y) &= \phi_1 \cos \alpha x \sin \beta y \\
\psi(x,y) &= \psi_1 \sin \alpha x \cos \beta y \\
w(x, y) &= w_1 \sin \alpha x \sin \beta y
\end{aligned}
\tag{29}
$$

Substitution of solution form given by Eq. (29) into governing equations (21)-(25) and setting value of mechanical load (q) zero, results into a system of the algebraic equations which can be written into a matrix form as follows:

$$
\begin{pmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\
K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\
K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\
K_{51} & K_{52} & K_{53} & K_{54} & K_{55}
\end{pmatrix}
\begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \phi_1 \\ \psi_1 \end{Bmatrix}
=
\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{Bmatrix}
\tag{30}
$$

The elements K_{ij} of stiffness matrix $[K]$ are given below:

$$
\begin{aligned}
K_{11} &= (A_{11}\alpha^2 + A_{66}\beta^2), \\
K_{22} &= -(A_{12}+A_{66})\alpha\beta, \\
K_{13} &= B_{11}\alpha^3+(B_{12}+B_{66})\alpha\beta^2, \\
K_{14} &= -(C_{11}\alpha^2+C_{66}\beta^2), \\
K_{15} &= -(C_{12}+C_{66})\alpha\beta, \\
K_{22} &= -(A_{22}\beta^2+A_{66}\alpha^2), \\
K_{23} &= B_{22}\beta^3+(B_{12}+B_{66})\alpha^2\beta, \\
K_{24} &= -(C_{12}+C_{66})\alpha\beta, \\
K_{25} &= -(C_{22}\beta^2+C_{66}\alpha^2), \\
K_{33} &= -(D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4), \\
K_{34} &= F_{11}\alpha^3+(F_{12}+2F_{66})\alpha\beta^2, \\
K_{35} &= F_{22}\alpha^3+(F_{12}+2F_{66})\alpha^2\beta, \\
K_{44} &= -(H_{11}\alpha^2+H_{66}\beta^2+I_{55}), \\
K_{45} &= -(H_{12}+H_{66})\alpha\beta, \\
K_{55} &= -(H_{22}\beta^2 + H_{66}\alpha^2 + I_{44})
\end{aligned}
\tag{31}
$$

The elements f_i of force vector $\{f\}$ are given below:

$$f_1 = (B_{x11} + B_{y12})T_0\alpha,$$
$$f_2 = (B_{x12} + B_{y22})T_0\beta,$$
$$f_3 = -(D_{x11} + D_{y11})T_0\alpha^2 - (D_{x12} + D_{y22})T_0\beta^2, \tag{32}$$
$$f_4 = (C_{x11} + C_{y12})T_0\alpha,$$
$$f_5 = (C_{x12} + B_{y22})T_0\beta$$

From the solution of Eq. (30), the unknown coefficients $u_1, v_1, w_1, \phi_1, \psi_1$ can be obtained readily. Substituting these coefficients into Eq. (29), displacements and rotations can be obtained, and subsequently, stresses can be obtained using Eqs. (1)-(4). Transverse shear stresses obtained by using constitutive relations are designated as $\tau_{xzn}^{CR}, \tau_{yzn}^{CR}$. To satisfy the continuity conditions at the layer interface, transverse shear stresses are also obtained by using following stress equilibrium equations of 3D elasticity theory and designated by $\tau_{xzn}^{EE}, \tau_{yzn}^{EE}$.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \text{ and } \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \tag{33}$$

Then further numerical results for the solution are obtained.

4. NUMERICAL RESULTS AND DISCUSSION

In this paper, displacements and stresses are obtained for square single layer plate subjected to linear thermal loading. The following properties of

$$\frac{E_1}{E_2} = 25, \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5, \frac{G_{23}}{E_2} = 0.25$$
$$\frac{\alpha_X}{\alpha_y} = \frac{1}{3}, \mu_{12} = 0.25, \mu_{21} = 0.01 \tag{34}$$

Where the subscripts 1, 2 and 3 denote properties associated with x, y and z-directions respectively. For the purpose of comparison, results are presented in the following normalized forms.

$$W_n = w\left(\frac{a}{2}, \frac{b}{2}\right)\frac{10}{\alpha_X T_0 b}$$

In the preceding section, detail solution procedure for the thermoelastic analysis of single layer plate has been discussed. The material properties used in all examples are given in Eq. (34). The results obtained are presented in normalized form given by Eq. (35). Normalized thermal displacement under sinusoidally distributed linear thermal load obtained for orthotropic plate using trigonometric shear deformation theory is presented in Table1.

Table 1. Results

b/h	Source	Un	Vn	Wn	xn	σyn	τxy	τxz	τyz
4	Present	0.4017	0.4281	1.0902	0.2767	2.2785	1.3834	0.0957	0.1169
4	A. S. Sayyad	0.4016	0.4887	1.0847	0.2660	2.2168	1.3985	0.1463	0.1507

5. CONCLUSION

Thermal response of single layer plate under non-linear thermal load across the thickness of plate has been studied by using present trigonometric shear deformation theory. The results are compared with higher order shear deformation theory.Present theory gives good prediction of the thermal response of laminated plates in respect of displacements and stresses. The effect of linear variation of thermal load through the thickness of laminated plate shows the significant effect on in-plane normal and transverse shear stresses as observed from this investigation which validates the efficacy of the present theory.

6. REFERENCES

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